PHYSICAL OPTICS

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PREFACE

TO

THE THIRD EDITION.

A FEW additions have been made to this third edition, to bring the work up to recent discoveries. Thus some account is given of the theory of the concave grating and of Hertz' experiments. In the main, however, there is not much change. The chapter dealing with Fresnel's theory of double refraction has been left unaltered. The subject is at present changing too rapidly to make it desirable to introduce here an account of more modern views, which would require analysis for their full explanation, and which very shortly may give way to more complete knowledge.

CAMBRIDGE: *September*, 1893.

PREFACE.

THE Science of Optics is generally divided into two parts, Geometrical and Physical. The former treats of the propagation, reflexion, and refraction of rays of light according to certain definite laws; the latter deduces those laws as consequences of a certain hypothesis as to the nature of light, and in addition explains numerous phenomena which geometrical optics leaves unaccounted for. The main subject of this Text-Book is Physical Optics, but throughout I have purposely avoided drawing a very decided line between the physical and geometrical portions. For this course there have been several reasons. The whole science of geometrical optics rests on the rectilinearity of rays of light. To explain the existence of these on the principles of the undulatory theory is one great difficulty of physical optics, and meets us at the very outset of the subject.

Many propositions deduced in books on geometrical optics from the laws of the reflexion and refraction of rays can equally well be established as direct consequences of the undulatory theory, and these have been treated somewhat fully in the chapter on prisms and lenses, which has thus travelled out of the region usually indicated by the title of the book.

Again, to understand at all the arrangements of apparatus requisite for most optical experiments it is necessary to have a clear idea of the action of various combinations of lenses, and this has led to a detailed account of at least two forms of telescope. Much that relates to the spectroscope and spectrum analysis is usually considered geometrical. The physical cause of the dark lines in the solar spectrum and the connexion between the various kinds of spectra, however, form part of my subject, and accordingly a considerable portion of the book has been devoted to dispersion, achromatism, and the geometrical theory of the spectroscope.

In only one or two sections of the book has any knowledge of mathematics beyond very simple trigonometry been assumed. As a necessary result of this some of the methods used will appear long and clumsy to those who are acquainted with the differential and integral calculus. But I wished to establish as rigidly as I could—considering the limits allowed me—the fundamental principles of the subject, and to escape the necessity of begging my readers to believe that, were those limits somewhat enlarged, I could prove much of what they were asked to assume in reading the book. Perhaps, however, it will be well for those who are approaching the subject for the first time to take for granted some of the propositions discussed in the earlier chapters. The rectilinear propagation of light with the laws of reflexion and refraction, and the explanation of the existence of rays, are among the most difficult points considered, and they meet us at the very threshold in Chapters II. and III. Those readers to whom these chapters present great difficulties will find it best to assume at first that we can show from the principles of the undulatory

theory why light travels in straight lines and is reflected and refracted according to certain laws. Fresnel's theory of double refraction too will, no doubt, present difficulties, and the section dealing with it has been written rather with the view of giving a physical idea of the subject to those who have already attacked it from the mathematical standpoint.

The proofs of the work have been read through and corrected by my friend Professor Forsyth, of the University College, Liverpool, and many improvements are due to his suggestions. Mr. Merrifield, the general editor of the series, has always been ready to help me with his kind advice; while the frequent references to the writings of Professor Stokes, at whose request I undertook the book, will show how much I am indebted to him.

Several sections that are new to the text-books have been taken from the optical papers of Lord Rayleigh, while for others I have to thank the MS. notes and lectures of my friend and tutor, Mr. T. Dale, Fellow of Trinity College, to whose teaching I owe it mainly that I have been able to write the book.

My thanks are also due to Mr. M. Miley of Trinity College, from whose photographs of apparatus in the Cavendish Laboratory some of the woodcuts have been taken.

TRINITY COLLEGE, CAMBRIDGE:
October 1882.

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PHYSICAL OPTICS.

CHAPTER I.

INTRODUCTION.—WAVE MOTION.

'Forms of energy' is a term used to denote most of the branches of physical science at the present time. Light, heat, and sound are forms of energy; forms, that is, in which energy is recognised by our different senses; and many of the laws of optics may be deduced from the fundamental principle of the conservation of energy. We must, therefore, endeavour at the outset to form a clear idea of what is meant by energy and the conservation of energy.

According to the usual definition, that which changes or tends to change the state of rest or motion of a body is called force. When a force is applied to a body, then it in general produces motion, and in so doing is said to do work. If the force is balanced by a system of other forces there is no motion, and hence no work. The work done by a force is measured by the product of the force multiplied by the distance resolved in the direction in which the force is acting through which it moves the point at which it is applied.

The power to do work—that is, to produce motion against resistance—is called energy; and a body may pos-

sess this power from various causes and in various forms. My arm is a source of energy. I can lift a weight from the ground and place it at a height above, and in so doing I shall have performed a definite amount of work—that is to say, an amount measured by a certain number of units of work; and when I have placed the weight at the required elevation, my body has lost exactly that amount of energy. In England the unit of work is generally taken as the amount required to raise one pound (avoirdupois) through one foot. If, then, we suppose the weight to be one pound, and that I have raised it by the aid of some machine through 32 feet, I have done thirty-two units of work in raising the weight, neglecting for the present the work done in raising my arm, and overcoming the friction of the apparatus, and so forth. But these thirty-two units of energy have not been lost. They exist in the weight and can be obtained from it by allowing it to fall to the earth again. When the weight is at rest at a height of 32 feet, its energy is said to be potential, so far at least as it depends simply on its position with reference to surrounding objects. But a body may possess energy from other causes besides its position. A ball fired from a cannon is a source of energy, and does work when it strikes the target at which it is aimed. Its energy depends on its momentum and its velocity, and is measured by half the product of these two. Energy in this form is said to be kinetic. All moving bodies possess kinetic energy. If we suppose the weight dropped in a vacuum, then at each instant until it reaches the ground it still possesses thirty-two units of energy; but its energy is no longer all potential. Some of it depends on the motion of the body; and this part is, we have seen, called kinetic. The rest is potential, and the sum of the two is always equal to the original energy of the weight, or thirty-two units. After the body has dropped one foot it has one unit of kinetic energy, thirty-one units of potential, and so on. If I had taken a weight of m lbs. and raised it to a height of h feet, the

energy of the weight would be measured by $m h$, or the product of the weight multiplied by the height through which it had been raised; and if the weight be now allowed to fall freely (and we neglect the resistance of the air), this will be the constant amount of the sum of the kinetic and potential energies. When the weight is just on the point of reaching the ground, the whole of its energy is kinetic, and is exactly equal to the potential energy of the body before it began to fall—that is, to $m h$.

But when the body has been reduced to rest by impact with the ground, it appears to have lost all its energy. It has no kinetic energy, apparently, for, so far at least as we can see, it is at rest; and it has no potential energy, for it can fall no further. Let us consider the question a little more closely. When the body struck the ground there was a considerable noise. Now sound being due to the small vibratory motions of the particles of the air, these moving particles possess energy, which they have acquired at the expense of the falling weight. Besides, heat has been produced by the impact, and heat is a form of energy, being due, in fact, to the small motions of the molecules of the heated body. Light, too, may have been emitted if the impact has been sufficiently severe. Thus the energy originally communicated to the weight by my arm has been changed, first into the visible form of kinetic energy, and then from that into the forms to which we give the names of heat, and sound, and light; and if we were to measure the energy in these three forms respectively, we should find that the sum was just equal to that originally expended in raising the body. The energy has been transformed, but remains unaltered in amount. And this is what is meant by the conservation of energy.

Professor Maxwell[1] states it thus: 'The total energy of a body or system of bodies is a quantity which can neither be increased nor diminished by any mutual action of these

[1] Heat: *Text-books of Science.*

bodies, though it may be transformed into any of the forms of which energy is susceptible.'

We are to consider light, then, as one mode in which the energy of a body becomes known to us; and luminous bodies are those which are of themselves capable of changing part of the energy they possess or receive into this form. Those bodies which possess the power of transmitting the energy of light are said to be transparent. No known body is perfectly transparent: all absorb some part of the light which enters them—that is to say, they change the light-energy into some other form, usually heat.

The energy of a system, we have seen, is either potential or kinetic. Potential energy is due to the configuration of the system and the forces between its various parts; kinetic to the motion of the parts. If, then, light is a form of energy, it must be kinetic or potential, or a combination of the two.

Now light is propagated from point to point in straight lines with a definite velocity. This was first discovered by a Danish astronomer, Olaus Roemer, in 1676, from observations on the times of eclipse of Jupiter's satellites. Motion of some kind, then, is involved, and light-energy is not purely potential. It is, therefore, either kinetic, or a combination of the two. There are two methods in which energy is usually transferred from point to point, and corresponding to these two methods two theories have been proposed to account for the phenomena of light.

A luminous body may be considered as a source of energy emitting in all directions a number of material particles which travel through space with a definite velocity, and carry with them their kinetic energy; endowed also, it may be, with potential energy from the forces which they exert on each other and on other forms of matter. We must further suppose that when these particles come in contact with the retina they give rise to the sensation of vision.

This is the basis of the emission theory, which was elaborated at length by Newton, and by means of which he was enabled to explain the phenomena of the rectilinear transmission, reflexion, and refraction of light.

It has also been applied to explain other optical problems, but each new explanation required additional hypotheses, until at last they grew so numerous that, to quote from Verdet's Introduction to the works of Augustin Fresnel, 'To overturn this laboriously built scaffolding (*pénible échafaudage*) of independent hypotheses, it is almost sufficient to look them in the face and try and understand them' (Verdet: 'Œuvres,' tome i. 344). In fact, the emission theory has failed to stand the test of a crucial experiment devised by Arago.

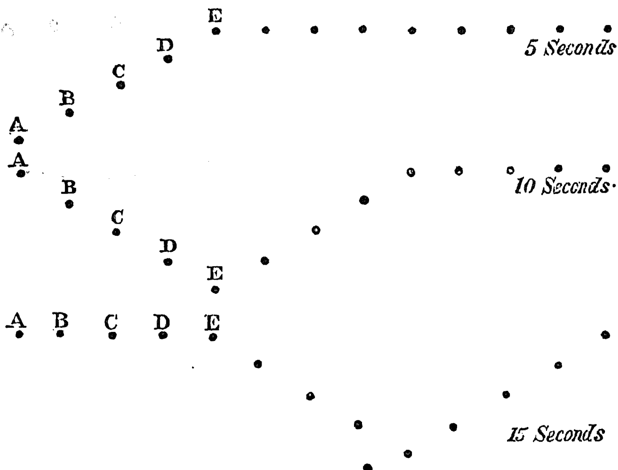
Another method, however, in which energy is transmitted, is known. Sound is a form of energy, and there are no moving particles emitted from the sounding body so as ultimately to reach the ear. The surface of a sounding body is in a state of rapid vibratory motion, each particle of which it is composed performing a series of small oscillations about its position of equilibrium. The motion is communicated to the layer of air in contact with the body, and from it to the next layer, and so on to the next until it reaches the ear; and setting the tympanum in vibration produces the sensation of sound. In this case, each particle of air moves backwards and forwards about its position of rest in the same direction as the sound travels.

This is not always the case in wave-motion; the directions of vibration and transmission are sometimes at right angles to each other. We will take an illustration. Suppose we have a file of men of the same height. Let the first man, A, bend his knees and move his head gradually downwards for 5″; then let him recover his original position during the next 5″. When he has been moving for 1″, let the next man, B, begin to do the same. After another second let C follow, and so on. And suppose that A B C, &c., return to

their original positions after moving each for 10″, and remain there.

Thus a depression is formed in the line of the men's heads which travels onwards so as to pass from one man to the next in one second. A wave is propagated along the column. Each man's head moves a short distance at right angles to the direction of wave propagation, and then returns to its original position.

FIG. 1.



Now let us suppose that instead of a number of independent beings, each moving in obedience to a common command, we have a series of material particles, A B C, lying initially at rest in a straight line. Let us move A in a direction perpendicular to this line, and suppose that there is some connection between A and B, and that when A has been in motion for 1″, it draws B after it, causing B to begin to move at right angles to the line A B C just as A had done 1″ previously. Suppose, further, that when B has been in motion for 1″ it produces the same effect on C. Our

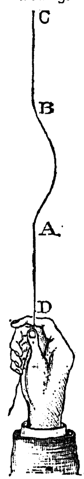
line of men is replaced by this series of particles, and fig. 1 represents the particles after 5″, 10″, 15″ respectively.

Thus in virtue of the action between the particles the wave of depression, carrying with it its energy, is transmitted along the line.

The state of things described in this illustration can be realised approximately at least by experiment. For take a long elastic tube and fill it with sand. Suspend one end from the ceiling and let it hang vertically, the other end being closed. The sand renders the motions of the tube slower, and so makes them better visible to the eye.

FIG. 2.

FIG. 3.

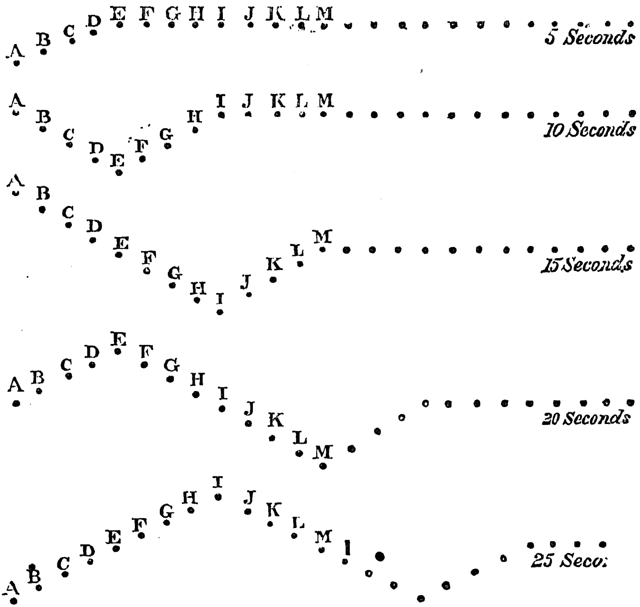


Take the lower end in the hand and stretch the tube slightly. Jerk the tube at right angles to its length, bringing the hand back to its original position. By this time a portion of the tube A B will be in motion, and the form will be as in fig. 2. If the hand remain at rest the displaced portion B A will appear to run along the tube; the form after an interval will be as in fig. 3. The space A B corresponds exactly to the depression in our column of heads or row of particles. A has moved exactly as the particle A in fig. 1, only probably at a greater rate, and its motion is followed by all the particles above it; the energy communicated to A exists partly as potential, partly as kinetic, in the space A B, and is transmitted along the tube.

So far we have supposed the head of our first man or our first particle A to remain at rest after the one excursion. Suppose now that instead of coming to rest the motion is in each case continued upwards in the same manner as previously, and let us consider the particles. The particle A reaches its original position after an interval of 10″; it

continues to move upwards for 5″, then comes to rest for an instant, and finally returns to its original position. At the end of the eleventh second B has reached its first position, but as before it is drawn on by A, and in its turn draws C on after it, repeating again the motion of A at an interval of 1″ later.

FIG. 4.



Thus the wave of depression is followed by one of elevation of exactly similar form, to be followed by a second depression, if we suppose the vibratory motion of A to continue. Fig. 4 gives the arrangements of the particles at intervals of 5″, and we see that the energy communicated to A is thus transmitted from particle to particle along the line. Each particle moves backwards and forwards through a small distance; the form of the system of particles travels

on uniformly in a direction at right angles to the motion of the individuals. We can realise this state of motion with our tube. For the present we will suppose it of very great length—the reason of this will appear shortly.

The first disturbance was caused by jerking the tube to the right. Instead of allowing the hand to remain at rest in its original position, let this jerk be followed by an exactly similar one to the left, occupying the same length of time. The first disturbance to the right will be followed by an equal one to the left, travelling up the tube at the same rate. This will be succeeded by a disturbance to the right, if we suppose the hand again moved in the same manner; and by thus moving the hand backwards and forwards in equal periods of time, a series of waves is propagated along the tube.

FIG. 5.

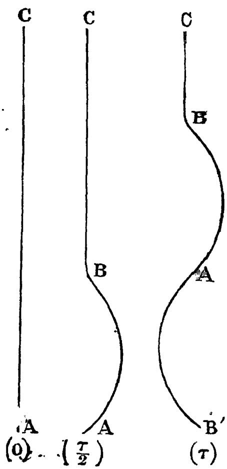
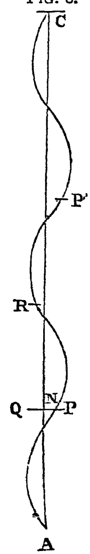


FIG. 6.



Let τ be the time of a complete oscillation; then fig. 5 gives the form of the tube at times— $0, \frac{\tau}{2}, \tau$. Each particle moves through a small distance about its position of initial rest; the form of the wave is transferred onward at a uniform rate, carrying with it the energy communicated to the end of the tube. This rate is the velocity of the wave. The wave length λ is the distance traversed by the disturbance in a complete time of vibration τ'' or BB′, fig. 5 (τ). Let us take any point P, fig. 6, on the tube at any instant, and let us move along the tube till we reach the next point P′, which is moving in exactly the same manner as P, then τ'',

previously the motion of P, was exactly the same as it is at the time considered, and this motion has been transferred to P′; therefore $PP' = \lambda$, and the wave length is the distance between any one point and the next point on the tube, which is always moving in exactly the same manner. Again, $\frac{\tau''}{2}$ previously to the given instant the motion of P was exactly opposite to that which it has at that instant, so that if we draw PN perpendicular to the undisturbed position of the line, and produce it to Q, making $NQ = PN$; and suppose that P is moving in direction NP, then $\frac{\tau''}{2}$ previously P was at Q, and was moving in direction NQ. By the time considered this motion of P has travelled a distance $\frac{\lambda}{2}$ up the tube. If then R be a point midway between P and P′ the motion of R is exactly opposite to that of P or P′. Thus in wave motion the displacements and velocities of two points at distance λ apart in the direction in which the wave is travelling are always the same in amount and direction, while the displacements and velocities of two points $\frac{\lambda}{2}$ apart are equal in amount but opposite in direction.

It follows, further, from this that two points separated by any multiple of a complete wave length are at each instant, after the disturbance has reached the more distant, moving in the same manner, while two points separated by an odd multiple of half the wave length are moving in opposite manners; that is, their displacements and velocities are equal in amount but opposite in direction. The amplitude of a vibration is the greatest distance which each particle moves from its first position. When two points are moving in exactly the same manner they are said to be in the same phase. Two points whose original distance apart is a wave-length, are always in the same phase. If the original

distance between the two is half a wave-length, the points are always in opposite phases.

It has been necessary to consider the tube as of indefinite length, for otherwise after no very long time the disturbance would reach the fixed end, and the motion would become more complicated. We could show that a certain condition must hold between the time of vibration and the length, thickness, and material of the tube, in order that such a system of waves as here described may continue to traverse it, but this is not requisite for our present purpose.

We have here considered the case of a wave travelling along a single tube or string in a direction transverse to that of the motion of the particles which compose the string.

Let us now suppose we have a large number of such tubes all exactly alike, suspended so as to hang vertically with their ends all lying in the same horizontal plane. Suppose, further, that these ends are connected to a plane piece of wood in such a way that the strings remain parallel. Make the wood oscillate backwards and forwards in time τ in its own plane, so that each point describes a small straight line. The end of each of the tubes will perform small oscillations perpendicular to the length of the tube, just as did the end of a single tube in the previous section. A series of waves such as is there described will run along each of the tubes.

The motion of any point on one of the tubes depends only on the time and its distance from the end. Hence if we take any horizontal plane the motion of the points in which this plane cuts the several tubes will be at each instant the same for all the tubes, for these points are all equidistant from the ends of their respective tubes. Such a plane constitutes a wave surface. Moreover, the tubes always remain parallel to each other—two points on the same horizontal plane on any two tubes are always the same distance apart. If then one tube were initially in contact with the next, each tube would move independently of the

other, but they would always remain in contact throughout their length; and if P, P′ be contiguous particles on two tubes at first, P, P′ remain contiguous. If we therefore suppose the whole space above our horizontal board filled with these tubes, the motion continues as before; and if, further, the contiguous tubes become united along their length, this will not alter the effect. We have thus arrived at the conception of a rectangular prism of elastic material, bounded in one direction by an end perpendicular to its length, extending indefinitely in the other. The end of this prism is made to vibrate in its own plane in period τ. Each particle in the plane moves through the same amount and in the same direction, and in moving draws after it the particle which originally was immediately above it. The motion is thus communicated to the next layer of particles, and from that to the succeeding, and so on.

At any instant all the particles lying in any plane parallel to the end of the prism are moving in the same manner. A series of plane waves is said to be travelling through the prism in the direction of its length, and the direction of the displacement of each of the particles lies in the wave surface through the particle, and is perpendicular to the direction of propagation.

Moreover, if λ be the wave length or distance traversed by the disturbance in the time of vibration, the motion in any two planes parallel to the end of the prism separated by a multiple of λ is the same at each instant, while the motion throughout two planes separated by an odd multiple of $\frac{\lambda}{2}$ is at each instant the same in amount though opposite in direction.

The energy communicated to the board exists at any future time throughout the column of elastic substance, and travels along it at a uniform rate.

We have thus been led to recognise a second method in which energy may be transmitted from point to point—that

of 'Wave Motion.' Sound energy travels thus. The sequel will show us that light also travels in waves.

We have already considered two kinds of waves. In the one, sound waves in the air, each particle oscillates in the direction in which the wave is moving. In the second the motion of the particles is perpendicular to that direction. Light waves, as we shall see, are of this latter class.

For simplicity we supposed that each particle moved backwards and forwards in a straight line. This is by no means necessary, and in fact is not usually the case. In waves in water the various particles describe curves, frequently circles or ellipses in vertical planes, parallel to the direction of propagation. In light waves the motion of any particle is confined to the plane perpendicular to the direction of propagation, which passes through the particle, but may be any small vibration in that plane. To recur to our vertical tube, instead of making my hand move backwards and forwards in a short horizontal straight line, I may make it describe a small horizontal ellipse or circle or other curve. Each succeeding particle above will describe a similar curve in due time, and the transmission of the energy will take place as before.

CHAPTER II.

THE RECTILINEAR PROPAGATION OF WAVES.

WE have said that the phenomena of light may be explained on the hypothesis that they are due to wave motion; and we are met with the question, What is the substance whose motion constitutes light?

Sound waves are transmitted by the air, but light travels through space unoccupied by air or indeed by any other medium the presence of which we can recognise directly.

We therefore make the hypothesis that there exists every-

where a medium capable of transmitting light vibratic and this we call the ether. We are not sensible of its prese by its taste, or touch, or smell, or by any direct experime We cannot see it, although we believe that without it th would be no seeing. We cannot weigh it, neither can say it has no weight. It exists, we shall suppose, between and the furthest star. We see a star because it has set ether round it in a state of vibration, communicating to energy which has travelled in waves in all directions fr the star. Some portion of this energy reaches our e and thence our brain, producing the sensation of sig Further on we may discuss the reasons we have for believi in the existence of the ether. For the present we shall tr it as an hypothesis, and attempt to explain by it the resu of various optical experiments.

This is the method we adopt in Physical Science. F example, we have no direct proof of Newton's laws motion. Our first rough experiments lead us to infer tl they are probably true. We apply the laws to the soluti of more difficult problems and compare our results w observation, as for instance in the calculation of eclips If in a large majority of cases we find agreement betwe theory and experiment, our belief in the foundations of t theory is strengthened, and may for all practical purpos become a certainty.

And so for the present we start with this hypothes We shall, moreover, assume that the motion which co stitutes light takes place entirely in the front of the wa that is, perpendicularly to the direction in which the light travelling. Later on we shall be able to show that if light due to a vibratory motion at all this must be the ca The phenomena to which we shall first apply the theo can be explained equally well without this second suppo tion ; it is true, however, and serves to keep our ideas cle We therefore make it.

This ether, then, is a substance filling space and capab

of transmitting wave motion. Its properties are modified to some degree by those of the material substance which exists with it in any portion of space. There is ether in what we call a vacuum; there is ether equally in air or glass, the ether helping to fill up the interstices between the molecules of air or glass. Some of the properties of the ether differ in the air and in the glass, among them the rate at which light energy is transmitted through it; still it is the same ether. In a perfectly opaque body, the ether, if it exists, has ceased to have the power to transmit light vibrations; at present we confine ourselves to transparent media.

A luminous body sets the ether round it in vibration; this moving layer communicates its motion to the next, and so on, and the light is thus transmitted from point to point with a definite velocity.

Our earliest observations teach us that light travels in straight lines. An opaque obstacle placed between our eye and a source of light obscures the light and casts a shadow, and if the source of light be small and we draw straight lines passing through it and the edge of the obstacle, and produce them to cut a screen behind the obstacle, we shall get very approximately at least the outline of the shadow which is thrown upon the screen.

How does the wave theory account for this, the rectilinear propagation of light? Why does not the light bend round corners as a sound does? A beam of light coming through a hole in the shutter in a darkened room casts a bright spot of light on the opposite wall and leaves the rest almost as dark as before, while a sound made outside is heard almost equally well at all points in the room. If the two—the light and the sound—are both forms of energy, transmitted by wave motion, how is it that there is this great difference? Huyghens, the real founder of the undulatory theory, failed to explain this, and it was Newton's great objection. It was this difficulty which led him to espouse so warmly the cause of the emission theory. Perhaps further

illustration will make the difficulty more apparent. For this purpose let us return to our tube. If I cause the free end to vibrate slightly, this motion is taken up by the next particle and transmitted in a straight line it is true; but suppose I have a large number of these tubes hanging side by side, if they are all independent the motion of the end of any one is transmitted, as before, along that tube. Is this necessarily the case when the tubes are all connected? Let A B, fig. 7, be two adjacent points in the horizontal plane

FIG. 8.

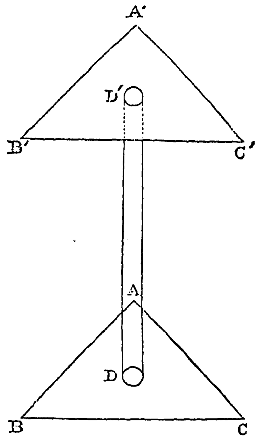
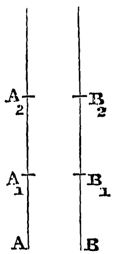


FIG. 7.

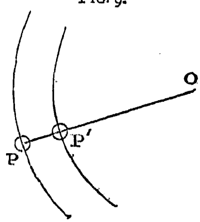


which forms the ends of the tubes, the ends of two adjacent tubes in fact, A_1 A_2, etc., B_1 B_2 points on the tubes vertically above A and B respectively. If A and B are unconnected the motion of A is transferred in order to A_1 A_2, that of B to B_1 B_2. Is this still the case if A B, A_1 B_1, etc., all form parts of the same connected body? or is the motion of A_1 affected by that of B, and the motion of B_1 by that of A, and if so, in what way?

Or again, consider a plane of particles A B C, fig. 8. Suppose every particle in that plane is vibrating in the same

way, after a definite time the motion of the particles will have been transferred to a parallel plane A′ B′ C′. Suppose that by some means I stop the motion of all the particles in a small area D of the plane A B C, what portion of A′ B′ C′ will this effect? Shall I be right in drawing a line from D perpendicular to the planes to meet A′ B′ C′ in D′, and saying that there will be no motion near D′? This would undoubtedly be the case if the motion of each point were transmitted perpendicularly to the wave surface, so that the motion round D′ would be due entirely to the motion of D, but it is not necessarily so if the motion at D′ is partly due to that of the rest of the original plane. Or suppose we have but one particle of ether originally set in motion and made to vibrate. It is surrounded on all sides by other particles, and in moving must affect them all in some way. Its motion is transferred to all surrounding it, and they begin to move. These particles all lie, it may be supposed, on a very small sphere, with the original particle as centre. From this sphere the disturbance is transmitted to an adjacent one concentric with it, and so travels outwards in spherical waves. All the particles which at any instant lie on a sphere with the original particle as centre are moving in the same manner. They are therefore on the same wave surface.

FIG. 9.



Now take at any instant a point P, fig. 9, on one of these spheres. Let O be the original particle and let O P cut an inner sphere in P′. The motion of P is due in some way to the motion which at some previous time existed throughout the surface of the inner sphere, but is it due to the motion of P′ alone, or have the other portions of the inner sphere each contributed their effect to the disturbance at P?

A single particle at O gives rise to these spherical waves travelling outwards. Any one particle on the inner sphere, as P′, resembles O in being in vibration. We ought then in

justice to consider it as a centre of disturbance emitting spherical waves, and contributing its share to the motion of P. That motion then depends not on P′ only, but on all points of the inner sphere.

In our illustration we considered a point as connected with a series of others all lying on a straight line along which alone the disturbance travelled. We ought really to treat each point as being connected with every other point in space. The particles lying on a certain wave front are in motion, and we wish to find the effect somewhere else, at a point P say. We must suppose, as it were, a series of strings drawn from this point to each particle on the wave front. A wave such as we have been considering travels down each string, and we must combine the effect of the whole to find the total disturbance at P.

Now suppose O to be a luminous object emitting approximately spherical waves, and P my eye, and suppose that I cannot get quite close to O to find out what is happening there, but that I do know all about the motion of every particle of ether lying on a certain sphere with O as centre, the radius of the sphere being less than O P. To find the effect at P, I suppose each element of this sphere to be a centre of disturbance sending waves to P. Now the effect produced is that I see O in the direction P O. The light appears o me to come along O P only, and not along a number of lines from all points in the inner sphere. This sphere is invisible; it is not luminous. If the effect at P is really due to the disturbance at all points of the inner sphere, and the effect of a vibration of the ether is to produce vision, how is it that I see, not this sphere, but only the point O? The wave theory apparently leads me to treat the disturbance at P as due to something at all points of this inner sphere—that is to say, all points of this sphere send light to P, and yet I do not see it.

But further. Suppose O P cuts the inner sphere in P′, and I place there an opaque obstacle. This quenches the

vibrations of P′, and prevents them being transmitted; but it does not affect the motion at other points of the sphere. These all send their effects unaltered to P. It would appear that the result at P could not be much different from what it previously was. The effect at P, we have seen, is the sum of the effects from all the very small elements of the sphere. I have destroyed a few of these elements it is true, but only a few. We should apparently be led to expect that I should only have altered by a small fraction the effect at P. But the most superficial observation teaches us that I have changed the whole effect. When I place the small obstacle between my eye and the source of light, I cease to see that source. Destroying P′ I destroy the whole. The effect of the rest of the sphere appears to be nothing compared with that of P′. How, then, can light be energy transmitted by wave-motion? Had O been a source of sound and P my ear; P′ an obstacle to sound—a sand-bag, for example—the result would have been different. The interposition of P′ would have produced but little effect. The ear at P is conscious of a sound coming to it in a general direction, but not from a particular point. The sound may be due to the motion of all points on the inner sphere. Can this be the case with the light?

Light casts shadows; sound, apparently, does not. Are they both transmitted by wave-motion? This difficulty, then, of the rectilinear propagation of light must be explained before we can proceed further.

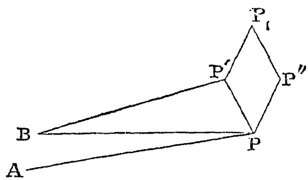
We have seen that we ought to consider the total effect at any point as the resultant of the effects produced by all the elements of a previous wave-surface.

How, then, are we to find this resultant? P is a point to which various centres of disturbance are sending waves. What is the motion of P?

Let us suppose that we have but two centres of disturbance, A and B (fig. 10), and that, under the influence of A alone, at a certain instant P would be at P′; while, if B

only had produced its effect, P'' would at that instant be the position of P; PP', PP'' will generally be in different directions. Let P_1 be the actual position of P. The displacement of P, due to either A or B, depends on the time and the distance of P from A or B respectively. Now the motion considered is very small, in the case of light not more than ·00005 centimetres; so that PP', PP'', are small compared

FIG. 10.



to AP or BP. Hence, BP and BP' are very nearly equal, so nearly that we can neglect their difference when compared with either of them. Now PP'', $P'P_1$ represent respectively the effects due to B in the first case when A does not produce any effect, and in the second when it does.

$P'P_1$ depends on BP' in just the same way that PP'' does on BP. In mathematical language $P'P_1$, PP'' are the same functions of BP' and BP respectively, but BP' is almost equal to BP. Hence PP'' and $P'P_1$ are nearly equal and parallel, and $PP'P_1P''$ is a parallelogram very approximately. Thus, to find the effect due to the two centres of disturbance, determine the displacements at any instant due to the two centres separately and complete the parallelogram, of which they are adjacent sides; then that diagonal of the parallelogram which passes through the position of rest of the particles gives the resultant effect. We combine these displacements as we do forces in statics, and velocities or accelerations in dynamics. If there be three or more centres of disturbance we proceed in the same way.

We have considered the general case first, in which the displacements due to A and B respectively are inclined to each other at any angle. Let us now limit this, and suppose that these two displacements are in the same straight line. If they be in the same direction, the resultant will be the sum of the two; if in opposite directions, the resultant will

be the difference. Let us call displacements in one direction positive, in the other negative; then the effect produced by the combination is always the algebraic sum of the individual displacements. If under the influence of A only, the particle would, at a given instant, be at a distance of one inch from its original position; while at the same instant, under the influence of B, its displacement would have been half-an-inch, then the resultant displacement is an inch and a half.

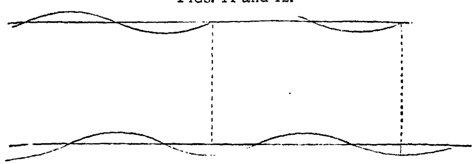
So far A and B have been any two centres of disturbance emitting waves. Let us now suppose the time of vibration and wave-length to be the same in the two. Then the velocity of propagation will also be the same. Let λ be the wave-length, τ the periodic time. Recurring to our former figure, after an interval τ, the displacement due to A is again represented by $P P'$; that due to B by $P P''$. The resultant displacement of the particle is, therefore, again $P P_1$, and its velocity is the same as at the beginning of the interval τ. The same is the case after a second interval τ. Thus the motion of the particle repeats itself at equal intervals of time, τ. The resultant motion is a small oscillation. Two waves of given period and wave-length combined produce at the point P the effect of a single wave of the same period and length. If the displacements due to A and B be not in the same straight line, the resultant motion will not in general be rectilinear; P_1 will never coincide with P, but will describe a small curve, which we may show is an ellipse, round P as centre. If, however, $P P'$, $P P''$, be in the same straight line, the resultant motion is in that line also; and, as before, the motion is the algebraical sum of the motions due to A and B respectively.

If we consider the position of the particles initially lying on the line A P, they will at any instant form a curve as in fig. 11.

After an interval $\frac{\tau}{4}$, fig. 12 will give their positions.

The curve is the same as fig. 11, but it has been p
forward a distance $\frac{\lambda}{4}$, and it has travelled this distan
time $\frac{\tau}{4}$. It has then travelled on with a velocity $\frac{\lambda}{\tau}$
if we know the form of the curve at any moment, w

FIGS. 11 and 12.

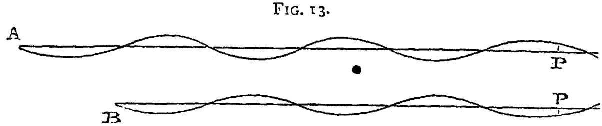


find its form after an interval of t'' say, by supposing
be pushed onwards for the t'' with this velocity.

This curve is called the displacement curve. We
suppose it constructed for each instant of time during
interval considered. The ordinates of the several cu
corresponding to the same abscissa, A P, give the displ
ment of P at the instant for which the curve is drawn.

Let us construct, also, the displacement curve for
disturbance from B, corresponding to the same instant
time, and find the ordinates of the same point, P, at th

FIG. 13.



instants. Combining these with the results for the (
turbance from A, each with each, we get the resultant (
placement. This construction is true always.

If, now, we suppose that the waves emitted from B
the same in length, period, and also amplitude, as those fr
A, the displacement curve for B is the same in form as t
for A. The two, however, differ in position with respect
the point P. This is shown in fig. 13. The upper cu

gives the motion arising from A, the lower that from B. The two curves are the same in form. The second might have been obtained from the first by moving it vertically downwards, so as to bring the horizontal line A P to position B P; and then, moving it horizontally onwards, so as to bring A into coincidence with B, through a distance (x, suppose) equal to A P—B P. Now, if we know the motion of a point at any instant, we have seen that we can find its motion at a future time by drawing the displacement curve at the given instant, and supposing it to move forward with a velocity $\frac{\lambda}{\tau}$, so that it will move over a distance x in time $\frac{\tau x}{\lambda}$. The displacement curve for B is identical in form and position with that of A, at a time $\frac{\tau x}{\lambda}$ previously. That is to say, the motion of P due to B is at each instant the same as that due to A at time $\frac{\tau x}{\lambda}$ previously.

Now suppose $x = \frac{\lambda}{2}$, then $\frac{\tau x}{\lambda} = \frac{\tau}{2}$. Hence the motion due to B is the same as that due to A half a period previously. But the motions of a point at two instants differing by half a period are exactly opposite. Since, then, this distance x is equal to $\frac{\lambda}{2}$, the motion of P due to B is always exactly opposite to that due to A, and so, on the whole, P never moves.

Thus, provided A P — B P is equal to $\frac{\lambda}{2}$, the point P remains at rest, though the waves pass over it. The same is true clearly if A P — B P *is any odd multiple of* $\frac{\lambda}{2}$, for the motions of a point are opposite, at all times differing by an odd multiple of a complete period; but the motion due to B is the same as that due to A at a time $\frac{\tau x}{\lambda}$ previously.

If then $x = \frac{2n+1}{2}\lambda$, the motion of B is the same as that due to A at a time $\frac{2n+1}{2}\tau$ previously; that is, is opposite to that due to A at the instant considered. Hence on the whole P has no motion.

If again we had supposed that x or AP − BP was a multiple of λ, the motions arising from A and B respectively would have always been identical, and the resultant motion would have been double of either. In the general case the resulting motion (at a point) due to two centres of disturbance, each sending waves to that point, is to be found by determining the motion due to each separately, and compounding the two. If the displacements of the point due to each centre be in the same straight line, the resulting motion is the sum of the two individual motions. If, further, the two centres of disturbance be identical, the total disturbance may at most be double that due to either, and in this case the difference of the distances of the point considered from the two centres is a multiple of a wave length, or, which comes to the same thing, an even multiple of half a wave length, while if the difference of these two distances is an odd multiple of half a wave length, the resultant disturbance is nothing. In any case the two waves are said to *interfere*. The result of *interference* may be to augment the effect due to each source separately, or it may diminish the effect and in some cases destroy it altogether.

We must notice carefully that if at a point P the condition $AP - BP = \frac{2n+1}{2}\lambda$ is fulfilled, that point is always at rest; if $AP - BP = n\lambda$, then P oscillates in the same time as previously, only with twice the velocity, and through twice the distance.

We may realise this interference phenomena easily with a bowl of water, or, better still, of mercury. On making an indentation in the surface, or dropping some small object

in, we get a series of waves diverging in circles from the point. The surface of the liquid is ploughed out into a number of crests and furrows.

If we start two series of waves by making two indentations, the resultant motion will be due to the combined effects of the two. Where crest meets crest, or furrow furrow, the effect is doubled, at these points the elevations are twice as high, the depressions twice as deep as when we had but one series ; while at the points in which the crest of a wave coincides with the furrow of one from the other centre we have no effect, the level of the water is unchanged.

Let us call A and B the centres of disturbance, P any point ; then if λ be the wave length and P be such that

$$AP - BP = \frac{2n+1}{2}\lambda,$$ P is always at rest.

If we give n a definite value we know that P lies on an hyperbola with A and B as foci, and by putting n equal to 0, 1, 2, 3, 4, etc., in turn we get a series of hyperbolas which are all lines of no motion so long as the disturbance lasts.

In the space between these lines the surface fluid oscillates up and down, being at times above, at times below the mean level.

The motion is greatest along another series of hyperbolas given by $AP - BP = n\lambda$ when n has any integral value including zero. We get these interference phenomena without making a second disturbance on the surface between the waves reflected from the sides of the vessel, and the original system, but it is more difficult to calculate the form of the lines of rest.

The tides on the ocean are a series of waves of periodic time, about $12\frac{1}{2}$ hours, and at some ports the tidal effects are considerably modified by interference. This is notably the case at Batsha. The tidal wave reaches the place by two passages of so unequal a length that it occupies about six hours more in travelling through the one than through the other. The time of high water for the tide coming by one

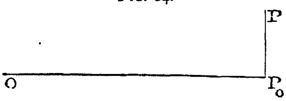
passage nearly coincides with that of low water for the other, and so the only effect is the difference of the two due to the two passages respectively. Now this difference is very small, and hence there is but little rise and fall of the water in the port.

As Professor Tyndall has pointed out, the rapids of the Niagara river some distance below the falls are an illustration of interference. At about two miles below the falls the river narrows considerably and the banks are high and steep. The water rushing down breaks against the rocks and boulders which cumber the sides. Waves are generated which would, were the mass of the water at rest, travel across the river in circles. This wave motion is compounded with the river motion, and in the centre interference-effects on a grand scale are produced between waves which have really been generated at the sides.

It is this principle of interference which forms the basis of the undulatory theory of optics, and to which we must turn for an explanation of the rectilinear propagation of light.

Before doing so, however, it may be useful to give the mathematical equations which express this vibratory motion, and to show how the laws of interference follow from them.

FIG. 14.



Let O, fig. 14, be a fixed point from which waves are being transmitted. Let P be any point initially at a distance x from O, and suppose that at time t, P has been displaced a distance y from its original position P_0 perpendicularly to the line O P_0. Then y depends on x and t or $y = f(x,t)$.

Now if v is the velocity with which the wave travels, the displacement at a point distance x' at time t' is the same as that at x at time t provided $x' - x = v(t' - t)$.

$$\therefore f(x, t) = f(x', t') = f\{x + v\,(t' - t), t'\} = f(x - vt + vt', t').$$

This is true for all values of t', and hence since $f(x,t)$ is independent of t', putting $t' = 0$, we get

$$f(x,t) = \phi(x - vt),$$

where $\phi(x - vt)$ means any function of $(x - vt)$. Thus in wave motion $y = \phi(x - vt)$.

Again y is a periodic function of x. It has the same value at distances x and $x + \lambda$ if λ be the wave-length, and equal and opposite values at distances x and $x + \frac{1}{2}\lambda$; and we know that any such periodic function of x of period λ may be expanded by Fourier's theorem, provided we choose rightly the point from which to measure x, in the form :

$$f(x) = a_1 \sin \frac{2\pi x}{\lambda} + a_3 \sin \frac{6\pi x}{\lambda} + \&c.$$

Hence

$$y = a_1 \sin \frac{2\pi}{\lambda}(vt - x) + a_3 \sin \frac{6\pi}{\lambda}(vt - x) + \&c.$$

Each of these terms gives an harmonic wave of length λ, $\frac{1}{3}\lambda$, &c. There is reason to believe that harmonic waves of different length produce different impressions on the eye; we restrict ourselves, therefore, to waves of a definite length, and for the present take as our expression

$$y = a \sin \frac{2\pi}{\lambda}(vt - x).$$

If another centre O′ at distance x' sends waves of the same length to P, we shall have for it

$$y' = a \sin \frac{2\pi}{\lambda}(vt - x'),$$

and the whole disturbance

$$\bar{y} = y + y' = a\left\{ \sin \frac{2\pi}{\lambda}(vt - x) + \sin \frac{2\pi}{\lambda}(vt - x') \right\}$$

$$= 2a \cos \frac{\pi}{\lambda}(x - x') \sin \frac{2\pi}{\lambda}\left(vt - \frac{x + x'}{2}\right)$$

This represents wave motion of which the amplitude is

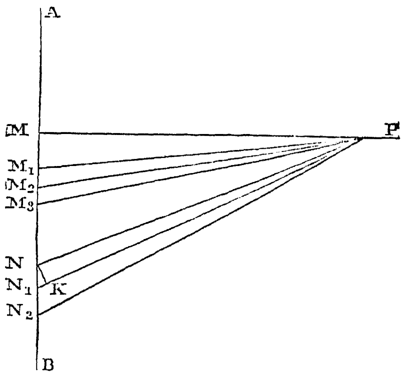
$$2a \cos \frac{\pi}{\lambda}(x - x').$$

If $x - x' = \frac{2n+1}{2}\lambda$, the amplitude is $2a \cos \frac{2n+1}{2}\pi$, or zero, so that there is no motion whenever

$$OP - O'P = \frac{2n+1}{2}\lambda.$$

We have now to show how interference accounts for the rectilinear propagation of light. Let us limit ourselves at first to the ether lying in the plane of the paper. Let A B (fig. 15) be a line of particles which are all moving in the same manner, a wave front in fact in the plane of the paper,

FIG. 15.



and suppose for the sake of clearness that the motion of each particle is perpendicular to the paper.

The motion in A B is communicated to the next layer, and so on from it to the next. Suppose, further, that the light is travelling from left to right.

We require to find the effect at any point P. For this purpose we are to divide A B into a series of small elements, find the effect of each element and add the results. We shall divide into elements in a special manner.

Let λ be the wave length.

Draw PM perpendicularly to AB and take in AB a series of points M_1, M_2, M_3, &c., such that

$$PM_1 - PM = \frac{\lambda}{2} \quad PM_2 - PM_1 = \frac{\lambda}{2} \quad PM_3 - PM_2 = \frac{\lambda}{2}$$

&c., join PM_1, PM_2, &c.

Then, since the conditions just investigated are satisfied, the light coming from M_1 is exactly opposite in phase to that coming from M, that from M_2 opposite to that from M_1, and so on; and if we consider points between M and M_1 they will send to P effects opposite in kind to those from points between M_1 and M_2.

Thus on the whole the effect of the line MM_1 is opposite to that of M_1M_2.

Now the wave length of light is exceedingly small between 7600 and 4000 tenth metres.[1] Thus the lines MM_1, M_1M_2, &c., are all exceedingly short.

The effect of a small area in illuminating a point will depend partly on the size of the area and partly on the angle which the line joining the point to the area makes with the area.

In the case we are considering, each of the pieces MM_1, M_1M_2 being very short, this angle is very nearly the same for two consecutive elements—such as M_1M_2, M_2M_3.

Thus the effects at P due to two consecutive elements are in amount proportional to the sizes of the elements, but opposite in kind. The effect of any one element is diminished by that due to the next; if the two elements are of the same size the effect of the second destroys entirely that of the first. If they differ in size, since the effects are proportional to the areas, the resultant effect at P will be of the same nature as that due to the greater area.

We are led, therefore, to compare the sizes of the elements MM_1, M_1M_2, &c. Now λ being very small we can neglect λ^2 unless it is multiplied by some large quantity.

[1] A tenth metre is $\frac{1 \text{ metre}}{10^{10}}$.

Let $$PM = b;$$

then $PM_1 = b + \frac{\lambda}{2}$; $PM_2 = b + \frac{2\lambda}{2}$ . . . $PM_n = b + \frac{n\lambda}{2}$.

And so

$$MM_1 = \sqrt{(PM_1{}^2 - PM^2)}$$

$$= \sqrt{\left\{\left(b + \frac{\lambda}{2}\right)^2 - b^2\right\}}$$

$$= \sqrt{\left\{\left(b\lambda + \frac{\lambda^2}{4}\right)\right\}} \quad = \sqrt{b\lambda} \text{ very nearly}$$

$$MM_2 = \sqrt{(PM_2{}^2 - PM^2)}$$

$$= \sqrt{\left\{\left(b + \frac{2\lambda}{2}\right)^2 - b^2\right\}} = \sqrt{2b\lambda}$$

$$MM_n = \sqrt{\left\{\left(b + \frac{n\lambda}{2}\right)^2 - b^2\right\}} = \sqrt{nb\lambda}$$

provided that $\frac{n^2\lambda^2}{4}$ is small enough to be neglected compared with λ.

Thus

$$MM_1 = \sqrt{b\lambda}$$

$$M_1M_2 = \sqrt{b\lambda} \quad (\sqrt{2} - \sqrt{1})$$

$$M_2M_3 = \sqrt{b\lambda}\,(\sqrt{3} - \sqrt{2}), \text{ \&c.}$$

Now $\sqrt{2} - 1$ is small compared with 1, $\sqrt{3} - \sqrt{2}$ is small compared with $\sqrt{2} - 1$, $\sqrt{4} - \sqrt{3}$ is small compared with $\sqrt{3} - \sqrt{2}$, and so on.

Thus at first the consecutive elements decrease rapidly; MM_1 is considerably greater than M_1M_2, M_1M_2 than M_2M_3, and so on.

But let us consider two consecutive elements NN_1, N_1N_2 situated so far from M that we may not neglect the term involving λ^2.

Let

$$PN = R - \frac{\lambda}{2}; \quad PN_1 = R; \quad PN_2 = R + \frac{\lambda}{2}.$$

Draw NK perpendicular to PN_1. The two right-angled triangles MPN_1 NKN_1 have the angle N_1 common, they are therefore similar.

$$\therefore \frac{NN_1}{N_1K} = \frac{PN_1}{MN_1} = \frac{R}{\sqrt{R^2 - b^2}}$$

The angle NPN_1 is very small, so that PNK is very nearly isosceles, and $PK = PN$ nearly.

$$\therefore N_1K = PN_1 - PK = PN_1 - PN \text{ nearly} = \frac{\lambda}{2}$$

$$\therefore NN_1 = \frac{\lambda}{2} \frac{R}{\sqrt{R^2 - b^2}}$$

If we were to find $N_1 N_2$, we should have to change R into $R + \dfrac{\lambda}{2}$.

This would alter $\dfrac{R}{\sqrt{R^2 - b^2}}$ by a quantity proportional to λ. But $NN_1 = \dfrac{\lambda}{2} \dfrac{R}{\sqrt{R^2 - b^2}}$; hence the difference between NN_1 and N_1N_2 depends on λ^2, and is therefore very small. Thus if we go some little distance from M the consecutive elements are equal in length; the effects which they send to P, therefore, are equal in amount, but opposite in kind. They therefore destroy each other entirely. The same is the case, of course, if we travel from M in the other direction along the wave. This does not happen, it is true, until we have passed over a very large number of elements, but then each element is so very small that a small space contains a very large number, and hence the consecutive elements destroy each other's effects, except those just in the neighbourhood of the point M.

Moreover, the effect due to MM_1 is very large compared with that due to an element NN_1 at some distance away.

If we call the effects due to the consecutive elements m_1, m_2, &c., in order, the whole effect is

$$m_1 - m_2 + m_3 - + \ldots \pm m_n + \text{\&c.}$$

and the series is convergent.

After a large but finite number of terms, n suppose, the consecutive terms become nearly equal. The value of this latter portion of the series is $\pm \frac{1}{2} (m_n \pm m_\infty)$, m_∞ being used to denote the last term.

This is very small compared with the part of the series before m_n. Hence the resultant effect at P is comparable with that produced by the elements of the wave in the neighbourhood of M.

If my eye were at P, that part of the wave which is near to M alone would appear to produce any effect, that is, the light would seem to me to come in the direction M P.

All this depends on the fact that λ is so small that I may neglect λ^2. But if A B were a wave of sound it would not be true to say that the effect at P was due to the part of the wave about M only, so that I should hear the sound only in the direction M P. Thus we have explained how a line of light is propagated forward through space, and why it is that light appears to travel in rays. Of course, if I block out by an obstacle the portion near M; I remove the only part of the wave which produces any visible effect at P, and therefore P ceases to be illuminated. The effect at P is due to the effects sent from all the elements of the original wave, but with the exception of the elements near M_1 the motion sent from any element is exactly opposite to that sent from a consecutive one, and the effect is the same as if we supposed it to arise from the elements near M only.

Now let us consider a plane wave, and endeavour to see how it is that light from it as well as from the linear wave appears to travel in straight lines.

To estimate the effect of the wave A B C at the point P

we must divide A B C into a number of small elements or parts, calculate the effect of each element at P, and find the algebraical sum of the whole. Consider then a small area α round any point N, the effect of α depends on its area, the distance N P, the angle between N P and the direction of vibration at N, and the angle N P makes with the wave normal at N.

It is a somewhat complicated function of these quantities, and advanced mathematics are required for its calculation. The problem has been discussed by Professor Stokes in his paper on the 'Dynamical Theory of Diffraction.' Let P M be perpendicular on the wave front and let t'' be the time occupied by the wave in travelling a distance M P. Then it follows from Professor Stokes's results that the disturbance at P at any instant is the same as that which t'' previously existed at M. The light reaching P has appeared to travel along M P. By blocking out a small portion of the wave round M in general we produce darkness at P.

We may give a general though hardly completely satisfactory explanation of the way in which this comes about as follows.

Consider the plane wave as made up of a series of parallel straight lines, each constituting a linear wave such as has just been discussed. Let P M be perpendicular to the wave and $M M_1 M_2$, &c., be a line in the wave front perpendicular to the series of parallel lines. The effect on P of any one of the linear waves is very nearly the same as that of a certain small portion of the line on either side of the point in which it is met by a perpendicular from P. The effective portion, therefore, of the plane is limited to a narrow band lying on either side of the line $M M_1 M_2$, &c.

But we might have divided our plane by a series of parallel lines in some other direction. The effective portion then would have been limited to another narrow band, also passing through M, but differing in direction from the first. We can thus reduce the effect to that of a narrow band of

the wave through M, but in any direction. It follows then that the effective portion must be that part of the wave front which is common to all these bands, that is, an element of the wave in the neighbourhood of M, and we are led to the same result as previously.

I have called this explanation unsatisfactory, for it appears at first sight to be independent of the angles between the direction in which the disturbance due to any element is being transmitted, and the wave normal and direction of vibration respectively. Now, in considering the effect of a linear wave in the previous section, we supposed the vibrations to be perpendicular to the paper; the angle between the directions of vibration and transmission is thus the same for all the elements, and affects all equally. Hence, so far as this is concerned, the results of the section still hold.

But in the case of our plane wave this angle varies from point to point, and is not in general the same along any one of the linear elements. Let N be any point of the wave. Let γ be the angle between N P and the direction of vibration θ between N P and the wave normal. We have seen that the effects of variations in γ have been neglected. The same is the case with θ. Even in our previous investigation θ is a variable, and this fact modifies slightly the conclusions arrived at. It follows from the more rigid investigation that so far as variations in the value of θ are concerned, the effect of an element as N is greatest when θ is nothing, and decreases as θ increases. (A factor $(1 + \cos \theta)$ is introduced into the expression.)

Now the results already arrived at depend in great measure on the fact that the ratio m_n to m_1 is small when n is considerable. The variation of θ tends still further to diminish this ratio, and the conclusions still hold in the main. Again, much of the argument depends on the ratio of the effects due to two consecutive elements. But in passing from one element to the next, θ and γ are approximately unchanged in value, and the ratio of the effects of

the two is therefore not much altered by taking into account the variations of θ and γ; so that we may consider the argument of the previous pages as affording a general explanation of the rectilinear propagation of waves. It is, however, of importance to remember that the whole depends on the smallness of the wave length compared with the other quantities involved.

Let us suppose, now, that a plane wave is divided into elements in the manner just described, and consider any one element, *e.g.* that nearest the pole. The phase of the disturbance received from this element changes gradually by $\frac{1}{2}\lambda$ as we pass from one edge to the other, and the phase of the resulting disturbance will be something between the phases due to its two edges.

If we represent the effect of an indefinitely small area a at the pole M (fig. 15) by $a \sin \frac{2\pi}{\lambda}(vt - b)$, then the effect due to an equal area a, near M_1, will be $a \sin \frac{2\pi}{\lambda}(vt - b - \frac{1}{2}\lambda)$; the phase changes gradually from $vt - b$ to $vt - b - \frac{1}{2}\lambda$.

The phase of the resultant effect is somewhere between these, and if A_1 represents the whole first half-period element, the disturbance due to it is $\text{A}_1 \sin \frac{2\pi}{\lambda}(vt - b - \epsilon_1)$, where ϵ_1 is some quantity which can be found by a simple application of the integral calculus. Again, for the second half-period element the phases of its indefinitely small parts lie between $vt - b - \frac{1}{2}\lambda$ and $vt - b - \lambda$, and the resultant effect is $\text{A}_2 \sin \frac{2\pi}{\lambda}(vt - b - \frac{1}{2}\lambda - \epsilon_2)$.

Now in what precedes we have assumed that these two elements are exactly opposite in phase, *i.e.* that ϵ_1 and ϵ_2 are equal. Dr. Schuster ('Phil. Mag.' February 1891) has called attention to the fact that this is not the case. The value of $2\pi\epsilon_1/\lambda$ is found to be $53^\circ\ 20'$; that of $2\pi\epsilon_2/\lambda$ is $80^\circ\ 18'$, while as we proceed along the wave the values

of $2\pi\epsilon/\lambda$ converge to 90°. It is not true, therefore, that the values of M_1, M_2, &c., are proportional to th of the corresponding elements except as a rough appr tion, and accordingly when a numerical calculation tempted the results do not accurately agree with th the complete investigation. Dr. Schuster, howeve shown that by a slight modification of the method of d into zones results much closer to the truth may be obt Instead of putting $PM_1 = PM + \frac{1}{2}\lambda$, as above, Dr. Sc takes

$$PM_1 = PM + \tfrac{3}{8}\lambda,$$

and then procedes according to the same law as for putting

$$PM_2 = PM_1 + \tfrac{1}{2}\lambda = PM + \tfrac{3}{8}\lambda + \tfrac{1}{2}\lambda,$$
$$PM_3 = PM_2 + \tfrac{1}{2}\lambda = PM + \tfrac{3}{8}\lambda + \lambda.$$

When this is done the values of $2\pi\epsilon/\lambda$ for the firs zones are found to be much more nearly equal than viously; they are 42° 40′, 38° 15′, 44° 20′, &c., conve to 45°; thus it is in this case much more nearly ex say that the effects of these elements are opposite in and proportional to the areas of the zones in amount. areas of the zones will of course differ from those give p. 30, but will follow the same general law, decre rapidly at first and then becoming nearly equal to other and very small in comparison with those of the few. Thus by adopting this method we are led to same general considerations as to the rectilinear prop tion, but are able to obtain in diffraction problems exact numerical results. Various examples of the met of doing this will be found in Dr. Schuster's paper.

The method adopted for dividing up the front into ments, which at first sight appears arbitrary, was sugge by an inquiry into the law of the variation of the posi of the elements when at a distance from the pole, in o that the effect of one element might exactly annul tha the next.

This was found to lead to the law

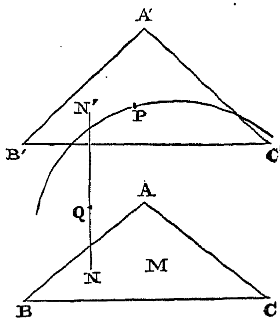
$$PM_n = PM + \tfrac{3}{8}\lambda + \frac{n-1}{2}\lambda = PM + \frac{4n-1}{8}\lambda.$$

when n is large, and then this law was assumed to hold for all values of n.

Having thus explained how to estimate the effect of a plane wave by breaking it up into elements, we proceed to apply the method to the explanation of various optical phenomena.

Let A B C (fig. 16) be a plane front, and let v be the velocity with which the wave is being propagated. After an interval of time t, the wave front, will have advanced a distance vt to the position A′ B′ C′. Each element of the surface A B C is to be considered as a centre of disturbances from which spherical waves emanate, travelling outwards with velocity v. At the end of the time t the radius of each sphere will be vt. They will all touch the new position A′ B′ C′ of the wave front. It is the envelope of the spheres.

FIG. 16.



Let P be any point on A′ B′ C′. Let M be the centre of the sphere which touches A′ B′ C′ in P. Then we have seen that the only portion of A B C which is effective in illuminating P is confined to the element near M. And conversely the only part of A′ B′ C′ which M is effective in illuminating is the element near P.

Thus the disturbance at any point of the original wave, as M, has travelled to the point in which the envelope of the system of spheres is touched by the sphere whose centre is M. Take now a point Q between A B C, A′ B′ C′, and draw N′ Q N perpendicular to A B C, meeting it in N. N is the only portion of A B C which would be effective in illuminating Q. But since Q N is less than vt, the disturbance from N has already passed over Q.

In each case we have supposed our linear wave front divided into a series of elements in such a way that the distances of any two consecutive elements from the point at which we are considering the illumination differ by half a wave length.

When a wave of light passes from one medium into another the velocity of propagation and the wave length are both altered. The time of vibration of the molecules of ether is, however, the same in the two media.

For take a molecule just on the bounding surface, it is made to vibrate by the action of contiguous molecules in the first medium, and its period must be the same as those of the particles which cause it to move. This molecule in its turn causes the neighbouring particles in the second medium to vibrate, and that in the same time as itself. Hence the time of a complete vibration is the same in the two media. Let us call this time a period. Now in half a period the disturbance travels through half a wave length.

Instead, then, of saying that the distances of two consecutive elements from P differ by half a wave length, we may say that the times taken by the disturbances to travel from the two elements respectively to P differ by half a period, and this statement of the fact will in future be the more useful.

When a wave front has been thus divided we shall speak of it as divided into half-period elements with reference to the point.

We have considered hitherto only a plane wave front, a similar method will apply to a wave of any form.

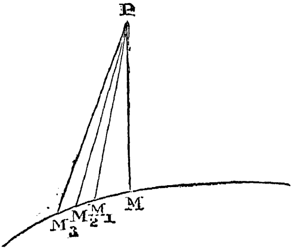
To determine the illumination at P due to a series of given waves; as previously we considered first the effect of a wave front in the form of a straight line let us now take any curve (fig. 17).

Let M be a point on the curve such that the time taken by the disturbance in travelling from M to P is a maximum or minimum. We shall call M a pole of P.

In any isotropic medium as air or water the distance M P is a maximum or minimum; it is not so in all bodies.

Take a series of points M_1, M_2, &c., beginning from M so as to divide the wave into half-period elements. The effects sent by any two consecutive elements to P are opposite. Then, as before, we may show that the first element $M M_1$ is greater than any of the others; that at first the elements decrease rapidly; that after travelling along the wave for only a short distance from M the effects of consecutive elements become very nearly equal and very small compared with that of the first, so that in this case also the effect of the whole wave practically depends on a small portion of it in the neighbourhood of the pole. By placing an opaque obstacle at the pole we in general destroy the whole effect at P.

FIG. 17.



If we have a wave front in the form of any surface we can treat it as made up of a series of curves, and show here too that the portion of the wave front which is effective in illuminating a given point is limited to a small area in the neighbourhood of the pole of the point. We can, moreover, determine the position of the wave front at any instant from that at a previous time by a construction identical with that used for plane waves.

We shall first explain the term wave surface. If one particle in the medium be made to vibrate, in general the motion travels outwards in all directions. As we go along any line from the particle there will, at any given instant, be a point beyond which the motion has not penetrated. These points corresponding to all possible directions lie on a surface; and this surface is called a wave surface.

The wave surface then is the surface passing through the

points, which are at the moment considered just beginning to move. In an isotropic medium the disturbance during a given interval travels an equal distance in all directions ; the wave surface is a sphere. In a crystal it has a different form.

To determine then the position of a wave front after an interval t'', its position at the beginning of the interval being known, divide the original wave into a series of elements. Describe a series of wave surfaces corresponding to the time and with each of these elements respectively as centre. The surface which envelopes all these wave surfaces is the new position of the wave front, and, as previously, we may show that the disturbance at any point of the original wave, as M, has travelled to the point of the new wave front in which it is touched by the surface centre M.

Wave surfaces described in this manner will frequently be referred to as secondary waves, to distinguish them from their envelope, which will be spoken of as the wave front.

In all cases of the transmission of light the effect at any point P depends on that portion of the original wave front from which the light can be propagated to P in the least time.

CHAPTER III.

REFLEXION AND REFRACTION.

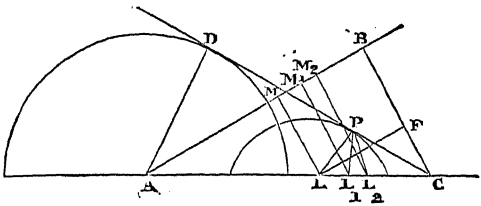
WE have seen that the hypothesis of transverse waves of very short wave-length travelling in the ether, combined with the principle of interference, serves to account for the rectilinear propagation of light. Now, when a wave of light in one medium is incident on the surface of a second and different medium, part of the light is reflected back into the first medium, part is refracted into the second, each according to definite laws. For two isotropic media the laws are as follows :—

The incident and reflected or refracted rays lie in a plane which also contains the normal at the point of incidence, and on opposite sides of the normal. The reflected and incident rays make equal angles with the normal at the point of incidence. The refracted and incident rays make with the normal at the point of incidence angles the ratio of whose sines depends only on the two media and the nature of the light.

Let us proceed to deduce these laws as consequences of the wave theory.

Each element of the bounding surface, as the incident wave reaches it, becomes in its turn a centre of disturbance, and from it secondary waves are propagated in both media.

FIG. 18.



The effect at any point P is the resultant of the disturbances sent to P from all the elements of the surface.

The secondary waves, propagated back into the first medium, produce the reflected wave; those in the second give rise to the refracted wave.

We will consider in detail the case of a plane-wave incident on a plane surface and suppose that both media are isotropic, so that the secondary wave surfaces are spheres. As previously, let the wave front taken first be a straight line, A B (fig. 18), cutting the surface in A. Let A C be the surface. Let B C be perpendicular to A B, V the velocity of light in the first medium, t the time it takes the disturbance to travel from B to C, τ the time of a complete period; then $BC = Vt$.

The disturbance of the surface in the neighbourho of C is due to a small portion of the original wave about B.

As soon as the wave cut the reflecting surface at A, became a centre of disturbance, from which a second wave travelled outwards with velocity V. At the end the interval t, when the disturbance at B has reached C, t radius of this spherical wave will be V t. Let C D be a li in the plane of the paper touching this sphere in D. Th A D = B C. Let M be any point in the original wave. Draw M perpendicular to A B to meet the surface in L. At some insta during the interval t the point L became a centre of distur ance, which has also travelled outwards in a spherical wav

The time occupied by the disturbance at M in reachi L is $\frac{ML}{V}$. Therefore the time during which the seconda wave centre L has been travelling is $t - \frac{ML}{V}$, and since velocity is V its radius is V t − M L, or B C − M L.

Draw L F perpendicular to B C, L P perpendicular to C Then the triangles F C L, P L C, are equal in all respects:—

$$\therefore PL = CF = BC - BF = BC - ML.$$

∴ P is on the secondary wave arising from L, and C touches that wave.

Thus C D touches all the secondary waves. We sh prove that it is the front of the reflected wave.

We have $\frac{PL}{V} + \frac{LM}{V} = t.$

Let M_1, M_2, &c., be a series of points in A B. Draw M_1 L M_2 L_2, &c., parallel to M L, and join L_1 P, L_2 P, &c. Th time taken by the light in travelling from M_1 to P along th lines M_1 L_1, L_1 P is clearly $\frac{PL_1}{V} + \frac{L_1M_1}{V}$.

Let us take M_1, M_2, &c., so that

$$\frac{PL_1}{V} + \frac{L_1M_1}{V} = t + \frac{\tau}{2},$$

$$\frac{\text{P L}_2}{\text{V}} + \frac{\text{L}_2\text{M}_2}{\text{V}} = t + \tau,$$

and so on.

The portion $L L_1$ of the reflecting surface is illuminated by the element $M M_1$ of the original wave, $L_1 L_2$ by $M_1 M_2$, and so on. All these elements, $L_1 L_2$, &c., have become in their turn centres of disturbance and from them secondary waves have been emitted, which help in disturbing the point P.

The vibrations are in the same phase at starting over the two elements, $M M_1$, $M_1 M_2$. The difference in their phases, then, when reaching P depends on the difference in the time taken by the light in travelling along $M L + L P$, and $M_1 L_1 + L_1 P_1$ respectively; but by our construction this difference is $\frac{\tau}{2}$, or half a period. In fact, $M M_1$, $M_1 M_2$, &c., are half-period elements with regard to P, so that the disturbances from consecutive elements arrive at P in opposite phases, and tend to destroy each other.

Now we may show as before that the first element, $L L_1$ is the greatest; that, at first, they decrease very rapidly; that, after going but a very short distance on either side of L, the consecutive elements became approximately equal, and very small compared with $L L_1$. Thus the effect at P arises from the action of a small portion of the surface near L; but the disturbance over this element was transmitted to it from M. The disturbance at M has travelled to L, been there reflected, and by the end of the time t has arrived at P. P is a point on the reflected wave at the time t; the point which the disturbance coming from M has reached.

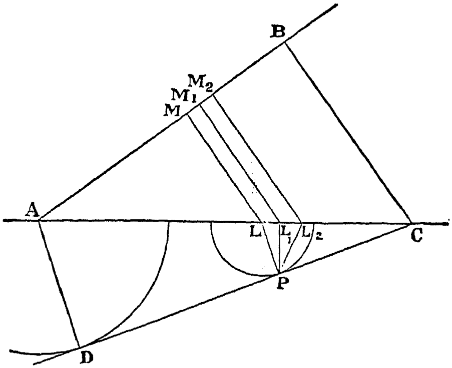
Similarly, any other point on C D is on the reflected wave.

L P is the reflected ray corresponding to an incident ray M L. But M L and L P are in the same plane with the normal to the surface and on opposite sides of it; also the angles which M L and L P make with the normal are equal

from the construction. Thus, the two laws of reflection follow in this case from our theory.

For the refracted wave (fig. 19) let V' be the velocity of light in the second medium and let the other symbols have the same meaning as before.

FIG. 19.



A is now to be taken as the centre of a wave diverging with velocity V' in the second medium. The radius of this secondary wave at the end of the time t is $V' t$, or $\frac{V'}{V}$ BC.

Let CD touch this wave. Any other point, L, has at some instant during the period become a centre of disturbance. The radius of the secondary wave to which it has given rise is—

$$\frac{V'}{V}(BC - ML),$$

and, as before, we may show that this wave touches CD in some point, P, suppose.

Also
$$\frac{PL}{V'} + \frac{ML}{V} = t.$$

Divide the original wave in $M_1 M_2$. Draw $M_1 L_1$, $M_2 L_2$, parallel to ML, and join $L_1 P$, $L_2 P$, &c.

Take M_1, M_2, &c., so that—

$$\frac{M_1 L_1}{V} + \frac{P L_1}{V'} = t + \frac{\tau}{2}$$

$$\frac{M_2 L_2}{V} + \frac{P L_2}{V'} = t + \tau.$$

Then in the same manner as for reflection, it follows that P is on the refracted wave, and that L P is the refracted ray corresponding to an incident ray M L.

Let ϕ, ϕ' be the angles between the normal at L, and M L, L P respectively. Then, since L M A and L P C are right angles, B A C $= \phi$, A C D $= \phi'$. M L and L P are in the same plane with the normal at L and on opposite sides.

Also

$$\frac{\sin \phi}{\sin \phi'} = \frac{\frac{BC}{AC}}{\frac{AD}{AC}} = \frac{BC}{AD} = \frac{V}{V'}$$

and $\frac{V}{V'}$ is a constant, depending on the media and the nature of the light.

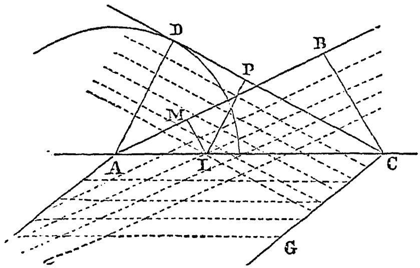
Thus, the laws of refraction have been deduced from the theory.

If instead of a linear wave we have a plane one, let us suppose the plane of the paper to be perpendicular to the wave and the reflecting or refracting surface. Let A B (fig. 20) be the trace of the wave-front, A C that of the surface, and let the other symbols have the same meaning as above. Describe the secondary wave centre A, in the first medium, corresponding to the time t. Draw through C a line C G parallel to that in which the wave-front cuts the surface. Through C G draw a plane touching the secondary wave centre A in D. Let C D be the trace of this on the plane of the paper. This tangent plane is the front of the reflected wave.

For, divide the reflecting surface into narrow strips per-

pendicular to A C. Then the disturbance is in the same phase throughout any one of these strips; and we know, therefore, that the effect of each strip at a point P in C D is the same as that of a small portion of the strip in the neighbourhood

FIG. 20.



of the point in which it is met by a line from P, perpendicular to its length. But the feet of all such perpendiculars lie on A C.

The effect, therefore, of the whole surface reduces to that of a narrow band on either side of A C, and this again is the same as that of a small element of the surface in the neighbourhood of the point L, in which a line from P, perpendicular to C D, cuts A C. It is true that, when treating of a linear wave, the line A C is of the same breadth throughout, while the band here considered increases in breadth as we go away in either direction from the point L. A strict investigation, however, shows that the statements there made as to the relative sizes of the consecutive elements still hold. Let L M be perpendicular to A B, then the disturbance at L has reached it from M. Hence, the effect at P reduces to that of a small area in the neighbourhood of M. And the laws of reflection hold for the case of a plane wave incident on a plane surface.

If we had taken P in the second medium and constructed as above for the refracted wave, a similar argument would have shown the truth of the laws of refraction.

Let us now consider the general case. Let the primary wave-front, A B (fig. 21), and the reflecting or refracting surface, C D, each have any form.

Let M be a point on the primary wave, and let M L, normal to the wave, meet the surface in L. Draw L P, making with the normal to the surface the same angle as M L.

The element about L may be treated as a portion either of the reflecting surface or its tangent plane (provided at least its principal radii of curvature are great compared with the wave length of light). In the latter case we have seen that the reflected ray is in the direction L P, and since the

FIG. 21.

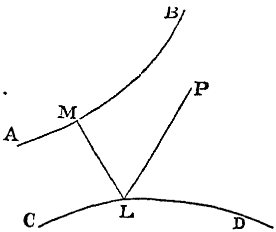
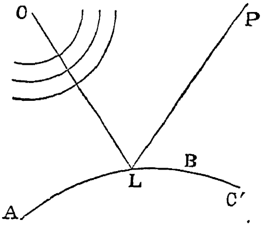


FIG. 21*a*.



effect of L is the same in the two cases, the ray incident along M L is reflected along L P. Similarly for refraction.

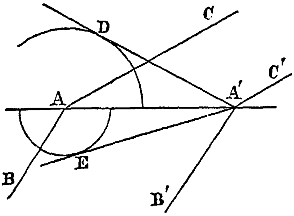
In the propagation, refraction, and reflexion of plane waves we have seen that the effect at any point P is the same as that arising from a small element of the original wave in the neighbourhood of that point from which the disturbance could reach P in the shortest possible time. Moreover, the path of the ray is the same as that which a point travelling in the two media with the velocity of the light would take so as to reach P as quickly as possible. And this is true always. Let O (fig. 21*a*) be an origin of light from which spherical waves diverge, A B C any reflecting or refracting surface, P any point. To determine the path of the light from O to P, find a point L on the surface such that the time

occupied by the light in going from O to L and L to P i
minimum, then O L P is the path of the ray. It may happ
that there is more than one path of minimum time, if
light will appear to reach O in more than one direction.

If a plane wave is incident on the plane surface
separation of two media we have then the following co
struction to determine the position of the reflected ar
refracted waves.

Let A B C (fig. 22) be the position of the incident wave
a given instant, cutting the separating surface in A B. L
A′ B′ C′ be its position after a
interval of time t. A′ B′
parallel to A B. With A a
centre describe the respectiv
wave surfaces in the tw
media, corresponding to tim
t. Through A′ B′ draw plane
touching these surfaces in I
and E. These planes D A′ B′
E A′ B′ give the positions of the reflected and refracted waves
at the end of the interval t.

FIG. 22.



The construction is perfectly general, and applies whatever be the shape of the wave surfaces. For isotropic
media they are of course both spheres.

If ϕ, ϕ' be the angles which the incident and refracted waves make with the refracting surface, V and V′ the
velocities of light, in the two media we have seen that

$$\frac{\sin \phi}{\sin \phi'} = \frac{V}{V'}$$

ϕ, ϕ' are of course also the angles which the normals to the
incident and refracted waves respectively make with the
normal to the surface.

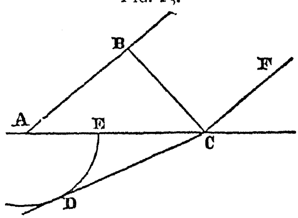
Now experiment proves that when light passes from one
medium into a denser ϕ is greater than ϕ'. Therefore,
according to our theory, the velocity of light is less the

greater the density of the medium in which the light travels. We shall see afterwards that this result is verified by direct experiment.

Let us consider again the refraction of a plane wave at a plane surface.

Let the plane of the paper be perpendicular to the wave fronts and the surface of separation. Let A B (fig. 23) be the trace of the incident wave, A C the surface of separation. Let C F be the trace of the incident wave after time t. Draw C B perpendicular to A B, then C B $= \text{v}t$. With A as centre and radius $\text{v}'t$ describe a sphere; let this cut the paper in the circle E D, E being a point on A C.

FIG. 23.



Draw C D to touch the sphere, C D is the trace of the refracted wave. Now A E : C B :: v′ : v, and if v′ is less than v, A E is less than B C. A E is therefore *à fortiori* less than A C. C is always external to the secondary wave centre A, and a tangent C D can always be drawn to this wave. The refracted wave always exists. Light will pass from one medium to a denser at any angle of incidence.

But suppose the second medium to be the less dense so that v′ is greater than v. Then A E will be greater than C B. A E may therefore be equal to or greater than A C.

If A E = A C so that C and E coincide, the refracted wave is perpendicular to A C, and the refracted light travels along the surface of separation.

If A E is greater than A C then C lies within the secondary wave centre A, and no tangent to this wave can be drawn through C. In this case there is no refracted wave.

When E coincides with C we have A C = A E $= \text{v}'t$, C B $= \text{v}t$,

$$\therefore \sin\phi = \frac{\text{B C}}{\text{C A}} = \frac{\text{v}}{\text{v}'}.$$

If the value of ϕ be greater than that given by this equation then A E is greater than A C. If light therefore, travelling in one medium, falls on the surface of one less dense, there will be no refracted wave unless the angle of incidence be not greater than $\sin^{-1}\frac{v}{v'}$, v being the velocity in the denser medium.

If the angle of incidence be equal to $\sin^{-1}\frac{v}{v'}$ the refracted wave just grazes the surface. This angle is known as the critical angle for the two media.

When a wave falls on a refracting surface at an angle greater than the critical angle, so that there is no refracted wave, the whole of the incident light is reflected and the reflecting surface appears very bright. The light is said to undergo total reflexion. Total reflexion, it must be remembered, only occurs when light travelling in a dense medium is incident on the surface of one less dense, at an angle of incidence greater than $\sin^{-1}\left(\frac{v}{v'}\right)$.

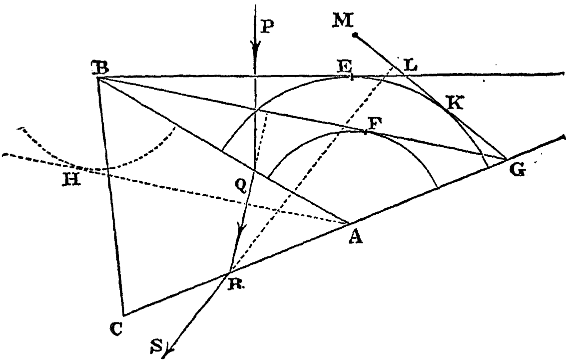
We have now to consider the refraction of light at two surfaces inclined to each other at a known angle.

A figure bounded by a series of planes all parallel to the same straight line is called a prism. A plane perpendicular to the faces of the prism is called a principal plane.

Let A B C (fig. 24) be a triangle. Through A B, B C, C A draw planes perpendicular to the paper. The figure thus formed is a prism. Suppose the space within these planes is occupied by a transparent medium, and that a plane wave of light is incident on one of the faces. Some of the light is there refracted into the prism, and falls on one or other of the other faces; and after a second refraction emerges into the first medium. The emergent wave will still be plane, but, as we shall see, inclined to the incident one at an angle, which depends on the nature of the two media and of the light.

Let us suppose for simplicity that the incident wave as well as the faces of the prism is perpendicular to the paper; the wave normals will then all be in a principal plane. Let A B be the face of incidence, and let us confine our attention to that part of the refracted wave which falls on the face A C. Let i be the angle B A C, ϕ, ϕ' the angles of incidence, and refraction at A B. ψ', ψ the angles of incidence on and emergence from A C, V, V′ the velocities of light without and within the prism respectively, and let the

FIG. 24.



prism be more dense than the surrounding medium, so that V is greater than V′. Let $\mu = \frac{V}{V'}$; μ is called the refractive index between the two media for the light considered.

Let B E be the trace of the incident wave front. With A as centre describe a circle touching B E in E. The disturbance at E would reach A after a time t such that A E $=$ V t. With A as centre and V′t as radius, describe in the first medium a circle. Draw B F G touching this circle in F and cutting C A in G. Then B F G is parallel to the refracted wave; for to determine the position of the refracted wave at the time t we should, as we have seen, have described with B as centre a sphere radius V′t and drawn A H to touch

this sphere in H. A H would be the required position. But A H is clearly parallel to B F. Thus B F is parallel to the refracted wave. Draw G K to touch the sphere radius V t in K. Then, similarly, G K is parallel to the emergent wave.

Also, let P Q be a normal to the incident wave perpendicular to B E cutting A B in Q; draw Q R, perpendicular to B G, to cut A C in R, and R S perpendicular to G K, then Q R is the refracted, R S the emergent, wave normal corresponding to P Q. Moreover, we notice that S R has been turned from the direction P Q towards the thick end of the prism, that is, away from the edge. This is the case always when light passes through a prism denser than the surrounding medium.

Let G K meet B E in L, and produce G L to M. In passing through the prism the wave front has been turned from the direction B L to the direction L G, that is, through the angle M L B. The angle which the emergent wave makes with the incident is called the deviation; let us denote it by D. Then in the figure

$$\text{BLM} = \text{D}, \quad \text{LBA} = \phi, \quad \text{FBA} = \phi', \quad \text{LGA} = \psi,$$
$$\text{FGA} = \psi', \quad \text{BAC} = i.$$

Now from the triangle B A G

$$\text{Angle BAC} = \text{ABF} + \text{AGF}$$
$$\text{or } i = \phi' + \psi' \quad . \quad . \quad (1)$$

From the triangle L B G

$$\begin{aligned} \text{BLM} &= \text{LBG} + \text{LGB} \\ &= \text{LBA} - \text{GBA} + \text{LGA} - \text{BGA} \\ \text{or D} &= \phi - \phi' + \psi - \psi' \\ &= \phi + \psi - i \quad . \quad . \quad . \quad . \quad (2) \end{aligned}$$

$$\text{Also } \frac{\sin \phi}{\sin \phi'} = \mu = \frac{\sin \psi}{\sin \psi'} \quad . \quad . \quad . \quad (3)$$

Suppose we know the angle of the prism and the angle of incidence, also the value of μ. Then equation (3) will give us ϕ', since ϕ and μ are known; then from equation (1) we get ψ'. Substituting in (3) we find ψ, and then ϕ, ψ and i being known, we have D from (2). Again, from the formula

$\frac{\sin \phi}{\sin \phi'} = \mu$ we see that if μ is increased, ϕ remaining the same, ϕ is decreased, but $\psi' = i - \phi'$ $\therefore$ ψ' is increased. Also $\sin \psi = \mu \sin \psi'$. Thus ψ is *à fortiori* increased, and since $D = \phi + \psi - i$, D is increased. Thus by increasing the refractive index we increase the deviation.

In the figure G falls outside the secondary wave surface centre A, belonging to the first medium. It is possible therefore to draw a tangent to that surface. G might, however, have been within, and in that case there would have been no wave emergent from the second surface. All the light would have been totally reflected there. If G coincide with the point in which this secondary wave-surface cuts C A produced, the emergent light will just graze A C. In this case

$$AG = Vt \text{ but } AF = V't$$

$$\sin \psi' = \frac{AF}{AG} = \frac{V'}{V} = \frac{1}{\mu}$$

$$\therefore \psi' = \sin^{-1} \frac{1}{\mu}$$

But always $\phi' = i - \psi'$.

Therefore in this case $\phi' = i - \sin^{-1} \frac{1}{\mu}$.

If ψ' be $> \sin^{-1} \frac{1}{\mu}$ the light is totally reflected.

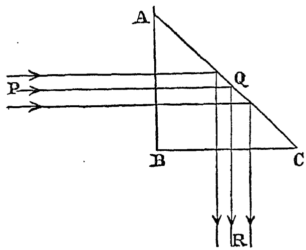
Thus there is total reflection at the second surface of a prism whenever ϕ' is less than $i - \sin^{-1} \left(\frac{1}{\mu}\right)$.

This is the case when the angle of the prism is 45° and the light is incident perpendicularly to one face. Then $\phi = 0$ $\phi' = 0$ $\psi' = 45°$. But the critical angle from glass to air is about 40° 30′. Thus all the light is totally reflected, and since the angle of incidence on the reflecting face is 45°, so also is that of reflexion. Thus the light is turned through a right angle.

If our prism B A C (fig. 25) have a third face, B C, perpendicular to A B, the light is incident on this normally, and passes out without having its direction altered.

Thus the effect of the prism is to turn the direction of the light through a right angle, and for this purpose such an arrangement is often useful.

FIG. 25.



Hitherto we have considered only the case of a plane wave incident on the prism. If our wave be a curved surface we must, as before, treat each element as an element of the tangent plane.

The effect of the two elements is identical, so that the future path of the ray from the element is obtained by a construction similar to that proved for a plane wave. It may be useful to give a construction to determine the form of the emergent wave.

Let P be any point on the incident wave, then P may be treated as an element of the plane which touches this wave at P. Draw the tangent plane B E, and construct exactly as in fig. 24 for a plane wave.

Let S T parallel to G K be the position of the emergent wave corresponding to an incident plane wave B E at an interval t after the disturbance has left P, and let S be the point in which the ray coming from P cuts the wave.

For every point, such as P on the incident wave, we can find a corresponding point S. Since originally the disturbance was in the same phase at two points P and P′ on the incident wave, and has taken the same time t in travelling from P to S as from P′ to S′, the disturbance is at the end of the time t in the same phase at S and S′. Thus S and S′ lie on the same wave front at the time t. The emergent wave front then at the time t is the surface which passes through all such points as S, and the path of the light in travelling

from P to S is that which requires least time. In general the form of this emergent wave is complicated. We shall, however, discuss later some simpler cases in which it may be a plane or a sphere.

We have seen that when a ray of light falls on a prism it is bent out of its original course, away from the edge of the prism, and that the amount of deviation depends on the angle of incidence and the value of the velocities of the light in the air and the prism respectively.

Suppose we allow a beam of sunlight to pass through a hole in the shutter of a darkened room. We should get a bright spot of light on the opposite wall. Place a prism in the path of the light with its refracting edge horizontal and parallel to the wall and its vertex downwards. Then from what precedes we should expect the beam to be bent upwards so that the position of our bright spot would be transferred from A to B.

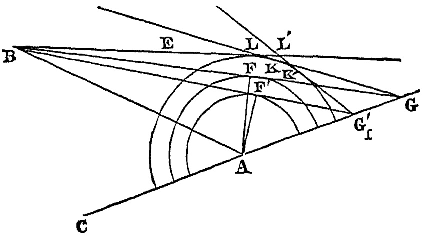
Instead, however, of seeing on the wall simply a bright white spot, we find a coloured band, the horizontal breadth of the band being equal to the diameter of the spot, while the length, which in this case is in a vertical direction, is considerably greater. The band is red at the lower end and passes through orange, yellow, green, blue, and indigo, to violet at the upper.

Let us fix our attention on two points in the band. Suppose they are P and Q in the violet and red respectively. From the sun to the prism the path of the light coming to P and Q is the same. Since P is above Q the violet light has been more bent from its original course, *i.e.*, the deviation of the violet is the greater. The angle of incidence of the two rays on the prism is the same. Thus the velocities of the red and violet light must be different in the prism, or, more strictly, the ratio of the velocity in glass to the velocity in air must be different for the two colours. Experiment shows that the velocity in air is very nearly the same for the two; we will therefore treat it as such at present. Now let

us construct a figure to determine the position of the emergent red wave. Let it be parallel to L K G (fig. 26), B E L being the incident wave.

The violet wave on emergence touches the circle E K K′, the secondary wave surface for the first medium, but is inclined to E L at a greater angle than the red. It falls, therefore, in some position such as L′ K′ G′, L′ being on B L, G′ on A G. Thus B G, B G′ are parallel to the red and violet waves in the prism. Draw A F, A F′ perpendicular to B G, B G′ respectively. Then since G′ falls between A and G, A F′ is less than A F. But A F, A F′ are respectively the radii of the

FIG. 26.



secondary wave for the red and violet, and since A F is greater than A F′ the velocity of the red wave in the prism is greater than that of the violet. Thus a ray of sunlight is composed of a number of rays of different colours, each of which travels with a different velocity in the prism.

The velocity in a given medium is greatest for the red light, and decreases as we approach the violet. In a substance such as glass violet light moves less rapidly than red. Some experiments of Arago show us that this is not the case in free space. Let us consider a star which has been eclipsed and becomes suddenly visible. If the red light travelled more quickly the red waves from the star would reach us first, and when first seen it would appear red, then as in turn the yellow, green, and blue waves came, its tint would change till at last it became white. If it were suddenly

eclipsed again, the red waves would cease to reach us first, the violet last, and the star would again change its colour, dying away as violet. Arago made a series of observations on Algol in Perseus, a variable star, but could observe no change in colour.

The velocity of light in free space is thus the same for all colours. But in this case it follows that the wave length and time of vibration must be different. For if the wave length is the same for red and violet light, so also is the time of vibration, since it is the time required for the disturbance to travel through a wave length, and we have seen that the velocity of propagation is the same. But if the wave length, velocity, and time of vibration be the same for red and violet light in free space, we have nothing left to distinguish the red from the violet. Of two given waves we cannot say why on entering a medium, as glass, one should prove to be red, the other violet. We must thus admit that the wave length, and therefore also the time of vibration, is different for light of different colours.

By a ray of definite colour we shall in future mean a ray in which the period in free space has a certain definite value.

We have already seen that the period is not altered by reflexion or refraction. The time of vibration of such a ray is therefore always the same.

But, as we have seen, the velocity of propagation is different in different media. Now the wave length is the distance the disturbance passes over in a complete period.

Hence the wave length of the ray also changes as we pass from medium to medium, the changes taking place in such a manner that the ratio of the wave length to the velocity remains the same, for this ratio measures the time of vibration.

As yet we do not know whether the red or the violet waves are the shortest; all we can say is that the shorter the wave length in a vacuum the shorter also is the period, and therefore the greater is the number of vibrations per-

formed in a given time. Further experiments will show us that the wave length in vacuo of violet light is less than that of red, and therefore the time of vibration of the violet waves is less than that of the red. In fact the extreme red waves make about 395 billions of vibrations in a second; their period is about one 395 billionth part of a second ($\frac{1}{395,000,000,000,000}$), while the period of the extreme violet is about one 763 billionth part of a second ($\frac{1}{763,000,000,000,000}$). These measurements, it is true, only tell us the wave lengths in air directly; to find those in vacuo we have to multiply the values in air by the refractive index of air. This is very nearly the same for all colours of the spectrum, and therefore that colour whose wave length in air is the greatest has also the greatest wave length in vacuo.

CHAPTER IV.

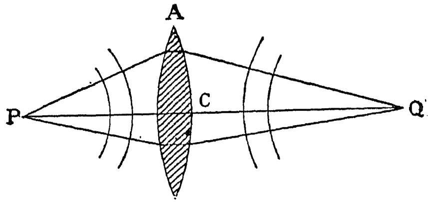
PRISMS AND LENSES.

WE will now consider a little more fully the circumstances of the reflexion and refraction of the waves of light proceeding from a point; for simplicity we shall suppose all the media we treat of to be isotropic.

Two courses are open to us. We may single out a ray of light and investigate its future history, determining its direction after each reflexion or refraction from the laws we have stated above; this is the method followed in text-books on geometrical optics. Or we may determine the form of the wave front which at any future time is passing over a given point from the condition that it passes through all points at which the disturbance is at that instant in the same phase as at the given point, so that the time taken by the light to travel from the luminous point to any other is the same for all points on the wave front which at the moment considered is passing over the given point. Since the light

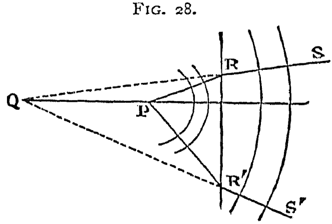
diverges from a point the wave front is initially spherical. In general, after reflexion or refraction it loses its spherical form. Suppose, however, that in the case we are considering this form is still retained, and that we find that at a given instant the wave front is a sphere centre Q (fig. 27) convex to the direction from which the light is coming. Then the disturbance from each element of the sphere will reach the centre simultaneously; every point of the sphere is a pole of Q. The effect at Q is that due to any unit of area of the sphere multiplied by the number of units of area in the wave; thus it is very great compared with that at any other point. The light coming from the luminous point P is concentrated at Q. Q is an image of the point P formed by the system of reflexions and refractions we are considering.

FIG. 27.



Since the time occupied by the light in reaching Q from any point of the sphere centre Q is the same, it follows that the time from P to Q is the same by all paths which it is possible for the light to take.

FIG. 28.



Again, the light from P all passes through Q. Q is said to be a real image of P. Clearly, if Q were a source of light, P would be its image. P and Q are therefore called conjugate foci of the system.

Suppose, however, that the spherical waves centre Q are concave to the direction from which the light has come. Then so long as the light continues to travel in the same medium it will proceed as though it came from Q, and in this case Q is said to be a virtual image.

Let us distinguish the various media by the suffix 1, 2, 3, &c. Let l_1, l_2, l_3, &c., be the lengths of path of ray in the media, v_1, v_2, v_3, &c., the velocities. Let be the source of light, and P any point which lig reaches after passing through the media. To determine tl form of the wave surface through P we have the equatic $\frac{l_1}{v_1} + \frac{l_2}{v_2} + \dots =$ a constant, for $\frac{l_1}{v_1}$ is the time occ pied by the light in traversing its path in the first mediur and so on, and the equation expresses the fact that the tin taken by the disturbance to travel from S to any point the wave front through P is constant.

Suppose we have two media, and we wish to find tl form of the bounding surface in order that light divergir from a point S may after reflexion converge to a point S Let P be any point on the surface, V be velocity of the ligh Then the time occupied by the disturbance from S in reacl ing S′ after reflexion at P is $\frac{SP}{V} + \frac{S'P}{V}$ and this is constar for all positions of P. Thus $SP + S'P =$ a constant. Bu the curve given by this equation is, we know, an ellipse wit S, S′ (fig. 29) as foci, and the surface required will be obtaine by making this curve revolve round S, S′.

If the emergent rays are to be all parallel so that S′ is a an infinite distance, the ellipse becomes a parabola with as focus and axis parallel to the direction of the emergen rays.

If the light is to appear after reflexion to diverge fron S′ so that S′ is a virtual image of S, let Q (fig. 30) be any point in the first medium. Join S′Q cutting the surface in P, join SP. Then for all points on the wave front through Q

$$\frac{SP}{V} + \frac{QP}{V} = \text{constant}.$$

But $QP = S'Q - S'P$, and $S'Q$ is constant, since the wave front is a sphere, with S′ as centre. Therefore $SP - S'P$ $=$ constant.

This curve is an hyperbola with S and S′ as foci, and the surface required is produced by rotation, the curve round the line joining the foci S and S′.

If it happens that the constant is zero, then $SP = S'P$, the hyperbola becomes a straight line, and the surface a plane bisecting SS' at right angles. Thus light diverging from a point appears after reflexion at a plane surface to

FIG. 29.

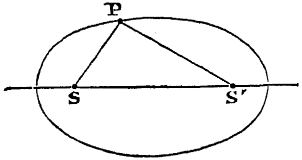
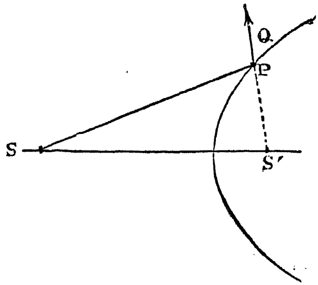


FIG. 30.



diverge from a point on the opposite side of the surface, and equidistant from it with the first point, the line joining the two being perpendicular to the surface.

Suppose that instead of being reflected at the surface of the second medium the light is refracted into it, then our equations to determine the wave surface become respectively

$$\frac{SP}{V} + \frac{S'P}{V'} = \text{constant},$$

$$\frac{SP}{V} - \frac{S'P}{V'} = \text{constant};$$

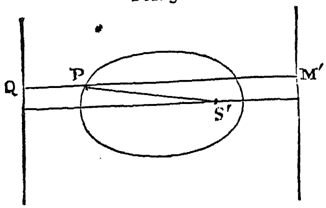
or if we put μ for $\frac{V}{V'}$

$$SP + \mu\, S'P = \text{constant},$$
$$\text{and } SP - \mu\, S'P = \text{constant}.$$

These curves are called Cartesian ovals.

If again the incident wave be plane so that S′ is at an infinite distance, we must alter the construction slightly. Suppose the plane of the paper to be perpendicular to the

FIG. 31.



wave and pass through Q and S′ (fig. 31). Let Q be any point on the wave, and let QP, perpendicular to the wave, meet the surface in P. Join S′P.

$$\text{Then } \frac{QP}{V} + \frac{S'P}{V'} = \text{constant},$$

$$\text{or } QP + \mu\, S'P = a, \text{—suppose.}$$

$$\text{Therefore } \mu\, S'P = a - QP.$$

Produce QP to M making $QM = a$, then M lies on a fixed plane parallel to the incident wave, and this cuts the paper in a fixed line.

$$\text{Also } PM = a - PQ.$$

$$\text{Therefore } S'P = \frac{1}{\mu}\, PM.$$

Thus P lies on a conic with S′ as focus and $\frac{1}{\mu}$ for eccentricity. If μ be greater than unity, *i. e.*, if the second medium be the denser, the conic will be an ellipse. The incident light is parallel to the major axis of the conic, and falls on the part of the curve remote from the focus S′.

If $\mu = 1$ so that the two media are identical, the conic becomes a parabola; the further focus is at an infinite distance from the vertex in a direction opposite to that of the incident light, so that the light in order to reach it continues

in its course without deviation. There is therefore no refraction. These surfaces are called aplanatic, and the points S, S′ are called the foci.

Let us now suppose further that the light, after passing through the second medium, again emerges into the first, and that we wish to discover the form of the surfaces of the second medium in order that the emergent wave front may be spherical and the refracted rays converge to or diverge from a focus.

The problem becomes more complicated, and its general solution would involve considerable mathematical know-

FIG. 32.

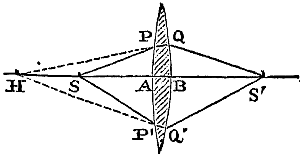
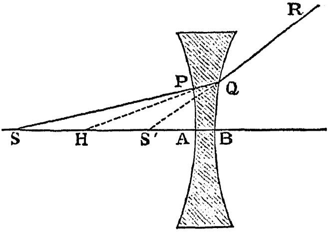


FIG. 33.



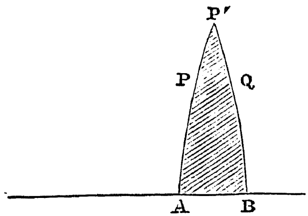
ledge. We can, however, construct several cases in which the conditions are satisfied.

For example, let H be any point in S S′, and consider as previously the section of the surface by the plane of the paper which contains the line S S′ we suppose. We shall call S S′ the axis. Construct an aplanatic curve P A P′, with S and H as foci, and then a second with Q B Q′, with H and S′ as foci. Let the curves cut the axis in A and B, and let the second curve be drawn so that B lies between A and S′. Let the space between the two curves be occupied by the second medium, and suppose two surfaces produced by rotating the figure round S S′, then light diverging in spherical waves from S will, after passing through the second medium, converge to or diverge from S′. Figures 32, 33 give the two cases.

A portion of a transparent medium bounded by two surfaces of this sort is called an aplanatic lens.

Let A P and B Q (fig. 34) be two curves such that the tangents at A and B are both perpendicular to A B; let these curves revolve round A B as an axis. This produces two surfaces called surfaces of revolution. Let the space between these two surfaces be filled with a transparent medium. Then this portion of the medium is called a lens, and A B is its axis.

FIG. 34.



All lenses are not aplanatic, that is, rays diverging from a point on the axis of the lens are not in general refracted so as to all pass through another point on the axis; moreover, a lens which is aplanatic for one pair of points may not be so for another. Again, the form of an aplanatic surface with two given foci depends on μ, and therefore a lens which is aplanatic for light of one definite colour need not be so for any other colour.

We have seen already that a pencil of rays diverging from a point will appear after reflexion at a plane surface to diverge from a point behind the surface at the same distance from it as the origin of light. These two points, the origin of light and its reflected image, are known as conjugate foci.

Let us now consider the case of reflexion at a spherical surface. We shall suppose we have a small pencil of rays emerging from a point forming a small cone with that point as vertex; the axis of the cone is called the axis of the pencil. When the axis of the pencil coincides with the normal to the surface at the point of incidence, the incidence is said to be direct, in other cases it is oblique. Consider first the case of direct incidence.

Let Q be the origin of light; Q O A the axis of the

incident pencil, O being the centre of the reflecting sphere, and A the point in which the axis meets the surface.

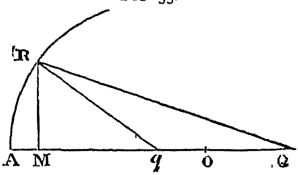
The incident waves are a system of spheres with Q as centre, let us suppose, if possible, that the reflected waves form a system of spheres with some other point on the axis as centre. Let q be that point, Q R any incident ray, R q the reflected ray. Then the light takes the same time to travel from Q to the surface and thence to q by all possible paths.

$$\text{Thus } QR + Rq = QA + Aq$$

$$\text{Let } AO = r, \quad AQ = u, \quad Aq = v,$$

Suppose the surface to be concave (fig. 35).

FIG. 35.



Draw R M perpendicular to A O and let R M $= y$.
Then we know that ultimately when R is very close to A

$$AM = \frac{y^2}{2r}$$

$$\text{Now } RQ = \left\{ RM^2 + MQ^2 \right\}^{\frac{1}{2}}$$

$$= \left\{ y^2 + \left(u - \frac{y^2}{2r}\right)^2 \right\}^{\frac{1}{2}}$$

$$= \left\{ u^2 + y^2\left(1 - \frac{u}{r}\right) \right\}^{\frac{1}{2}}$$

neglecting y^4, which will be very small compared with y^2. Expanding the right hand side by the binomial theorem, we have to the same approximation

$$RQ = u + \frac{y^2}{2u}\left(1 - \frac{u}{r}\right)$$

Similarly, $\text{R}q = v + \dfrac{y^2}{2v}\left(1 - \dfrac{v}{r}\right)$

Hence $u + v = \text{AQ} + \text{A}q = \text{RQ} + \text{R}q$

$$= u + v + \frac{y^2}{2}\left(\frac{1}{u} - \frac{1}{r} + \frac{1}{v} - \frac{1}{r}\right)$$

$$\therefore \frac{1}{v} + \frac{1}{u} = \frac{2}{r}.$$

Now let us suppose that our incident light is refracted at the spherical surface into another medium of refractive index μ, and that after refraction the waves form a series of spheres with q as centre.

Then we have seen that

$$\begin{aligned} \text{QR} - \mu q\text{R} &= \text{constant} \\ &= \text{QA} - \mu q\text{A} \end{aligned}$$

The values of QR and qR are given above. Substituting, we have

$$u - \mu v = u + \frac{y^2}{2}\left(\frac{1}{u} - \frac{1}{r}\right) - \mu\left\{v + \frac{y^2}{2}\left(\frac{1}{v} - \frac{1}{r}\right)\right\}$$

$$\therefore \frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{r}.$$

In both these formulæ we have taken A as our origin from which distances are measured. The light is supposed to come from the right. Lines drawn to the right from A are considered positive, those drawn to the left are negative. If in any case to which we apply the formulæ we find v is negative, this will simply mean that q is to the left of A.

Both these formulæ can be obtained by the first method, viz., by considering an isolated ray and determining its path; but we must remember that a ray of light can only exist in conjunction with other rays. If we attempt to isolate one ray from the rest of a pencil by passing the light through a very small hole, we shall not succeed; however, we may consider the path of one ray, remembering that it follows that path because of the others which surround it. Let SR (fig. 36) be a ray incident at R on a spherical re-

flecting surface. Let R K be the reflected ray cutting the axis in K; let S′ be the geometrical focus, then when the incidence becomes direct so that R moves close up to A, K

FIG. 36.

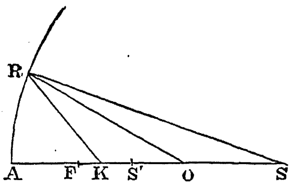
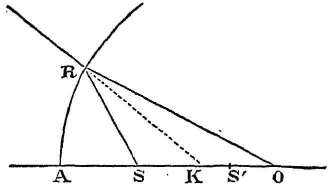


FIG. 37.



moves up to and ultimately coincides with S′. Let R O be the normal at R, O the centre; let $AS = u$, $AS' = v$, $AO = r$. Then since R O bisects the angle S R K we have (Euclid VI.) $OS : OK = SR : RK$. Therefore ultimately, when R and K coincide—

$$OS : OS' = SA : S'A$$

$$\text{or } r - u : v - r = u : v$$

$$\therefore u(v - r) = v(r - u)$$

$$\therefore r(u + v) = 2uv$$

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{r}.$$

For the case of refraction (fig. 37) with the same notation, let $SRO = \phi$, $KRO = \phi'$, $ROA = \theta$.

In the triangle OSR $\quad \dfrac{\sin\phi}{\sin\theta} = \dfrac{OS}{SR}.$

In the triangle OKR $\quad \dfrac{\sin\phi'}{\sin\theta} = \dfrac{OK}{KR}.$

$$\therefore \frac{OS}{SR} \; \frac{KR}{OK} = \frac{\sin\phi}{\sin\phi'} = \mu,$$

and ultimately when the incidence becomes direct—

$$OS \,.\, AS' = \mu \,.\, SA \,.\, OS'$$

$$\text{or } (r - u)\, v = \mu\,(r - v)\, u$$

$$\therefore r(\mu u - v) = (\mu - 1)\, uv$$

$$\therefore \frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{r}$$

Thus we have determined the geometrical focus of a pencil of rays directly reflected or refracted at a spherical surface.

We will discuss next the various cases that may arise in the application of these formulæ. Take the first one—

$$\frac{1}{v} + \frac{1}{u} = \frac{2}{r}.$$

Let r be positive, *i.e.*, let the surface be concave to the light. Let $u = \infty$ so that the incident rays are parallel.

$$\text{Then } \frac{1}{u} = 0, \text{ and therefore } v = \frac{r}{2}.$$

Thus S′ is midway between between O and A, or a pencil of parallel rays is reflected to a point lying halfway between the centre and surface, on a line through the centre parallel to the rays.

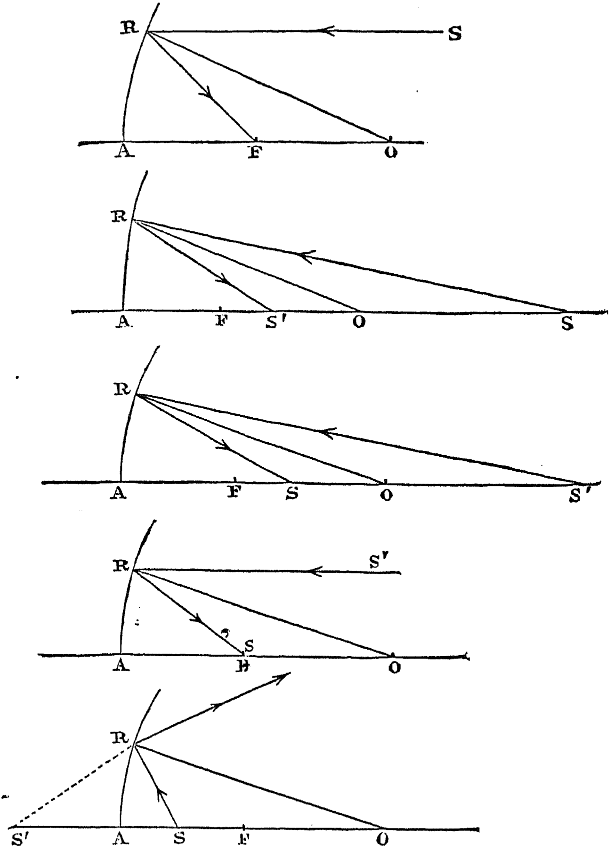
This point is called the principal focus; let us denote it by F. The distance A F is called the focal length of the mirror. Thus the focal length of a concave mirror is positive and is equal to half the radius.

Again, since $\frac{1}{v} + \frac{1}{u}$ is a constant; as u decreases remaining positive, v increases, or S and S′ move in opposite directions along the line A O. Thus if the pencil becomes divergent S moves towards A, S′ from A, and they meet at O. If S still moves towards A, S′ moves on from A, and when S arrives at F so that $u = \frac{r}{2}$, v is infinite, or the reflected pencil consists of parallel rays.

If S still moves towards A then u becomes less than $\frac{r}{2}$ and, since $\frac{1}{v} = \frac{2}{r} - \frac{1}{u}$, v becomes negative, thus S′ will be to the left of A, and the reflected pencil will appear to diverge from a virtual focus. When S gets very close to A on the positive side, S′ is very close on the other. If S is to the

left of A so that the incident pencil is convergent, S′ lies between A and F, and as S moves in the negative direction

FIG. 38.



away from A, S′ moves towards F, coinciding with it, when the incident pencil becomes parallel.

These changes are shown in fig. 38.

Now let us consider a refracted pencil. We have the formula—

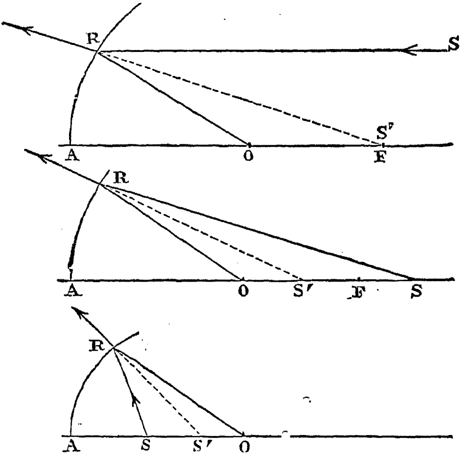
$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{r}.$$

Let the incident pencil consist of parallel rays so that u is ∞.

Then $$v = \frac{\mu r}{\mu - 1}$$

and the refracted rays appear to diverge from a point F, on

FIG. 39.



the axis at this distance from A. F is the principal focus, and A F the focal length of the surface, and since μ is greater than 1 we see that the focal length is greater than the radius. Clearly the image formed at F is virtual, for the refracted rays do not pass through it, but only appear to diverge from it.

This will be the case whenever v is positive.

Also, since $\frac{\mu}{v} - \frac{1}{u}$ is a constant, u and v increase or

decrease together, so that S and S′ move in the same direction along the axis. Thus, if the pencil becomes divergent, so that S approaches A, S′ also approaches A, though slowly, and the two coincide at O. Again, S′ always lies between O and S; hence, if S continues to move towards A, so that u becomes less than r, v becomes greater than u, but less than r; and S′ also moves towards A. When S gets very close to A, so that $\frac{1}{u}$ is infinite, so also is $\frac{1}{v}$, and S′ is close to A.

These cases are shown in fig. 39.

If the incident pencil is convergent, so that S is to the left of A, u is negative, and the formula becomes—

$$\frac{\mu}{v} = \frac{\mu - 1}{r} - \frac{1}{u}.$$

Thus v is negative, and S′ to the left of A, so long as $\frac{1}{u}$ is greater than $\frac{\mu - 1}{r}$, *i.e.* so long as u is less than $\frac{r}{\mu - 1}$. v, however, is greater than u, so that S′ is further from A than S; and when u becomes equal to $\frac{r}{\mu - 1}$, v becomes infinite, and the refracted pencil consists of parallel rays.

As S continues to move on from this point to the left of A, u being negative but numerically greater than $\frac{r}{\mu - 1}$, v becomes positive and continually decreases, S′ moves up on the positive side of A from an infinite distance towards A, and when u becomes infinite and the pencil parallel, S′ again coincides with F. These cases are shown in fig. 39*a*.

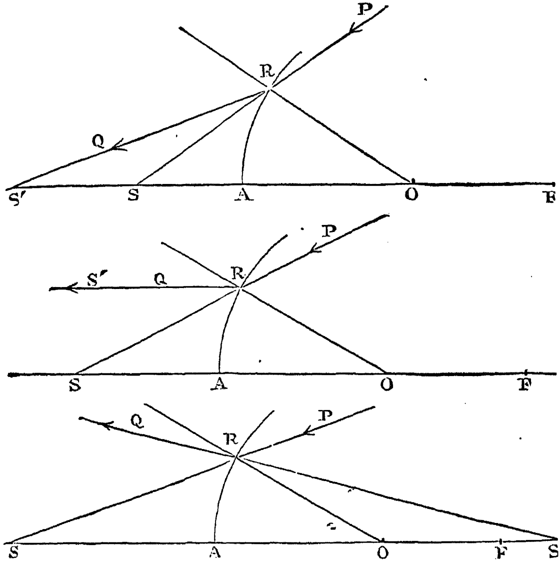
Had our surface been convex, we should have had to alter the sign of r. The focal length of a convex mirror would then be $-\frac{r}{2}$, and the image at the principal focus virtual, while for refraction the focal length would be

$-\dfrac{\mu r}{\mu - 1}$; and, the principal focus being behind the face, the light would pass through it, and the image the: real.

We can trace the changes of this conjugate foci similar manner.

If, again, our surface be plane, so that r is equ:

FIG. 39*a*.



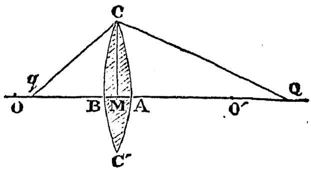
infinity, for reflection we have $v = -u$, and this is whether the incidence be direct or not; and for re tion $v = \mu u$.

These formulæ might be proved independently.

Let us now suppose that, instead of a single refrac surface, we have two constituting a lens, and let us de mine the position of the focus of a pencil after refrac through it.

Let the two surfaces be spheres, the first being convex towards the incident light, the second concave. Let O, O′ (fig. 40), be their centres, r and s their radii.

FIG. 40.



Let O O′ meet the surfaces in A and B respectively, Q being the point from which the spherical waves diverge, q that to which they converge. Let the surfaces of the two spheres meet in C, C′; then, since the time it takes the light to travel from Q to q is the same for all paths,

$$QA + \mu AB + Bq = QC + Cq.$$

Let CM, perpendicular to the axis meeting it in M, be equal to y, $QA = u$, $qB = v$.

$$\text{Then } AM = \frac{y^2}{2r}, BM = \frac{y^2}{2s}.$$

$$\text{Therefore } AB = \frac{y^2}{2}\left(\frac{1}{r} + \frac{1}{s}\right).$$

As we found in the case of simple reflection or refraction—

$$QC = u + \frac{y^2}{2}\left(\frac{1}{u} + \frac{1}{r}\right)$$

$$qC = v + \frac{y^2}{2}\left(\frac{1}{v} + \frac{1}{s}\right)$$

$$\therefore\ u + v + \frac{y^2}{2}\left(\frac{1}{u} + \frac{1}{v} + \frac{1}{r} + \frac{1}{s}\right)$$

$$= u + v + \frac{\mu y^2}{2}\left(\frac{1}{r} + \frac{1}{s}\right)$$

$$\therefore\ \frac{1}{v} + \frac{1}{u} = (\mu - 1)\left(\frac{1}{r} + \frac{1}{s}\right).$$

But we must remember that, in proving this formula, q and O being to the left of A and B, we should have assigned

a negative sign to v and r, and in this case our formula becomes

$$\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right)$$

and if we put

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right)$$

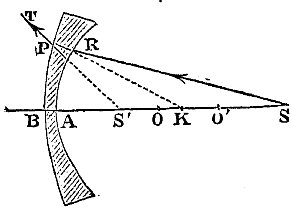
we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

The sign of f depends on that of $\frac{1}{r} - \frac{1}{s}$, and the value of f is constant for any given lens. f is called the focal length of the lens.

If in the formula we give f its proper sign and magnitude, we shall find a value for v corresponding to any value of u, that is, to any position of the luminous point on the axis.

FIG. 41.



The magnitude of v tells us the distance of q from the lens while its sign indicates on which side of the lens it lies. If, as before, we say that lines are positive which are drawn from the lens opposite to the incident light, and that light comes from the right, then, when v is positive, q is to the right of the lens; when negative, it is to the left.

As in the case of refraction at a single surface, the formula may be obtained geometrically by tracing a single ray.

Let us take a lens in which the two surfaces are spheres, with their concavities turned in the same direction towards the incident light; and let us suppose, further, that the radius of the second surface is greater than that of the first.

Let O′ O (fig. 41), the line joining these centres, meet the surfaces in A and B, and let r and s be the radii of the first

and second surfaces respectively. A and B are called the centres of the surfaces. Any ray, as S R, incident directly at R on the first surface, is refracted while passing through the lens. It appears to come from the geometrical focus of S with reference to the first surface. Let this be K. Light apparently coming from K is incident at Q on the second surface, and after refraction there appears to come from S'. K and S' are conjugate foci for the second surface. If μ is the index of refraction from air into the lens, $\frac{1}{\mu}$ is the index for the second refraction out again. Let $AB = t$; t is thus the thickness of the lens.

Let $$AS = u, \quad BS' = v.$$

For the first refraction we have, since K is conjugate to S,

$$\frac{\mu}{AK} - \frac{1}{u} = \frac{\mu - 1}{r} \quad . \quad . \quad (1)$$

for the second, since S' and K are conjugate, and the index is $\frac{1}{\mu}$,

$$\frac{\frac{1}{\mu}}{v} - \frac{1}{BK} = \frac{\frac{1}{\mu} - 1}{s}$$

$$\therefore \frac{1}{v} - \frac{\mu}{BK} = -\frac{\mu - 1}{s} \quad . \quad . \quad (2)$$

Now, frequently the thickness of a lens is very small compared with the other quantities involved, so that we may neglect it and consider A K as equal to B K, and u and v as measured indifferently from A or B. In this case formula (2) becomes

$$\frac{1}{v} - \frac{\mu}{AK} = -\frac{\mu - 1}{s} \quad . \quad . \quad (3)$$

Adding (1) and (3) we get—

$$\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right) \quad (4)$$

But

$$(\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right) = \frac{1}{f} \quad . \quad . \quad . \quad (5)$$

then we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad . \quad . \quad . \quad (6)$$

If we wish to find a more correct formula, we know that

$$\text{BK} - \text{AK} = t \quad . \quad . \quad . \quad (7)$$

but (2) gives us BK and (1) AK; substituting in (7) we get—

$$\frac{1}{\frac{1}{v} + \frac{\mu - 1}{s}} - \frac{1}{\frac{1}{u} + \frac{\mu - 1}{r}} = \frac{t}{\mu} \qquad (8)$$

We may apply formula (6) to trace the relative changes in the position of the conjugate foci. These, however, depend on the value of f, which may be positive or negative, according to the form of the lens. For example, if r and s are positive, f is positive or negative according as r is less or greater than s. If r is negative and s positive, f is negative. We may, however, show that f is positive or negative according as the lens is thinnest or thickest at the centres of its faces.

A lens in which f is positive, which is thinnest therefore at AB, is called a concave lens.

A lens in which f is negative, which is thickest therefore at AB, is called a convex lens. f is called the focal length of the lens.

The focal length of a concave lens is positive, that of a convex lens negative. Suppose the incident light to consist of rays parallel to the axis, then $u = \infty$, and from formula (6) $v = f$. This position of S′ is called the principal focus of the lens, and we see, hence, that the focal length of a thin lens is the distance between the lens and its principal focus.

We will state very briefly the results arising from a discussion of the formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

Then v and u increase or decrease together algebraically; that is, S and S′ always move in the same direction.

Take two points, F, F_1, each at a distance f from the lens on opposite sides, and let F be on the side from which the light is coming. Let the lens be concave, then f is positive—

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

If $u = \infty$, $v = f$, the emergent light appears to diverge from F. As S moves towards A, so does S′, and they coincide at A. If S lies between ∞ and A, the image at S′ is virtual. If S moves to the left of A, so that the incident light is convergent, u becomes negative, and v is negative and greater than u. So long as u is numerically less than f, the image at S′ is real. When $u = -f$, so that the incident light converges to F_1, v is infinite and the emergent pencil parallel. As S moves still further to the left, v becomes positive and decreases; S′ moves from infinity towards A in the direction of the incident light, and when u has become infinite again, S′ coincides with F. The image again is virtual.

If the lens be convex, its focal length is negative; let f be its numerical value, then our formula becomes—

$$\frac{1}{v} - \frac{1}{u} = -\frac{1}{f}.$$

Let $u = \infty$, $v = -f$; the emergent light converges to F_1. As u decreases v is negative until S coincides with F. Then $u = f$, $v = \infty$, and the emergent pencil is parallel.

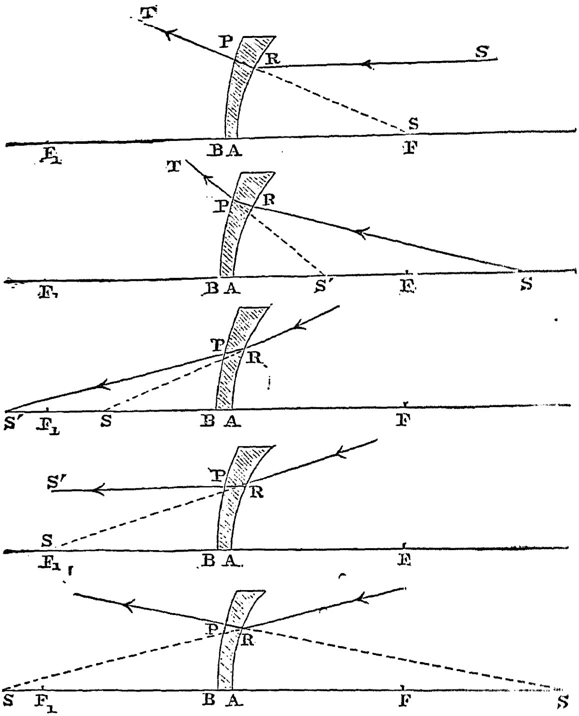
As u changes from ∞ to f, the image formed by the lens is always real.

If u be less than f, v is positive, and as S moves towards A, S′ follows it, and the two coincide at A. When S is be-

tween F and A, the image formed is virtual, and the emergent pencil divergent.

When u becomes negative so also does v, and as S moves from A to an infinite distance, S′ moves from A to F_1.

FIG. 42.



Both pencils in this case are convergent, and the image formed is real. Figs. 42 and 43 show the path of the rays in the different cases.

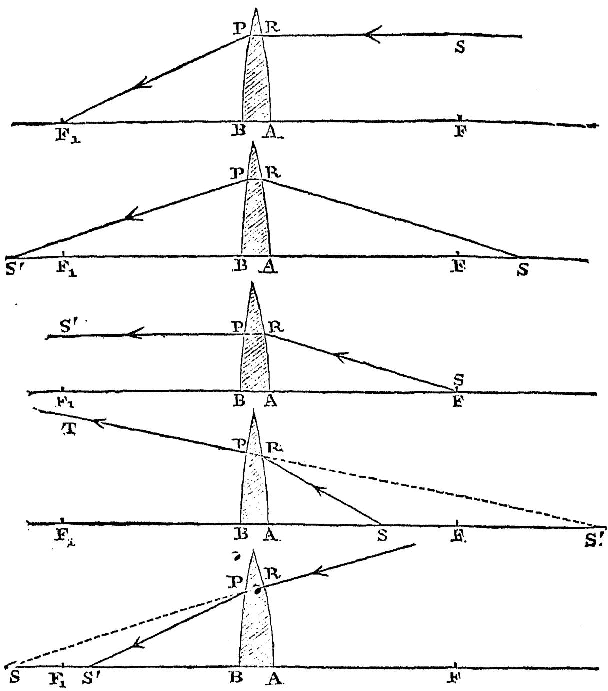
Frequently, without use of the formula, a simple inspection of the figure will enable us to judge of the effect a lens will have on a pencil of rays.

Let S P Q S′ (fig. 44) be any ray passing through the lens.

Draw tangents to the lens at P and Q, meeting at T.

The refraction at P depends simply on the nature of the media and the angle which S P makes with the normal to the

FIG. 43.



refracting surface. This angle is the same for the lens and its tangent plane. If, then, we suppose P T, instead of the surface of the lens, to bound the refracting medium, the refraction at P will be the same. P Q will still be its path.

Similarly, we may suppose Q T to be the other surface. Thus, the lens produces on the ray the same effect as the

prism PTQ. Now, a prism always turns an incident ray towards its thick end. If our lens be concave, the thick end of the prism is remote from the axis; the emergent light is turned away from the axis. If the lens be convex, the thick end is towards the axis, and the light is bent in that direction.

We have called A and B the centres of the faces of the

FIG. 44.

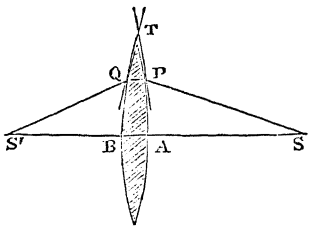
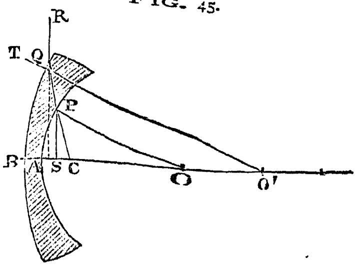


FIG. 45.



lens. We must now explain what is meant by the centre of the lens.

Let P (fig. 45) be any point on the first surface, join OP and draw O′Q to the second surface parallel to it; join QP, and produce it to meet the axis in C, then C is a fixed point and is called the centre of the lens.

For by construction the triangles COP, CO′Q are similar.

$$\therefore \frac{CO}{CO} = \frac{OP}{O'Q} = \frac{r}{s}$$

Also

$$CO = r - AC$$

$$CO' = s - t - AC.$$

$$\therefore \frac{r - AC}{s - t - AC} = \frac{r}{s}$$

$$\therefore AC = \frac{rt}{s - r}.$$

Thus C is a fixed point, and if we are neglecting the thickness of the lens C coincides with A or B.

Now let $SPQR$ be a ray incident at P in such a manner that its direction in the lens passes through C, produce $O'Q$ to T, and then by the definition of the centre of a lens OP is parallel to $O'Q$.

$$\therefore \text{ angle } CPO = CQO'.$$

$$\text{Also } \sin SPO = \mu \sin CPO$$

$$\sin RQT = \mu \sin CQO'$$

$$\therefore SPO = RQT$$

$$\therefore SP \text{ is parallel to } QR.$$

Hence if the path of a ray within the lens passes through the centre of the lens, the ray is not deviated by the lens. The emergent ray is parallel to the incident ray.

If we are neglecting the thickness of a lens its centre coincides with the centre of either of its faces, and hence we see that a ray incident on the centre of the face of a thin lens passes straight through without deviation. When the path of the ray in the lens passes through its centre the incidence is said to be centrical, otherwise it is called excentrical.

The formulæ proved above to determine the relative position of the conjugate foci depend on the fact that the incidence is direct. We will consider shortly the case of oblique incidence.

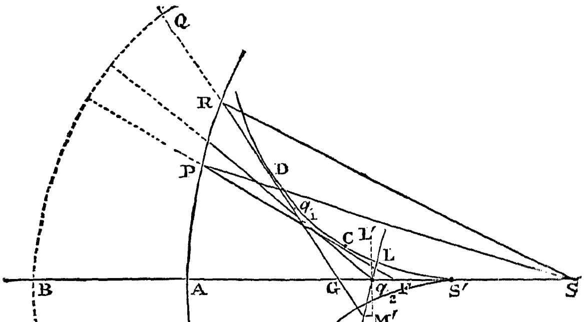
We started with the supposition that the emergent wave front was spherical and the surfaces considered aplanatic, and we saw that for direct incidence we might treat a sphere as such.

Let us suppose now that this is no longer the case, and let $P'AP$ (fig. 46) be a section of the reflecting or refracting surface by a plane through the axis, $Q'BQ$ a section of the emergent wave front cutting the axis in B. Since the origin of light S lies on the axis of the surface which is symmetrical with regard to it, the emergent wave also is symmetrical and the portion BQ' is a counterpart of BQ. Any incident ray lies in a plane through the axis, and the corresponding reflected

or refracted ray is in the same plane, which is called the primary plane for that ray.

The emergent rays in the plane of the paper are normal to the curve B Q. Any normal to a curve in general intersects the consecutive normal, and these points of intersection lie on a curve which is called the evolute of the given curve. Moreover, the normals all touch this evolute. Thus in the case before us any reflected or refracted ray intersects the next; the curve on which these points lie, which is the

FIG. 46.



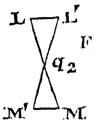


FIG. 46*a*.

evolute of the wave front, is known as the caustic curve, and all the rays are tangents to it. The form is shown in the figure in the curve S C D, S′ is the geometrical focus of S, and we may prove that the caustic curve has a cusp at S′.

By rotating the whole figure round S A, we should get a reflecting or refracting surface, a wave surface and a caustic surface instead of curves.

Now consider a very small portion C D of the caustic curve. The tangents at C and D are two of the emergent rays; let them meet the surface in P and R, and the axis in

F and G respectively. Join S P, S R: S P and S R are two rays incident obliquely on the surface. They give rise to P C and R D. Rays incident on the curve between P and R will give rise to rays meeting the caustic curve between C and D, and the axis between F and G.

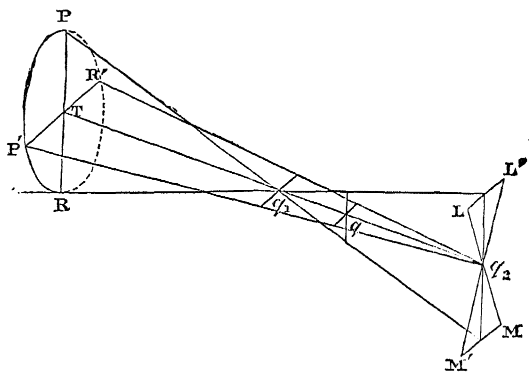
Let q_1 be the point in which a ray coming from the middle point of P R touches the caustic, and let this ray cut the axis in q_2. Let L q_2 M be parallel to the tangent to the reflecting or refracting surface at the point in which $q_2 q_1$ meets it, and consider only the small pencil of rays falling obliquely on P R; the reflected or refracted rays all pass very approximately through the point q_1, and accurately through a small portion of the line F G. Now turn the whole figure round the axis through a small angle. The portion of the reflecting or refracting curve P R becomes a part of the surface on which a pencil of rays falls obliquely.

The point q_1 traces out a small arc of a circle with its centre in the axis of the surface, and through this all the reflected or refracted rays pass very approximately. This may be treated as a small straight line nearly perpendicular to the plane of the paper, and is called the primary focal line. If we call the ray incident on the middle point of the area PR R′P′, the axis of the oblique pencil, the primary focal line is perpendicular to the primary plane of the axis of the oblique pencil. This plane we may speak of as the primary plane of the pencil.

The line L q_2 M comes into the position L′ q_2 M′, and traces out as small area as in fig. 46*a*, somewhat like an elongated figure of eight. Through this area all the emergent rays pass, and it may be treated as a small line in the primary plane. It is called the secondary focal line. Thus when a small pencil is reflected or refracted obliquely from a surface the reflected or refracted rays pass very approximately through two small focal lines at right angles to each other. One of these lines, the primary, goes through the point in which the axis of the reflected or refracted pencil

touches the caustic surface, and is perpendicular to the plane containing the axis of the incident and reflected or refracted pencils. This plane is called the primary plane. The other focal line, the secondary, lies in the primary plane and passes through the point in which the axis of the reflected or refracted pencil cuts the axis of the surface. It is parallel to the tangent plane to the reflecting or refracting surface at the point in which the axis of the pencil meets it. Again, consider a section of the pencil by a plane parallel to this tangent plane. When the cutting plane passes through

FIG. 47.



the primary focal line, the section consists of this line; its breadth in the primary plane is nothing. As the cutting plane moves parallel to itself towards the secondary line the breadth in the primary plane increases, that perpendicular to the primary plane diminishes, and at last, when we reach the secondary line, vanishes.

There must therefore be some point, q suppose, figure 47, between the two focal lines at which the breadth of the pencil will be the same in and perpendicular to the primary plane.

When the cutting plane passes through q the section of

the pencil will be a small oval curve with two diameters in and perpendicular to the primary plane respectively equal to each other. Such a curve cannot differ much from a circle. We shall treat it as one; it is called the circle of least confusion. Thus in addition to two focal lines the pencil after oblique reflexion or refraction passes through an oval curve approximately circular called the circle of least confusion. When light diverging from a point is obliquely reflected or refracted, the circle of least confusion is the nearest approach to a point in the path of the emergent pencil.

The positions of the primary and secondary focal lines, and of the circle of least confusion, can be calculated in terms of the angles of incidence and refraction, the distance from the surface of the origin of light, the refractive index and the dimensions of the reflecting or refracting surface. In some cases the primary focal line is the nearer to the surface, in others the secondary.

Their positions can easily be seen by reflecting obliquely from a concave mirror light diverging from a bright point and receiving the reflected beam on a screen.

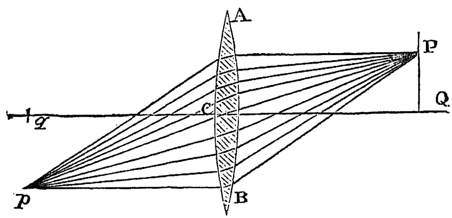
For one position of the screen we get a line of light at right angles to the plane passing through the axis of the surface and the luminous point; this is the primary line. As we move the screen further from the surface the line decreases in length and broadens out, and for one position of the screen its illuminated portion is very nearly circular; as the screen is moved still further, the longest diameter of the bright part is now perpendicular to the primary line, and after a time we obtain only a broad line in that direction. This is the secondary focal line.

When the incidence is direct the primary and secondary lines and also the circle of least confusion coincide at the geometrical focus. The distance between the two focal lines depends always on the square of the angle of incidence, and will therefore be very small even when the angle of inci-

dence is not exceedingly small, so that for rays incident at angles so small that we may neglect their square compared with the other quantities involved, we may very approximately use the formula for direct incidence.

Let us now suppose that we have a convex lens with an object before it emitting light. Corresponding to each point of the object we have its image at the conjugate focus of the point, and these images combined make up the image of the object. Let A C B (fig. 48) be the lens, and P Q the object. Consider any point P in the object. Rays diverging from it are incident at all points of the lens and are

FIG. 48.



refracted by it, the ray which passes through the centre C alone is unchanged in direction; thus p, the image of P, is somewhere on C P, or C P produced in either direction. If the incidence at C were direct, that is, if P were on the axis of the lens, the position of p would be given by the formula—

$$\frac{1}{Cp} - \frac{1}{CP} = -\frac{1}{f}$$

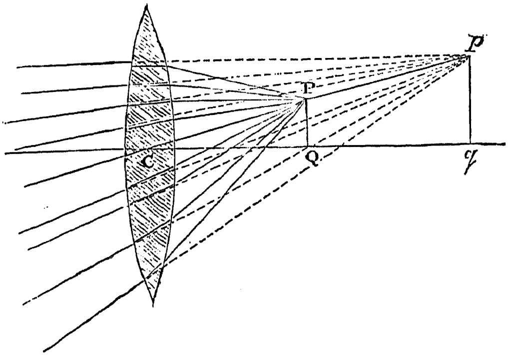
f being the focal length of the lens which is negative, since the lens is convex. If C P is greater than f, Cp is negative as in the figure. Thus all the rays from P converge to p. The locus pq of all points such as p gives us the image of the object P Q, and in the figure, since the rays pass through the image, it is a real one.

If, as in the figure, PQ is a straight line perpendicular to

the axis of the lens, $p\,q$ is very approximately another straight line also perpendicular to the axis of the lens; more strictly $p\,q$ is a conic with C as focus. Moreover, since P and p are on opposite sides of the axis Q $c\,q$, the image thus formed is 'inverted.'

If CP is less than f, Cp is positive but greater than CP, and the image is virtual but larger than the object. The rays coming from P are bent down towards the thicker portion of the lens, but not so much as in the first case, so that instead of converging to a point on the side of the lens

FIG. 49.



opposite to P they appear to diverge from a point on the same side. Fig. 49 gives this case.

Thus we may use a convex lens to give us an image of a luminous object in two ways. If the object be at a distance from the lens less than its focal length, and we look at it through the lens, the light will appear to come to us from a virtual image of the object further removed from our eye than the object itself, and of larger size. It will thus be of assistance to a long-sighted person. The convex lens thus used will magnify the object. This is the principle of the magnifying glass or simple microscope.

When we wish to use this instrument so as to look at an

object—a picture, for instance—we find generally that if we keep our eye and the object fixed there is one position of the lens in which we can see the object most distinctly. The reason of this is as follows. Our eye cannot see clearly objects too close to it, neither can it distinguish them if too far off; there is one distance at which we can see most clearly. Let us call this d, and let the object viewed through the lens be at a distance a from the eye; we have to find a position for our lens such that the image of the given object when seen through it may be at a distance d from the eye. Let us suppose the lens is at a distance x from the eye. Then the object is at a distance $a - x$, and the image at a distance $d - x$ from the lens; let f be the focal length of the lens; then we have

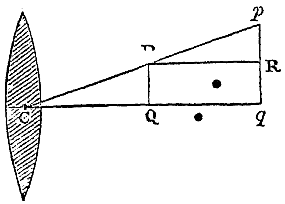
$$\frac{1}{d - x} - \frac{1}{a - x} = -\frac{1}{f};$$

$$\therefore f(a - d) + ad - x(a + d) + x^2 = 0;$$

and from this equation we can find the value of x.

To find the magnifying power of the lens used in this manner we must remember that in order to see the object

FIG. 49*b*.



distinctly we should have to put it at the same distance from our eye as the image is when viewed through the lens.

Draw PR (fig. 49*b*) parallel to CQq. Then Rq = PQ, and the effect of the lens is to increase the apparent size of the object in the ratio $\frac{pq}{Rq}$; thus if we call m the magnifying

power, we have $m = \frac{pq}{Rq} = \frac{pq}{PQ} = \frac{Cq}{CQ}$, by similar triangles CPQ, Cpq.

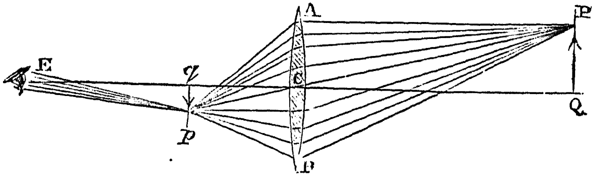
$$\text{Now } \frac{1}{Cq} - \frac{1}{CQ} = -\frac{1}{f}$$

$$\therefore \frac{Cq}{CQ} = 1 + \frac{Cq}{f}$$

$$\therefore m = 1 + \frac{Cq}{f}.$$

Let us now suppose we are looking through a convex lens at a distant object, the distance between the lens and object being greater than the focal length. Then we have

FIG. 50.



proved that the lens forms a real inverted image of the object at pq.

The rays from a point P (fig. 50) of the object converge to the point p of the image, and if we placed a screen in the position pq we should see on the screen an inverted image of our object. This is what is done by the lenses in a photographic camera. The sitter is the object. A system of two or more lenses equivalent to our convex lens in the figure forms a real inverted image of the sitter on the sensitised plate, and by the action of the light on the chemical substances with which it is coated an impression is produced on the plate which is rendered permanent by means of some other chemicals.

But the rays from P pass through the point p and form a divergent cone with it as vertex. If, then, we place our eye as at E to receive some of these rays, E being at a con-

venient distance from $p\,q$, we shall see the inverted image $p\,q$; the image $p\,q$, in fact, is just like a real object which is capable of emitting light, but only in certain definite directions, viz. those in which the rays from the various points of the object P Q travel.

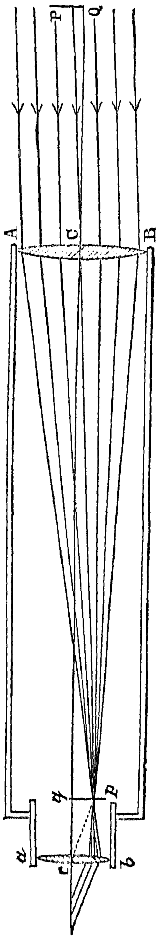


FIG. 51.

Now we have seen that by using a convex lens we can obtain a magnified image of any object. In general the inverted image of the distant object will be so small that we shall be unable to distinguish accurately its various parts; but by placing a convex lens behind it at a suitable distance, and in such a position that light coming from our object P Q may fall upon it, we can obtain a magnified image of the image $p\,q$. Two convex lenses arranged in this manner constitute the simplest form of the astronomical telescope (fig. 51). Of the two lenses, the one on which the light falls first is called the object-glass; the second is known as the eye-lens.

In the telescope these two lenses are fitted into tubes so that their axes coincide; the tube containing the eye-lens is movable in that containing the object-glass, and thus the distance between the two is capable of adjustment.

An inverted image of a distant object P Q is formed at $p\,q$ by the object-glass A C B; the rays from any point as P which fall on the object-glass are made to converge to p. The position of p is found by joining P to C the centre of the object-glass, and producing it to p, the distance C p being given by the

formula $\frac{1}{Cp} - \frac{1}{CP} = -\frac{1}{F}$; F being the focal length of the object-glass.

If the telescope is used to view a very distant object, CP is very great compared with F and Cp, and therefore $\frac{1}{CP}$ may be neglected, so that the distance between $p\,q$ and the object-glass is equal to the focal length of the glass; $p\,q$ is at the principal focus of the object-glass.

To determine the position of $a\,c\,b$ the eye-lens, let f be its focal length, then if the distance between the lens and $p\,q$ be f, so that $p\,q$ is at the principal focus of the eye-lens also, the rays from any point as p of the image will on emerging from the eye-lens be parallel. Now a pencil of parallel rays is capable of producing distinct vision for ordinary eyes. Thus if the lens $a\,c\,b$ have this position, an ordinary eye placed behind it will see an image of $p\,q$ distinctly. This image will be erect and virtual, but since $p\,q$ is an inverted image of P Q, the image seen by the eye is an inverted virtual image of P Q.

Moreover, this image subtends at the eye the angle $p\,c\,q$ very approximately (supposing the eye to be close up to the eye-lens). The angle subtended by P Q is P c Q, but since we suppose P Q to be a long way off compared with the length of the telescope, P c Q cannot differ much from P C Q. Now we estimate the size of objects at about the same distance by the angles which they subtend at the eye, and P Q and the image of $p\,q$ formed by the eye-lens are both at about the same distance from the eye. Thus if we call the magnifying power of the telescope m, m being the ratio of the apparent size of the image to the apparent size of the object, we have

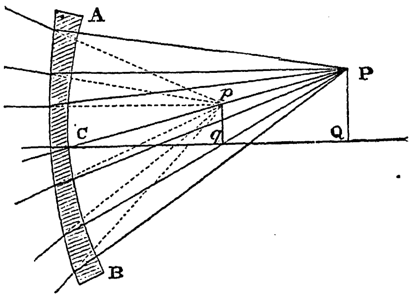
$$\frac{\text{PCQ}}{p\,c\,q} = \frac{p\,\text{C}\,q}{p\,c\,q} = \frac{\frac{p\,q}{\text{C}\,q}}{\frac{p\,q}{c\,q}} = \frac{\text{F}}{f};$$

for pcq and pCq being small angles, we may very approximately replace them by their tangents.

Thus the magnifying power of a simple astronomical telescope when used to view a distant object—the moon for example—is measured by the ratio of the focal length of the object-glass to the focal length of the eye-glass.

Let us now suppose we have a concave lens instead of a convex; let A C B (fig. 52) be the lens, C being its centre, P Q

FIG. 52.



the object as before. Then an image of P any point on the object is formed by the lens somewhere on the line C P; let it be at p, and let f be focal length of the lens, the distance Cp is given by the formula—

$$\frac{1}{Cp} - \frac{1}{CP} = \frac{1}{f}$$

$$\therefore \frac{1}{Cp} = \frac{1}{CP} + \frac{1}{f};$$

so that Cp is always positive and less than C P.

Thus the image formed is nearer the lens than the object, the magnifying power being the ratio of Cp to C P is equal to $1 - \frac{Cp}{f}$; and this being always less than unity, the image formed is diminished.

Thus the effect of a concave lens is to form an erect

virtual image of a distant object nearer to the eye than the object. It will therefore aid a short-sighted person.

Now let us suppose we have a pencil of rays which by some means have been made to converge towards a point, and that before reaching the point they fall upon a concave lens.

Let A C B (fig. 52) be the lens. We have seen that a pencil of rays diverging from a point as P appears to diverge, after refraction through the lens, from a point *p* lying on C P. If we reverse the direction in which a ray is travelling at any point of its course, it will pursue exactly the same path back; thus, if instead of considering our pencil after emerging from the lens as one diverging from P, we consider it as a pencil converging to *p* and incident on the lens, it will, after passing through, converge to P. If our original pencil had been one of parallel rays so that *p* is at the principal focus of the lens, then our reversed pencil, converging as it does to the principal focus, will on emergence consist of parallel rays, and may therefore be capable of giving distinct vision to an eye placed to receive it.

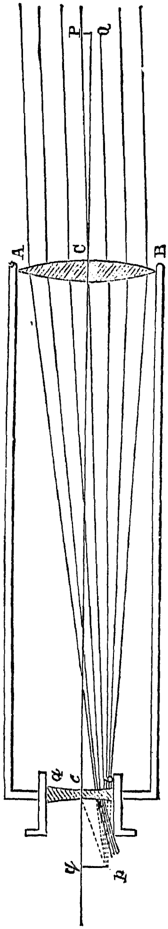


FIG. 53.

A concave lens is so used in the eye-piece of Galileo's telescope.

In this telescope (fig. 53) there are two lenses A C B, *a c b*, convex and concave respectively, having a common axis, and fitted in two tubes. The distance between their centres is capable of adjustment.

The convex lens would form an inverted real image of the object to which the telescope is directed. As before,

the image of a point P is on the line PC, and at a distance Cp from C given by

$$\frac{1}{\mathrm{C}p} - \frac{1}{\mathrm{CP}} = -\frac{1}{\mathrm{F}},$$

F being the focal length of the object-glass.

The eye-glass acb, a concave lens of focal length f, is placed between the object-glass and this image, and the rays converging to the various points of the image fall on the eye-lens. The eye-lens is so placed that the rays converging to any point as p of the image emerge parallel, and in this condition they fall on the eye of the observer placed behind the eye-glass, and thus an image of the point p is seen in the direction pc produced. The same holds for all points of the image pq; the image thus formed of pq is inverted, but pq being itself an inverted image of PQ, the image seen by the observer is an erect image of PQ.

In this telescope the distance between the lenses is approximately the difference of their focal lengths; in the astronomical it is the sum. The magnifying power is, we may show, measured, as in the astronomical telescope, by the ratio of the focal length of the object-glass to the focal length of the eye-glass. This arrangement of the lenses, invented by Galileo, is commonly used in opera and marine glasses.

For purposes of accurate measurement, however, it is necessary to use the astronomical telescope, or rather it or some other arrangement in which the image viewed by the eye-piece is real.

Suppose the astronomical telescope is adjusted to view a distant object PQ (fig. 51), and that pq is the image formed by the object-glass; then when the eye-lens is placed so that pq is clearly seen, any other object in a plane passing through pq perpendicular to the axis of the telescope will, if sufficiently illuminated, be distinct also.

In practice, two or more thin silk fibres or pieces of a

spider's-web are stretched across the telescope tube at this place. These are seen as dark lines crossing the field of view, and the point of intersection of two of the fibres forms a convenient mark of reference. For example, in using the telescope to determine the position of a star or some distant mark, it would be adjusted until the image of the star, formed by the object, coincided with the intersection of the cross wires. This of course would be the case when the images of the two, as seen by the eye-glass, appeared to coincide, and then we know that the line joining the intersection of the cross wires to the centre of the object glass passes through the star.

The position of this line is given by the graduations attached to the instrument, and hence the position of the star can be found.

Sometimes, instead of two cross wires, the point of a fine needle is used as a mark of reference.

In a telescope with cross wires, the eye-piece is usually capable of adjustment relatively to the wires, and then the wires and eye-piece together relatively to the object-glass. So that in using the telescope it is requisite first to place the eye-piece so that the cross wires are distinct—to focus the cross wires it is called—then to adjust cross wires and eye-piece together until the object looked at comes into clear view.

Instead of a single lens as an eye-glass, the astronomical telescope usually has two forming an *eye-piece*. Two arrangements of these lenses are common; the one is called Ramsden's, or the positive eye-piece, the other Huyghens', or the negative eye-piece. We can hardly discuss here the reasons for this arrangement, but proceed to describe the lenses.

Ramsden's eye-piece consists of two convex lenses of equal focal length f on the same axis, the distance between them being two-thirds of their focal length.

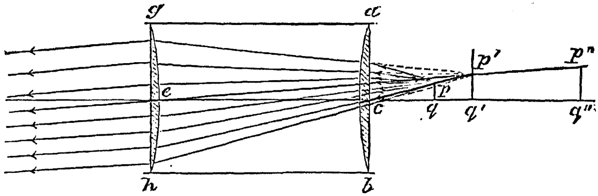
Let us trace a pencil of rays from an object $p\,q$ through such a system, let $a\,c\,b$, $g\,e\,h$ (fig. 54) be the lenses, $c\,e$ their

axis. Let the distance between c and q be less than f, the focal length of either of the lenses. $a\,c\,b$ is called the field-lens, $g\,e\,h$ the eye-lens. Rays diverging from p fall on the field-lens, join $c\,p$, and produce it to p' such that

$$\frac{1}{c\,p'} - \frac{1}{c\,p} = -\frac{1}{f}.$$

After passing through the field-lens the rays will appear to

FIG. 54.



diverge from p'; in this state they fall on the eye-lens $g\,e\,h$, join $e\,p'$, and on this or this produced take p'' such that

$$\frac{1}{e\,p''} - \frac{1}{e\,p'} = -\frac{1}{f}.$$

After passing the eye-lens the rays will appear to diverge from or converge to p'' according as it is the right or left of the eye-lens. If the distance between $p\,q$ and the field-lens is properly adjusted, these rays may emerge in a state capable of giving distinct vision to an eye placed so as to receive them. This will be the case if the rays from any one point emerge from the eye-glass in a state of parallelism, so that p'' is at an infinite distance off the lens.

When this is the case q' is at the principal focus of the eye-lens.

$$\therefore e\,q' = f.$$

$$\text{But } e\,c = \frac{2}{3}f; \quad \therefore c\,q' = \frac{1}{3}f.$$

$$\text{Also } \frac{1}{c\,q'} - \frac{1}{c\,q} = -\frac{1}{f};$$

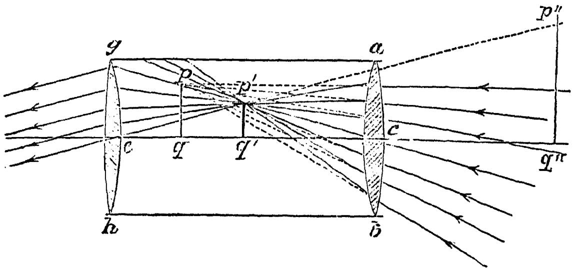
$$\therefore c\,q = \frac{f}{4}.$$

Thus to see an object distinctly through a Ramsden's eye-piece, the distance between the object and the field-lens must be one quarter of the focal length of the lenses.

When used with a telescope, the object viewed is the image formed by the object-glass and the cross wires; when these are clearly seen the distance between them and the field-lens is for ordinary eyes a quarter of the focal length of the field-lens.

Huyghens' eye-piece also consists of two convex lenses, a field-lens and an eye-lens. The focal length of the former

FIG. 55.



is three times that of the latter, and the distance between the lenses is the difference of their focal lengths.

This eye-piece is called negative because it cannot be used to view an object directly; as with the concave lens in Galileo's telescope so here we require, in order to produce distinct vision, to have a pencil of rays converging to a point behind the field-lens.

Let p (fig. 55) be this point, $a\,c\,b$ the field-lens focal length f_1; $g\,e\,h$ the eye-lens focal length f_2; then $f_1 = 3f_2$.

Join cp and in it take p' such that

$$-\frac{1}{cp'} + \frac{1}{cp} = -\frac{1}{f_1}.$$

We have $-\frac{1}{cp'}$ for p' is on the left of c by hypothesis,

and lines measured to the left of the origin are taken as negative.

Then rays converging to p converge after passing through the field-lens to p'; join ep', and in it produced take p'' such that

$$\frac{1}{ep''} - \frac{1}{ep'} = -\frac{1}{f_2}.$$

The rays diverging from p' after passing the eye-lens appear to diverge from p''.

If the adjustment is such as to produce distinct vision q'' is at an infinite distance off, and therefore $eq' = f_2$.

But $$ec = \frac{f_1 + f_2}{2} = 2f_2.$$

Thus $$cq' = f_2 \text{ and}$$

$$\frac{1}{cq} = \frac{1}{f_2} - \frac{1}{f_1} = \frac{2}{3f_2}$$

$$cq = \frac{3}{2}f_2 = \frac{1}{2}f_1$$

and q is just half-way between q' and e.

Hence if Huyghens' eye-piece be used with a telescope the image formed by the object-glass must be behind the field-lens, and at a distance of half the focal length of the lens from it.

Thus we cannot place a real object as the cross wires so as to be seen distinctly through the two lenses. If we wish to have cross wires we must place them at the principal focus of the eye-lens. They will then be seen distinctly by a person with normal vision, but the image with which they appear to coincide is no longer that formed by the object-glass simply, but the image of this image as formed by the field-lens of the eye-piece. By this second refraction the different parts of the image will be altered each in a different manner. The image will therefore be distorted and cannot be used for very accurate measurements. Thus for astronomical observations Ramsden's eye-piece is used.

The general principle of all telescopes is to form, by means of a lens or mirror, an image of a distant object and then by means of a lens or eye-piece to magnify that image. By making the object-glass or mirror large, a great number of rays can be concentrated into the image. When an object is magnified, if the amount of light which reaches the eye from it is not increased, the brightness of the object is diminished; the same amount of light being spread over a larger area, each point of that area is thereby rendered less bright. Thus in order to magnify highly a distant object we should use a large object-glass or reflector to form an image of it, and then since that image will be very bright, there being a large number of rays condensed into it, we can use an eye-piece of small focal length and magnify it greatly.

For an account of telescopes in which a reflector is used to form the image of the distant object which is then magnified, the reader is referred to books on geometrical optics.

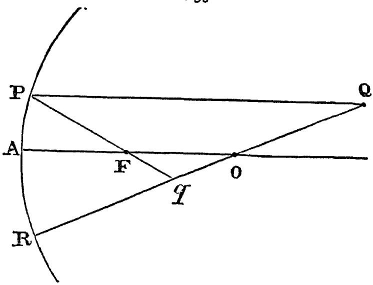
We have seen that in the case of reflection or refraction at a spherical surface, a ray incident so as to pass through the centre of the surface is reflected directly back or passes on into the refracting medium without deviation, while a ray incident parallel to the axis of the surface is reflected or refracted so as to pass through the principal focus. We will apply these results to determine geometrically the position of the image of a point formed by reflexion at such a surface.

Let Q (fig. 55a) be such a source of light, O the centre, O A the axis of a reflecting surface P A R, F the principal focus. Let Q O R be a ray passing through the centre O, it is reflected directly so that the image of Q will be somewhere in Q R. Let Q P be a second ray parallel to the axis O A, it is reflected so as to pass through the principal focus F; thus the image of Q is some point in P F, so that if P F and Q R intersect in q, this point must be the image of Q. An exactly similar construction holds for the case of refraction. So, too, for refraction through a lens; we know that a ray incident centrically passes through without deviation, while a

ray incident parallel to the axis is refracted so as to pass through the principal focus.

Let us take the case of a convex lens A C B (fig. 55*b*). P Q

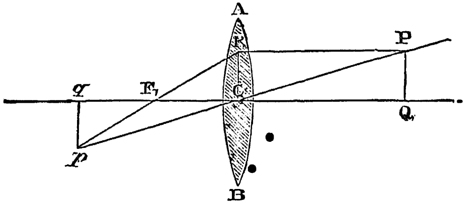
FIG. 55a.



being an object at some distance, the ray P C passing through the centre C is undeviated; the image of P is therefore on P C or P C produced.

Let P R be a ray incident parallel to the axis Q C q and let F_1 be the principal focus. Join R F_1 and produce it to

FIG. 55b.



meet P C produced in p. The ray P R is refracted along R F_1, the image of P must therefore be on R F_1; thus p is the image of P. Draw P Q, $p\,q$, and R C perpendicular to the axis. Let C Q $= u$, C $F_1 = f$, C $q = v$; we will proceed to find the relation between u, v and f. The triangles R C F_1, $p\,q\,F_1$, are similar.

$$\therefore \frac{p\,q}{\mathrm{R\,C}} = \frac{q\,\mathrm{F}_1}{\mathrm{C\,F}_1},$$

PQC and pqc are also similar.

$$\therefore \frac{pq}{PQ} = \frac{cq}{CQ}, \text{ but } PQ = RC.$$

$$\therefore \frac{qF_1}{CF_1} = \frac{cq}{CQ}, \text{ or } \frac{v-f}{f} = \frac{v}{u}.$$

Whence we find

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$$

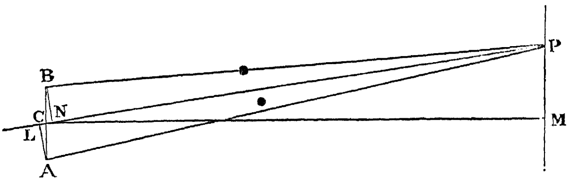
The case of any other form of lens could be solved similarly.

CHAPTER V.

INTERFERENCE.

WE have already (Chapter II.) explained what is meant by interference, and shown how it is connected with the rectilinear propagation of light. We have seen that if we have two sources of light, the effect produced at any point is the resultant of the effects due to each of the sources separately,

FIG. 56.



and we have shown that at certain points the effects of two exactly similar small sources of light will be to produce darkness. We turn now to various experimental methods of producing interference.

Let A and B (fig. 56) be two such sources, and suppose we try to find the appearance that ought to be presented on a screen on which the light falls. Let PM be a line in the

paper parallel to A B, and P be any point in the line, then the intensity of the illumination produced at P depends on the difference P A – P B. When P A – P B is equal to an even multiple of half a wave length, the effect is a maximum; when this difference is equal to an odd multiple of half a wave, it is zero.

Let C be the middle part of A B, and let C M perpendicular to A B and P M meet the screen in M. Join A P, B P, and C P. Let A B $= c$, C M $= a$, P M $= x$. Draw A L and B N perpendicular to P C or P C produced. Then A L is perpendicular to C P and A C to C M.

$\therefore$ the angle C A L = angle P C M.

Also A L C and P M C are both right angles.

$\therefore$ the triangles A L C and P M C are similar.

In a similar manner we may show that the triangles B N C, P M C, are similar

$$\therefore \frac{\mathrm{CL}}{\mathrm{CA}} = \frac{\mathrm{PM}}{\mathrm{PC}} = \frac{\mathrm{CN}}{\mathrm{CB}}, \text{ similarly.}$$

$$\text{Hence } \frac{\mathrm{PM}}{\mathrm{PC}} = \frac{\mathrm{CL} + \mathrm{CN}}{\mathrm{CA} + \mathrm{CB}} = \frac{\mathrm{NL}}{\mathrm{AB}}.$$

We shall now make the further assumption that A B and P M are both very small compared with C M. This being the case the angle P C M is very small, and C M P being a right angle, C P M is very nearly a right angle, so that the triangle C P M is very nearly isosceles; we may therefore put for C P the value C M or a.

In the same way P L is very nearly equal to P A, and P N to P B.

Thus N L = P L – P N = P A – P B. Hence we get on substituting for N L and C P

$$\frac{x}{a} = \frac{\mathrm{PA} - \mathrm{PB}}{c}$$

$$\therefore x = \frac{a}{c}(\mathrm{PA} - \mathrm{PB}).$$

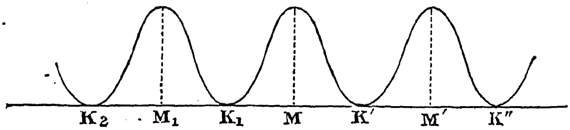
We have seen that the points of maximum brightness

are given by $PA - PB = n\lambda$, λ being the wave length, while those of darkness are given by $PA - PB = \frac{2n+1}{2}\lambda$.

Thus we shall have maximum brightness whenever $x = \frac{an\lambda}{c}$, and absolute darkness whenever $x = \frac{a(2n+1)\lambda}{2c}$, n being any integer whatever, including zero.

There will, therefore, be a series of very bright points, equidistant from each other, along the line PM, at distances 0, $\frac{a\lambda}{c}$, $\frac{2a\lambda}{c}$, &c., from M, and between these there will be a series of absolutely dark spots, also equidistant, at distances $\frac{a\lambda}{2c}$, $\frac{3a\lambda}{2c}$, $\frac{5a\lambda}{2c}$, &c., from M.

FIG. 57.



Between these points the light will change gradually. Beginning with maximum brightness at M it will fade away, reaching darkness at a point $\frac{a\lambda}{2c}$ from M, then it will increase again and become a maximum at a point at distance $\frac{a\lambda}{c}$, and so on. We can show that the intensity of the light at any point may be represented by the ordinates of a curve such as that in fig. 57.

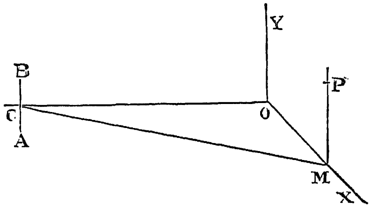
The points M, M_1, M′, &c., are those of maximum brightness, K_1 K′, &c., those of darkness, and each of the distances M_1K_1, MK_1, &c., is equal to $\frac{a\lambda}{2c}$. Thus if we take any line in the plane of the screen which is parallel to the line AB, we find a series of absolutely dark points on it. These

points are, so long as we keep to one of these lines, at equal distances apart.

If we consider the whole screen as made up of the various lines parallel to A B, these dark points form a series of dark bands crossing the screen in directions almost perpendicular to A B.

Let C O (fig. 58) perpendicular to the screen meet it in O.

FIG. 58.



Let O Y be a line in the screen parallel to A B, and O X perpendicular to O Y.

Let M be any point in O X, and P M a line perpendicular to O X, and therefore parallel to A B. P M corresponds to the line P M in fig. 56.

We have seen that there are a series of dark bands crossing the screen nearly parallel to O X, and we may show as follows that these dark bands are hyperbolas with their axes parallel to O X and O Y.

$$\text{CM} = a = \sqrt{\text{CO}^2 + \text{OM}^2}.$$

If P is on a dark band, say the first, then $\text{PM} = \dfrac{a\lambda}{2c}$.

Let $\text{CO} = b$.

$$\text{Then } \text{PM}^2 = \frac{\lambda^2}{4c^2}(b^2 + \text{OM}^2).$$

$$\therefore \frac{\text{PM}^2}{\dfrac{\lambda^2 b^2}{4c^2}} - \frac{\text{OM}^2}{\lambda^2 b^2} = 1.$$

Thus P lies on an hyperbola with O X and O Y as axes. Just near O these curves will not differ much from straight

lines, and the screen should appear to be crossed by a series of dark bars approximately parallel to each other, perpendicular to A B. In the space between these bars the light should increase gradually, reaching a maximum half way between each, and then decrease again. The dark bars should all be at equal distances apart, and that distance will be proportional to λ the wave length of the light.

This all depends on the hypothesis that the light is all of one wave length. Let us suppose that we have two systems of waves of different lengths coming from each source. Each system will produce its own interference effects, and the two effects will be superposed. It might happen that the dark bands of one system just corresponded with some of the bright bands of the other, and the visible effect will be much altered; we might get a series of bright and dark fringes, but not placed at equal distances apart.

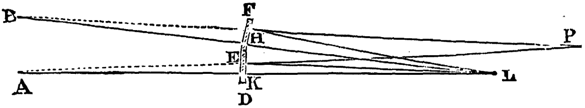
The central band on the screen being bright for both wave lengths, will give the combined effect due to the two. This would be equally true if we have three or more systems of waves instead of only two.

An experiment of this kind was first described by Grimaldi in 1665. He made two small holes close together in the shutter of a darkened chamber and allowed the sun to shine through them. Each hole gave rise to a spot of light on a screen placed to receive it, and the distance between the screen and the holes was so adjusted that the two spots overlapped. Grimaldi states that the portion common to the two spots was under certain circumstances sensibly darker than the rest; but it can be shown that the apparent size of the sun compared with the diameter of the holes will have considerable effect and that the appearances of Grimaldi's are much more complex in their nature than those we have been describing. Moreover he was quite unacquainted with the ideas of the wave theory, and merely advanced his experiment to show that, in the language of the schoolmen, light was an accident, not a substance.

The first to state clearly the principle of interference and establish it by means of experiment, was Thomas Young, of Emmanuel College, Cambridge.

He allowed the light from the sun to pass through a small hole in the shutter of a dark room, and then to fall on two small holes close together in a screen just behind the shutter; he received the rays from these holes on another screen at some distance off the first, and found that in the shadow of the dark portion of the first screen between the two holes, the second screen was crossed by a series of bands alternately light and dark. These disappeared when one of the holes was closed, or when for the narrow beam coming from the first hole the direct light of the sun or of a candle flame was substituted. The two holes constituted the

FIG. 59.



two small similar sources of light, and the bands were produced by their interference.

It is, however, known that the image of a small hole cast by the sun in this manner is surrounded by bands arising, it is true, from interference, but in a somewhat different manner. These are known as diffraction bands, and it was objected that these bands of Young's were merely a modification of the diffraction bands.

To remove this objection, a similar experiment was made by Fresnel in a somewhat different manner. The difficulty was to obtain two small and exactly similar sources of light without using two small holes. This he succeeded in doing in two different ways, which we will proceed to describe. Let L (fig. 59) be a small source of light, such as a small hole, on which an image of the sun or some bright light is cast by a lens of short focus. Let D E, E F be

two mirrors, so placed that the light from L falls on both of them. Suppose L lies in the plane of the paper, and that the mirrors are perpendicular to it, so that their line of junction is perpendicular to the paper, D E, E F being the lines in which the mirrors meet the paper. Draw L K perpendicular to D E, and produce it to A, making K A = K L. After reflection from the mirror, the light will appear to come from A. Let L H be perpendicular to the mirror E F, and make H B equal to H L, then the light reflected from the second mirror will appear to come from B. B and A are images of L, produced by reflection in the two mirrors.

If we make the angle between the two mirrors sufficiently nearly 180°, so that the two are very nearly in the same plane, the images A and B will be very close together, and, each being the image of L, they will be similar. Thus we have obtained two small similar sources of light close together. If, then, we receive the reflected rays on a screen as before, we ought to get interference effects.

For success in this experiment, very careful adjustment is necessary. It is important that the polished surfaces of the mirrors should extend right up to the line in which the two faces intersect. The mirrors should be metallic, or, if of glass, the glass should be black or silvered on the first face, otherwise the reflections from the other surface of the glass spoil the effect.

Fresnel's second experiment is more easy to carry out.

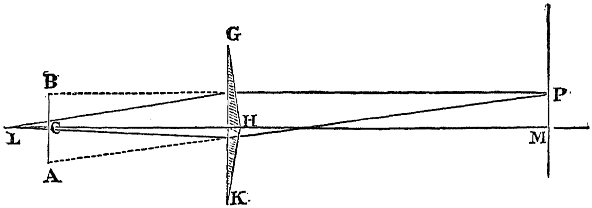
We have seen that a wave of light is bent on passing through a prism of refracting material, so that its course on emergence is inclined to that which it followed before entering the prism, at an angle depending on the material and angle of the prism and the angle of incidence.

Fresnel devised a piece of apparatus known as a biprism. It consists of a piece of glass, or other refracting substance, in the form of a prism.

The principal section, G H K (fig. 60), is an isosceles triangle, the angle G H K is very nearly 180°, so that each of

the angles at G and K is very small. The apparatus is like two prisms of small equal refracting angle G or K joined together at their bases. Waves of light, diverging from a point, L, fall on the face, G K, and enter the glass. Of these, some reach the face G H, others the face H K. Now, we know that a prism tends to turn a ray of light passing

FIG. 60.



through it towards its thicker end; the light then falling on the part G H, the upper portion of the bi-prism in the figure is bent downwards, and appears, after emergence, to come from the point B; while that which falls on H K appears to come from A. The amount through which each ray is turned depends on the angles at G and K; and thus, by decreasing these sufficiently, we can obtain two similar sources of light close together. If the light, after passing through the prism, fall on a screen, P M, interference effects should be produced.

In practice, instead of a point of light, a narrow line of light formed by a slit, with straight parallel sides, is used. The length of the slit must be parallel to the edge of the prism, or the line of intersection of the two plane mirrors. With this arrangement, theory shows that the bands should be straight lines parallel to the slit.

We proceed to discuss the results of either of these two experiments, reserving for the present the more detailed account of the apparatus used, and the precautions requisite to ensure success.

Let us suppose we are using white light from the sun or

a good lamp. Then there will be seen on the screen a white band crossing the centre of the field, which is bordered by a coloured fringe showing red on the side farthest from the centre. Then there comes a dark bar, and after that another iris coloured band, with violet on the inside, passing through green and yellow to red; then another dark band, followed by another band of colour, and so on.

Our experiments with a prism have shown us that a beam of white light is composed of a number of coloured beams, and in this experiment we have again split up white light into its various components.

Now, let us modify the experiment somewhat. Let us introduce between the source of light and the mirrors or bi-prism a piece of red glass. We shall then find that the screen is crossed by a series of parallel dark bands at equal distances apart. These are separated by a series of red bands. This is exactly what the theory would lead us to expect, if we suppose that the red light which passes the glass is all of the same wave-length; and if we call this wave-length λ_r, and the distance between the centres of two consecutive dark or bright bands, x_r, a and c, having the same meaning as before, then—

$$\lambda_r = \frac{c x_r}{a}$$

The quantities x_r, a and c can all be measured, and this will enable us to determine λ_r, the wave-length of the red light.

Now, let us substitute for the red glass a blue glass. The appearance on the screen will be similar to that previously observed, but the bands will be much closer together, and the bright bands will be blue.

We are thus led to infer that our blue glass transmits waves of the same length; and if λ_b be their length, and x_b the distance between two consecutive bands—

$$\lambda_b = \frac{c x_b}{a}$$

and, since x_b is less than x_r, λ_b is less than λ_r, or the wave-length of the blue light is less than that of the red. Now, let us remove our lamp and glass, and use instead a spirit-lamp with a salted wick, or a Bunsen gas-burner with a bead of salt on a piece of platinum. This gives a brilliant yellow flame, and on making our observations we find that the dark bands are still at equal distances apart, while the bright bands are now yellow. Moreover, the distance between the consecutive bands is intermediate in value between x_r and x_b, the distances when red and blue light was used respectively. The light emitted by the Bunsen flame with the salt in it is then of one wave-length which lies between those of red and blue light respectively.

Thus, our experiments verify the anticipations of theory, and teach us that, corresponding to variations in wave-lengths, we have variations in the sensation of colour which the light produces in our eye. Light of a definite wave-length produces a definite colour. The shorter waves produce the sensation of blue or violet, the longer waves that of red. The other colours of the spectrum, intermediate between the red and violet, correspond to waves of intermediate lengths.

The wave-lengths measured in this manner are those of the light when in air. Thus, red waves in air are longer than yellow, and these again are longer than green, blue, and violet. But we have seen that light of all colours travels with the same velocity in a vacuum, and with nearly the same velocity in air. Thus, ether waves of all lengths capable of affecting the eye travel with nearly the same velocity when in air.

But the velocity of a wave is measured by the ratio of the wave-length to the time of a vibration, and since the velocity of red light in a vacuum is the same as that of violet, while the wave-length of the red is the greater, it follows that the time of vibration of red light is greater than that of violet, while the times of vibration in the motions

which produce the sensations of the various colours intermediate between these are themselves intermediate between the periods for red and violet light. Thus, our experimental results are just what we should expect on the assumptions that light is a form of energy propagated by waves; that the period of these waves may vary, at any rate within certain limits; that waves of long period produce in the eye the sensation of red, while those of short period give rise to violet, so that the colour of an object depends on the period of the waves which it emits.

We have, moreover, been enabled to measure the lengths of the waves producing various colours. Later on we shall find other and better methods of measuring wave-length, and it is one of the proofs of the truth of our theory that the measurements of the wave-length of some monochromatic light—say that of a sodium-flame made by various methods—all agree, at least within the limits of error in the experiment.

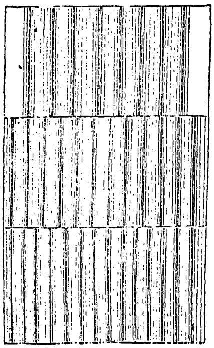
Newton's experiment with the prism showed us that a beam of white light consisted of an assemblage of beams of various colours. This experiment has taught us, further, that a wave of white light is an assemblage of a very large number of waves of different periods. Each of these, separately, would produce in the eye the sensation of a distinct colour. The sum of all these various sensations is the sensation we call white.

These considerations, too, explain the phenomena presented when we used sunlight or lamplight in our experiment. Each of the different waves in our beam produced its own system of bands on the screen. These were superposed, and thus gave rise to the variously-coloured appearance seen. The central line is a bright band for all colours; along it therefore, all the colours are superposed, and it is white. The central red bright band is broader than the violet, and therefore the edges of this white band are tinged with red. Then we have a dark space, since the

second bright band is nearer the centre in the violet than the red. A violet band comes next to this dark space. A little farther on we have the green and yellow bands coming in, in addition to the violet, and we get the colour produced by superposing these.

Going still farther from the centre, we come to the second dark space in the violet, but this corresponds to the farther edge of the second bright green band and the nearer edge of the red, so that we have a mixture of red and green and intermediate colours. This appears to be yellow, and thus we get the series of colours seen.

FIG. 61.



A figure may make it plainer. Fig. 61 gives the position of the bands in red, green, and violet light, drawn approximately to scale. To obtain the appearance actually presented we must superpose these. In the centre we shall get white: a little further on there will be no violet, only the mixture of green and red, which will give a yellow tinge, becoming more red as we go away from the centre, then a black band will come, and so on. At no very great distance from the centre, all regularity in the bands will cease, for the maxima in one colour will overlap sometimes maxima sometimes minima in the others.

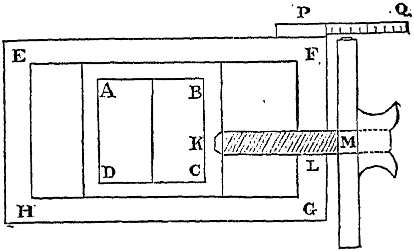
In any case, the system of bands will not extend very far from the centre of the field, for only the central portion of the screen receives light from both sources.

When working with monochromatic light, the edges of the field are usually seen to be bordered by another system of bands, broader and coarser than those in the centre. These arise from a different cause, and will be discussed later on.

Hitherto we have spoken of a screen on which the light

falls and the bands are produced, but this is not necessary. Whatever be the position of the screen behind the bi-prism, interference bands would be formed on it. If, then, we receive the light directly into our eye, interference bands are still formed, and we might see them. In both cases, however, whether formed on a screen or viewed directly, they would be exceedingly close together, and it would be difficult to see them clearly. We therefore use a magnifying glass, either a simple convex lens or a Ramsden's eye-piece, and view them through that. We have already proved that, to see an object distinctly through such a glass, it must be a

FIG. 61a.



certain distance before the glass. If we call the plane perpendicular to the axis of the lenses, which is at that distance, the focal plane, then the system of bands which we see looking directly through the eye-piece is that which is formed in the focal plane.

It remains to describe how the distances between the bands are to be measured. For this a micrometer eye-piece is used. This consists of a Ramsden's eye-piece, and in the focal plane there is a frame carrying a filament of a spider's web. This is known as a cross wire. This frame, A B C D, in fig. 61*a*, can slide in a fixed frame, E F G H, in a direction at right angles to the cross wire.

K L M is a fine, regularly cut screw, turning in a nut at L, which is part of the fixed frame, and so fixed at K that it

can turn round, while at the same time it draws the movable frame and cross wire with it; a graduated circle is fixed on to the screw at M. This is at right angles to the length of the screw and moves with it, and is divided into say one hundred parts.

P Q is a fixed straight edge parallel to the length of the screw, and graduated; the distance between the graduations is equal to that between two consecutive threads of the screw. Let us suppose that the edge of the screw head is opposite a certain division on the scale. Then one complete turn will bring it to the next division, and will draw the cross wire through a distance equal to the space between two divisions. If we suppose we are using a millimètre screw, then the distance between the divisions on the scale is a millimètre, and one turn draws the cross wire through a millimètre.

A fraction of a turn draws it through the same fraction of a millimètre. Suppose that a certain division, say 20, is opposite the straight edge, and that the screw is turned until division 72 comes opposite, that is, it is turned through 52-hundredths of a complete revolution, then the cross wire is drawn through ·52 millimètre.

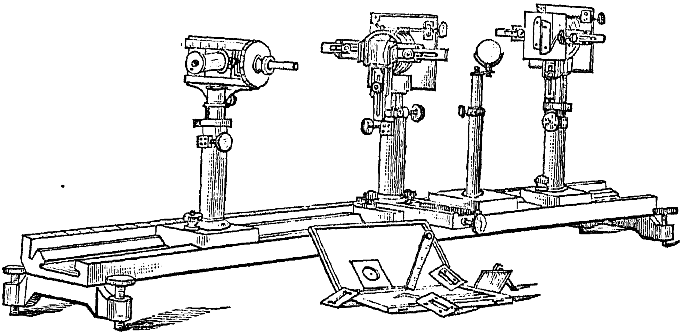
Some of the details given above vary somewhat in different instruments; for instance, the scale used to read the whole revolutions is frequently a straight edge fixed in the focal plane, with notches on it at certain distances apart. Every fifth notch is deeper than the others; another central notch is marked by a round hole below it. In other cases the scale is fixed above the tube containing the lenses, and a point attached to the movable frame is carried up to it. The fractions of a turn are read as before.

In some forms of eye-piece, and among them those fitted by Messrs. Elliott Brothers to their optical bench, the lenses move with the cross wire. This arrangement has the great advantage of keeping the object which is being measured always in the centre of the field, and allowing a greater range.

To measure the wave-length of a given monochromatic light by means of the interference bands, we need a slit supported on a suitable frame, a bi-prism, and a micrometer eye-piece. We must also be able to measure the distance between the slit and the focal plane of the eye-piece, and the distance between the two images of the slit given by the bi-prism.

The experiment is usually made on an optical bench. This consists of a horizontal stand carrying three uprights.[1] These can slide along the stand, which is graduated, and

FIG. 62.



attached to each of the uprights is a vernier, so that the distance between them can be accurately obtained. Each of these uprights has a head-piece, which can be raised or lowered at will, and also turned round a vertical axis parallel to the upright.

The first two uprights are fitted with jaws which can be made to grip any object they are required to hold, and by means of a tangent screw and worm wheel these jaws can be turned round a horizontal axis. A metal plate carrying the slit is placed in the first pair of jaws and the bi-prism in the second. The third upright carries a micrometer eye-piece with a vertical cross wire.

[1] The second upright from the right in the figure is an addition to be described later.

The axis of the eye-piece is always horizontal, but can be turned about in an horizontal plane, and in general the lenses move with the wire.

By means of a screw at its foot the second upright has a horizontal motion at right angles to the scale.

For success in making the measurements a bright light and a narrow slit are required. The axis of the eye-piece should be parallel to the scale along which the uprights slide; it should pass through the middle of the bi-prism and of the slit, and be perpendicular to the flat face of the bi-prism. The slit must be parallel to the edge of the bi-prism and to the cross wire.

All these adjustments but the last may be made with fair accuracy by eye. Where great care is required the following method will be found useful.

There is usually a flat plate of polished metal by which the eye-piece is attached to the upright, and which is generally sufficiently nearly perpendicular to the axis of the lenses to be treated as such. This will assist us to adjust the axis of the lenses. For, place a light behind the slit and somewhat to one side, and open the slit fairly wide; by means of a small mirror reflect the light through the slit, and at the same time look through it above or below the mirror towards the eye-piece. A reflection of the slit will be seen in the polished piece of metal. Move the metal round the axis of the upright until this image is just above or below the centre of the lenses. Then their axis passes through the slit. If greater accuracy is required, a reflexion from the surface of the lenses may be used, but this is fainter and more troublesome to get.

Now place the light behind the slit and look through the eye-piece; a bright blurred field will be seen. Adjust the lamp until the centre of the field is fairly uniformly illuminated, raising or lowering the slit or eye-piece if necessary. To adjust the slit parallel to the cross wire, introduce between the slit and eye-piece a convex lens of suitable

focal length, as shown in the figure, so as to form a real image of the slit in the focal plane of the eye-piece, and by means of the tangent screw turn the jaws and the slit until this image is parallel to the cross wire.

Fix the bi-prism in its jaws, with its edge approximately vertical, and turn the screw at the bottom of the upright stand so as to move it across the field, until a system of lines, more or less blurred perhaps, comes into view. Turn the tangent screw of the second upright until the lines are as distinct as possible, and bring the central line into the centre of the field by using the horizontal screw; move the lamp again to the side, and by means of the mirror reflect the light through the slit looking above it as before. Some of the light will be reflected from the first face of the bi-prism and an image seen. Turn the bi-prism round the vertical until this image appears to coincide with the centre of the lenses behind them, the axis of the lenses will then be perpendicular to the bi-prism. Bring the bands into the centre of the field again by means of the horizontal screw, and adjust the tangent screw till they are perfectly distinct.

Then, provided the axis of the eye-piece is sufficiently nearly parallel to the scale, the adjustments are complete.

To test for this move the third upright backwards and forwards along the scale, the bands should remain still in the centre of the field. If they do not, the eye-piece must be moved horizontally by means of the micrometer screw, and the prism by means of the horizontal screw, so as to keep the bands in the centre of the field until a longitudinal motion of the eye-piece along the scale produces no change in their position.

This adjustment will throw out that first made, so that the axis of the lenses will no longer be perpendicular to the bi-prism or pass through the slit, but if care has been taken initially to adjust the apparatus as well as possible by eye, the changes produced will be very small. For great accuracy, of course, the adjustments must be repeated.

The distance between the bands will be increased by decreasing the distance between the first two uprights carrying the slit and the bi-prism respectively, or by increasing that between the bi-prism and the eye-piece. Increasing this latter distance, however, renders the field less bright—the same amount of light is spread over a large area—and therefore it should not be too large. The other distance may be small; we must, however, remember that we cannot measure it accurately, and that probably the error actually made in measuring the small distance will be as large as that we should have made in measuring a larger distance, and this error being a larger fraction of the whole, will produce a larger percentage error in the result. This may perhaps be compensated for by the increased accuracy with which we can measure the distance between two dark lines; at any rate this consideration will prevent our making the distance between the slit and bi-prism very small.

We have now to measure the distances between the bi-prism and slit and bi-prism and focal plane of our eye-piece. The scale gives us the distance between two marks on the uprights carrying the slit and prism, but we do not know the horizontal distances between these marks and the slit or prism. To find this the simplest method is to take a rod of known length, let us suppose 10 centimetres; place one end against the slit, and move the stand carrying the bi-prism until its face is brought into contact with the other end of the rod. Then read on the scale the distance between the two marks on the uprights. Suppose it is 8·75 centimetres, the difference between this and the 10 centimetres is 1·25 c., and this must always be added to the reading as given by the scale to get the true distance apart. Thus suppose that when the observations are being made, the distance between the uprights is 10·34 c., then the true distance between the slit and bi-prism is 10·34 c. + 1·25 c., or 11·59 c.

A similar method must be used to find the distance between the bi-prism and the focal plane of the eye-piece. In general if the rod used be of length y, and the distance apart of the uprights as given by the scale and vernier be z, we must add to the distance between the uprights at any other time the length $y - z$.

To find the distance between the bands set the cross wire of the eye-piece as nearly as possible in the centre of a band (bright or dark), and read the scale and screw head, move the cross wire to the centre of the next band and do the same. The difference between the two readings gives in terms of the graduations of the scale the distance between two consecutive bands.

Proceed this way with a large number of lines and take the mean of all the values. Perhaps the best method of taking a set of readings is the following. Take the readings corresponding to some considerable number of bands, say ten. Subtract the first reading from the sixth, the second from the seventh, and so on; each of the values thus obtained gives the distance between five consecutive bands, divide each result by five, and take the mean; the result will be the most accurate value of distance between any two consecutive bands as given by the measurements.

If $2n$ measurements be taken, subtract the first from the $(n+1)$th, and so on. Divide each result by n, and take the mean.

We have now to find c, the distance between the two vertical images of the slit.

Let d be the distance between the slit L and the bi-prism H (fig. 60), and suppose that we know that D is the angle through which each of the prisms deviates light of the wave-length used, which is incident perpendicularly to one of its faces. Then, since L H is the direction of the light before incidence, and A H after emergence, angle L H A = D. Moreover, we can show that H A = H L = d very approximately.

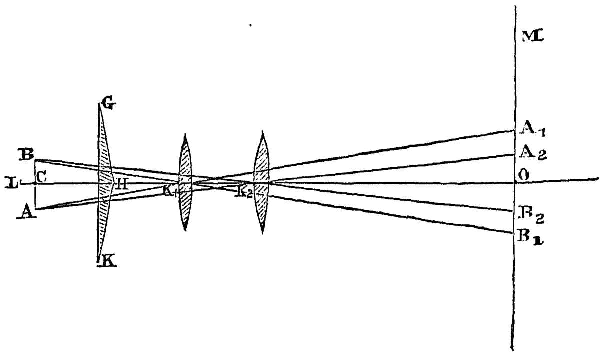
$$\therefore \quad AC = HA \sin AHL,$$
$$= d \sin D,$$
$$\text{and} \quad c = 2\, d \sin D.$$

D can be found by means of the spectrometer.

There is, however, a second method of finding c.

Let OM (fig. 63) be the focal plane of the eye-lens, and introduce a convex lens between the eye-piece and the prism. The lens will form an image of the two images, A and B. Move the lens along the scale until these images

FIG. 63.



are in the focal plane—that is, until they are seen sharp and clear on looking through the eye-piece. Let them be A_1, B_1, and let K_1 be the position of the lens. Then, if the distance between the eye-piece and prism be not too small, and the focal length of the lens be within certain limits, we can, without moving the slit, eye-piece, or prism, find another position, K_2, for the lens, in which we shall have two distinct images, A_2, B_2, of A and B, also in the focal plane.

By means of the micrometer wire we can measure $A_1 B_1$ and $A_2 B_2$. Let them be c_1, c_2 respectively. Of course, $A K_1 A_1$, $B K_1 B_1$, $A K_2 A_2$, $B K_2 B_2$, are all straight lines. Since

C and O are conjugate foci for the lens in the two positions, it is clear that $CK_1 = OK_2$, and $CK_2 = OK_1$, for

$$\frac{1}{CK_1} + \frac{1}{OK_1} = \frac{1}{CK_2} + \frac{1}{OK_2}$$

and

$$CK_1 + OK_1 = CK_2 + OK_2.$$

Thus, since CK_1 is not equal to CK_2, $CK_1 = OK_2$, $CK_2 = OK_1$. Also by similar triangles,

$$\frac{AB}{A_1B_1} = \frac{CK_1}{OK_1}$$

$$\frac{AB}{A_2B_2} = \frac{CK_2}{OK_2} = \frac{A_1B_1}{AB}$$

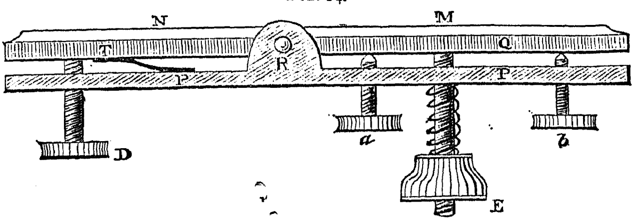
$$\therefore AB^2 = A_1B_1 \times A_2B_2$$

$$\therefore c = \sqrt{c_1 c_2}$$

Thus we can find c.

In this manner all the quantities in our formula can be

FIG. 64.



found, and we can get λ, the wave-length of the light we have been using.

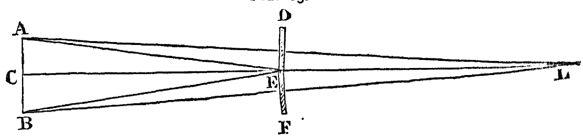
The adjustments necessary when using the two mirrors to produce interference bands are somewhat complex.

One mirror, M (fig. 64), is usually attached to a plate, P, which can be placed on one of the uprights of the optical bench; the other, N, can turn round an axis, R. This axis is fixed to the plate P, through which three screws a, b, c pass; c is not shown in the figure, being behind a, and on

these the mirror M rests. A third screw attached to the back of the mirror passes through the plate P; on this there works a nut E, and between E and the plate is a spiral spring which keeps the mirror pressed against the screws.

By means of the screws, *a b c*, we can adjust the mirror M, until the axis, R, is parallel to the edge of the mirror M, and the two mirrors are in the same plane. Then adjusting the screw D varies the angle between the mirrors.

FIG. 65.



In order to make sure that the mirrors are in the same plane before adjusting the screw D, look at the image of some straight line reflected from the two mirrors. This image should be a straight line.

We can find the distance A B (fig. 65) in terms of the distance between L and the mirrors, and the angle which they make with each other.

Let this angle be α, and let $\text{L E} = a$. Now $\text{L E} = \text{A E} = \text{B E}$. Let C be the middle point of A B.

Since L A and L B are perpendicular on the mirrors, the angle $\text{A L B} = \alpha$,

and

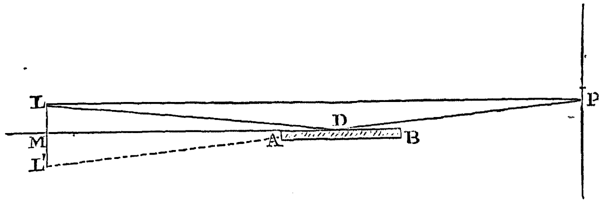
$$\text{A E B} = 2\alpha; \ \text{A E C} = \alpha$$

Therefore $$\text{A B} = 2\,\text{A E} \sin \text{A E C}$$
$$= 2\,a \sin \alpha$$

Of course, A B can be found experimentally, as with the bi-prism. In using the optical bench to make these measurements, it would be necessary to remove the upright which carries the slit and place it on one side of the apparatus.

Dr. Lloyd, of Dublin, suggested, in the 'Trans. Roy. Irish Acad.,' vol. xvii., another and simpler method of obtaining the interference bands. When light falls at a very oblique angle on a plane surface, nearly all is reflected, and appears to come from an image of the luminous point on the other side of the surface. Let L (fig. 66) be the source of light, A B the surface. Draw L M perpendicular to A B, and produce it to L′, making M L′ equal to L M, then L and L′ are two similar sources of light, and by making the angle of incidence very oblique we can bring them as close together as we please, and thus get interference bands. Of course, only one half the system will be formed; the central bright band will be the line in which A B produced cuts the

FIG. 66.



focal plane. The paths of the two rays which interfere at P will be L P and L D P, and we shall get that half of the system which lies on the same side of A B as L. The adjustments for this are very easily made.

In the first chapter we referred to an experiment of Arago's, which decided conclusively against the emission theory. This is but a slight modification of the fundamental interference experiment.

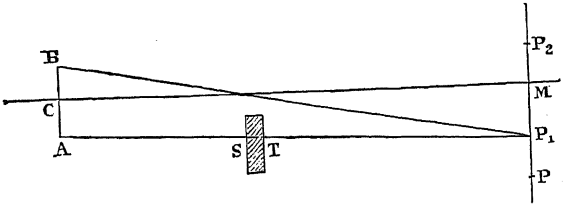
The central bright band is, we know, the locus of points, such that it takes the light the same time to reach them from either of the sources A or B. Let us introduce into the path of one of the interfering pencils a plate of glass or other transparent substance. Let A P_1 (fig. 67) cut this in S, T, P_1 being a point on the screen. Let v be the velocity in air, v' in the glass.

If P_1 coincides with M, the point in which a line through C perpendicular to A B meets the screen, $A P_1 = P_1 B$.

To get from B to P_1 the light travels through air only. To get from A to P_1 it travels partly through air partly through glass. The distance travelled is the same in both cases. Thus, if the velocity in air is greater than that in glass, the time of travelling from B to P_1 will be less than that from A to P_1; and conversely, if the velocity in air is less than that in glass, the time from B to P_1 will be the greater.

In the first case, the central band of the system will be shifted to P_1, on the same side of C M as the glass, so as to

FIG. 67.



decrease $A P_1$ and increase $B P_1$. In the second case it will be shifted to P_2 on the other side of C M.

Experiment shows that it is shifted towards the glass.

Thus, the velocity in air is greater than that in glass. Let ϕ, ϕ' be the angles of incidence and refraction on a piece of glass. Then, according to the undulatory theory,

$$\frac{\sin \phi}{\sin \phi'} = \frac{v}{v'};$$

but v is greater than v', $\therefore \phi$ is greater than ϕ', and this accords with experiment; but, according to the emission theory, we may show that,

$$\frac{\sin \phi}{\sin \phi'} = \frac{v'}{v},$$

or ϕ' should be greater than ϕ, which is contrary to experiment. Thus, this experiment, proving as it does that

the velocity in air is greater than that in glass, is fatal to the emission theory.

We have already seen that, at no very great distance from the centre of the field, the bands become invisible. It is therefore necessary in making Arago's experiment to use exceedingly thin glass, otherwise the shifting of the central band would be so great, that, for reasons which we shall consider shortly, it would cease to be visible. It is better, therefore, to cut two strips close together from the same piece of glass, and place one in path of each pencil, holding them both normally to the light. Then, the thickness of the glass traversed being the same for each, the bands will not be altered. Now, tilt one of the pieces of glass slightly. This increases somewhat the thickness of glass traversed by one of the pencils, and produces the same effect as if we had introduced a thin plate into its path. The bands are shifted towards that plate which has been turned round.

It remains to explain why we can only form a small number of interference bands, and this is intimately connected with the nature of the vibrations which constitute light.

We have already seen that the two sources which interfere must themselves be identical. Now, let P be a point on our screen, such that the distances A P and B P differ appreciably—that is, a point at some distance from the centre, and let us suppose that

$$A P - B P = \frac{2n + 1}{2} \lambda$$

n will be a large number.

Then, for all we have seen at present, we ought to get darkness at P. Let τ'' be the time of a complete vibration, and t' the time the light takes to traverse B P. Then, since light travels a distance λ in time τ'', the time it takes to travel from A to P is $t'' + \frac{2n + 1}{2} \tau''$. Thus, the light

which arrives at P at any moment from B comes from an image of L formed t'' previously, while that from A is from an image of L formed $t'' + \frac{2n+1}{2}\tau''$ previously, and it is quite possible that in the $\frac{2n+1}{2}\tau''$ between the formation of the two images L may have changed, so that the two sources of light, A and B, though at any given instant they are identical, vary together from time to time. The nature of the vibrations sent out from L may vary from instant to instant, so that, at any moment, we have no right to assume that the light coming from A is identical with that emitted by A or B some time previously.

The fact we have been discussing—namely, that if AP − BP is considerable we do not get interference, proves that the vibrations sent out by a source of light are not the same even for a portion of time which is itself not very great.

We have seen that the colour of the light depends on the time of vibration. Thus, if the light is such that its colour remains the same, the times of vibration of the various waves which compose it remain the same. Moreover, if the intensity of the light remains the same, the amplitude of the vibration must be unaltered.

But as yet we have said nothing about the directions in which the ether particles move, except that they must be in the front of the wave.

Our experiment, then, would show us that this direction does vary with the time.

We have, in fact, up to the present assumed that the directions in which each of our waves from A and B urged the particle at P, lay very nearly in the same straight line. Clearly, if the wave from A tends to make the particle at P move up and down, while that from B would produce motion to the right and left, even though the distances AP and BP do differ by an odd multiple of half a wave length,

we cannot expect to get darkness. We conclude then that we are to consider ordinary monochromatic light as a series of waves of constant length. The vibrations which constitute these waves all lie in the wave front, but their direction varies.

Since we do get a considerable number of interference bands under proper conditions, we conclude that the direction of the vibrations remains nearly the same for a considerable number of periods; or, in other words, that the rate at which it changes is slow.

Further investigation, in fact, will teach us that we may consider that any ether particle is describing a small ellipse in the plane of the wave, while at the same time that ellipse is more slowly turning round its centre in its own plane. The eccentricity of the ellipse may have any value, so that, in some cases, the particle may be moving in a circle; in others, its motion may be a straight line. Messrs. Fizeau and Foucault have made an elaborate series of experiments to test the question how long we have a right to consider the curve which an ether particle describes as the same.

CHAPTER VI.

COLOURS OF THIN PLATES.

WE turn now to another series of interference phenomena, generally known as the colours of thin plates.

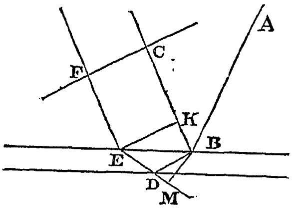
When white light falls on a thin film of transparent substance the reflected light is generally brilliantly coloured, and all the tints of the spectrum may at times be seen. This is shown well in soap films, or thin layers of oil or spirits floating in water.

Another method by which we can get these colours is to lay a thin plate of microscope glass on a thicker piece of glass, and press the two into contact. A series of dark

curves, separated by iris-coloured spaces, will be seen wh light is reflected from it. A thin film of air of varyin thickness is included between the glass plates, and th coloured bands are due to this. Of the light which falls c the upper surface of the film, some is reflected from it ar the rest enters the film. Some of this is reflected from th lower surface and reaches the eye after refraction ther Thus two streams of light coming from the same sourc reach the eye after having traversed slightly different path These two streams are in a condition to interfere, and owin to their interference the film may in parts appear dark.

For light of a given kind, incident at a given angle, th difference in the length of pat pursued depends on the thick ness of the film, and thus th form of the bright and dar curves seen depends on tha also.

FIG. 68.



Let us consider a very thi film of air bounded by tw plates of glass.

Let A B (fig. 68) be a ray in the upper glass incident a B, B C the reflected, B D the refracted ray. This is reflecte at D and refracted out again along E F parallel to B C.

Draw F C perpendicular to B C. If F C be a plane wave of light, the light from C will reach A pursuing the path C B A; that from F will reach A by the path F E D B A.

These two portions of light are in the same phase at starting. The effect at A depends on their difference of phase when they arrive there, and this again depends on the difference in the times of traversing the two paths.

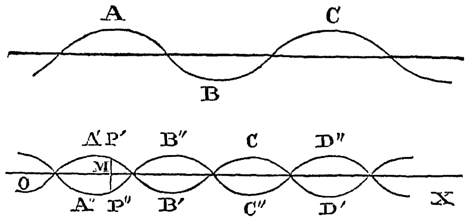
But this is not all; for if the difference in phase depends only on the lengths of the paths, when they coincide this difference will vanish, and the illumination produced at A will be a maximum. But the two paths coincide when the thickness of the film is reduced to nothing; so that if the

film be of variable thickness, at all points in which the thickness is zero, we should, if our reasoning be correct, get a bright point of light.

Now let us look at the question somewhat differently. If at any point the thickness is reduced to nothing, so that the two plates of glass are continuous, then is no reflection there; the light passes straight through and the point appears dark. Thus we have arrived at two opposite conclusions; and of these two, the last is obviously the right one. We are, in fact, wrong in supposing that the difference in phase depends merely on the length of the path.

The light reflected at B is travelling in a dense medium,

FIG. 68a.



and is reflected at the surface of a rarer one; that reflected at D is travelling in a rare medium, and is reflected at the surface of a denser. So far as the reflecting surface is concerned, the conditions under which the two reflections take place are reversed.

In neither case is the amplitude of the reflected light the same as that of the incident. If the amplitude of the vibrations in the incident light be unity, and we find in the one case that the amplitude of the reflected vibration is b, we might expect that that of the reflected vibration in the other would be $-b$. If in the one case the direction in which the particles at the point of incidence move in the reflected wave be the same as in the incident, in the other it will be opposite. Let A B C (fig. 68a) be the displacement

curve for the incident wave, then if in the one case that for the reflected wave be A′ B′ C′, in the other it will be A″ B″ C″. This second curve is exactly the same as A′ B′ C′, only drawn on the opposite side of the line of no displacement. Take any point P′ on A′ B′ C′, draw P′ M perpendicular to the line of no displacement O X, and produce it to P″, making M P″ equal to M P′, then P″ is a point on A′ B″ C″.

This may be illustrated by considering the impact of two elastic balls. If the heavier of two balls strike the other directly it drives that ball on, and continues to move in the same direction itself. If the striking ball be the lighter of the two, the other is driven on, but the motion of the striking ball is reversed, so that after impact it travels in the direction opposite to that of its original motion.

Let us now see how this affects the conditions of interference.

Let us suppose that after reflection at B the reflected vibration is in the same direction as the incident, then after reflection at D it will be in exactly the opposite direction.

If the amount of light in the two rays B A, D B be the same, and we call the amplitude of the vibrations along B A a, then we must call that of the vibrations along D B $-a$. Thus, even if the thickness of the film is nothing, the light coming by the two paths, C B A and F E D B A, would interfere, although the difference in path would be nothing. At all points in which the two surfaces touched we should get darkness. This change in the direction of vibration is in effect exactly equivalent to adding half a period to the time taken by one of the portions of light in travelling from the wave front to the eye.

Let t be the time from C to A, and t_1 from F to A, τ being a complete period, then there will be complete darkness at B if $t_1 + \frac{\tau}{2} - t = \frac{2n+1}{2}\tau$, or if $t_1 - t = n\tau$.

Let V be the velocity in air, V′ in the glass. Draw E K (fig. 68) perpendicular to B C. Let ϕ, ϕ', be the angles

between the wave normal and the normal to the surface in air and glass respectively, δ the thickness of the film, then

$$t = \frac{\text{CK} + \text{KB} + \text{BA}}{\text{V}'}$$

$$t_1 = \frac{\text{FE}}{\text{V}'} + \frac{\text{ED} + \text{DB}}{\text{V}} + \frac{\text{BA}}{\text{V}'}\text{; and FE} = \text{CK}$$

$$\therefore t_1 - t = \frac{\text{ED} + \text{DB}}{\text{V}} - \frac{\text{KB}}{\text{V}'}.$$

Draw BM perpendicular on ED produced, then by the law of refraction—

$$\frac{\text{KB}}{\text{V}'} = \frac{\text{EM}}{\text{V}}$$

Also $\quad \text{MDB} = 180° - 2\phi$

$$\therefore t_1 - t = \frac{\text{BD} - \text{DM}}{\text{V}} = \frac{\text{BD}(1 + \cos 2\phi)}{\text{V}}$$

$$= \frac{\delta(1 + \cos 2\phi)}{\text{V} \cos\phi} = \frac{2\delta}{\text{V}}\cos\phi.$$

Also if λ be the wave length in air $\tau = \frac{\lambda}{\text{V}}$; and thus our condition for interference becomes

$$2\delta\cos\phi = n\lambda$$

$$\text{or } \delta = \frac{n\lambda\sec\phi}{2}.$$

In this investigation we have assumed that the two streams of light coming along BA by the two paths have the same intensity. This, however, cannot be the case always; one stream has suffered one reflection only, the other two refractions and a reflection. By each of these operations the light is altered, and thus, in general, the two streams will differ. Our result, however, is quite correct, as a further investigation will show.

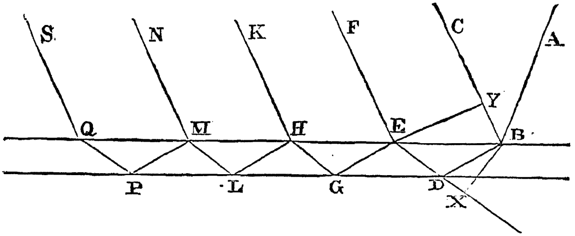
Let BQPD, fig. 69, be the thin film, and consider a ray of light AB incident at B; some is reflected along BC, the rest refracted along BD. Of this some is reflected along DE; on

incidence at E some is refracted out along E F, the rest reflected along E G, again reflected along G H, refracted along H K, reflected along H L, and so on.

And again, if A be the observer's eye, light would reach it which came from a distant object, as the sun, by all of the parallel paths S Q, N M, K H, F E, or C B, and the total illumination at B would be that due to all these different rays.

Moreover, the light coming along F E D B A must have left the source somewhat earlier than that coming along C B A. Let us suppose t'' earlier, that coming along K H G E D B A

FIG. 69.



will have left $2\,t''$ earlier, and so on; let T be the time at which the last wave started. We have to consider the effect of a series of waves which left the source at times $T + t''$, $T + 2\,t''$, $T + 3\,t''$, and so on. But this is not all; the light coming by the path C B A has been reflected at B, and its intensity will therefore have been altered somewhat, for some of the incident light will have been refracted into the film. If we call the amplitude of the incident light a, that of the reflected cannot be a. Let us suppose that to find it we have to multiply a by some quantity, say b, so that the amplitude is $a \times b$. At present we do not know what b is; it depends on the nature of the substance and the angle of incidence.

Again, the light coming by the path F E D B A has been refracted twice at E and B, and reflected at D. By each of

these operations the intensity of the light, and therefore the amplitude of the vibrations, is altered.

Let us suppose that we have a thin film of air between two plates of glass. Just at present we treat it as if our eye at D were in the glass. Moreover, suppose that when the light goes from glass to air we have to multiply the amplitude of the incident vibration by c to get that of the refracted vibration, and that when light goes from air to glass we multiply the incident vibration by e to get the reflected, and by f to get the refracted. The amplitude of the vibration along F E is a. For that along E D then we have $a\,c$; along D B, $a\,c\,e$; and, finally, along B A, $a\,c\,e\,f$. The amplitude of the light coming along K H G E D B A will be on emergence $a\,c\,e^3\,f$; for the next ray we shall have $a\,c\,e^5\,f$, and so on.

These, then, are the amplitudes of the vibrations which arrive at A at a certain moment, having left the source at times T, T + t, T + 2 t, &c.

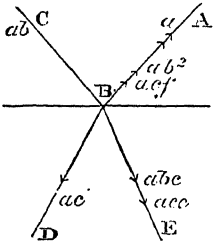
Thus the whole effect at A will be that due to the sum of a long series of terms arising from these different waves, and to find it we have to sum the series.

Each of the quantities, b, c, e, f, must be less than unity. and we can prove that they are connected together by the equations—

$$b + e = 0$$

$$cf = 1 - e^2 = 1 - b^2$$

FIG. 70.



For, let A B (fig. 70) be an incident ray in which the amplitude is a, it gives rise to a reflected ray B C, and a refracted one B D.

In the former of these the amplitude is $a\,b$, in the latter it is $a\,c$. We assume that if the light travelling along these two rays be reversed in direction, the two combined will produce the incident wave.

Now a ray of amplitude $a\,b$ along C B will produce a re-

flected ray $a b^2$ along B A, and a refracted ray $a b c$ along B E, B E being inclined to the surface at the same angle as B D. A ray $a c$ along C B will produce a reflected ray of amplitude $a c e$ along B E, and a refracted one of amplitude $a c f$ along B A.

Thus by reversing the two rays which do travel along B C and B D we should get a ray of amplitude $a b^2 + a c f$ along B A, and one of amplitude $a b c + a c e$ along B E. These two are by our supposition the same in effect as the original incident light, reversed in direction, but this consists of a ray of amplitude a along A B, and none along B E. Thus we get the equations

$$a b^2 + a c f = a$$
$$a c b + a c e = 0$$

or
$$b + e = 0 \quad c f = 1 - b^2.$$

The co-efficient of the first term in the series spoken of above was $a b$, of the next $a c e f$, of the third $a c e^3 f$, and so on. Thus the co-efficients are, substituting for e and $c f$, $a b, - a b (1 - b^2), - a b^3 (1 - b^2)$, and so on. Each of these has to be multiplied by a quantity depending on the time which has elapsed since the wave to which they respectively refer left the source, and the sum of the series thus formed gives the total effect at A.

The calculation of the sum of this series involves somewhat complicated mathematics. On performing the necessary operations we find that if I be the intensity of the light received by the eye

$$I = \frac{4 a^2 b^2 \sin^2 \frac{\pi V t}{\lambda}}{(1 - b^2)^2 + 4 b^2 \sin^2 \frac{\pi V t}{\lambda}}$$

t, it will be remembered, is the interval which elapses between the times at which two different waves which reach the eye simultaneously leave the source. This has already been found in the previous section, and we have from page

$$t = \frac{2 \delta \cos \phi}{V},$$

where δ, ϕ, and v have the same meaning and t is put for $t_1 - t$.

Thus the expression for the intensity is

$$\frac{4\, a^2 b^2 \sin^2 \dfrac{2\pi\delta\cos\phi}{\lambda}}{(1 - b^2)^2 + 4\, b^2 \sin^2 \dfrac{2\pi\delta\cos\phi}{\lambda}}.$$

This vanishes whenever

$$\frac{2\pi\delta\cos\phi}{\lambda} = n\pi\,;$$

that is whenever

$$\delta = \frac{n\lambda\sec\phi}{2}.$$

We may show that the intensity is a maximum when

$$\frac{2\pi\delta\cos\phi}{\lambda} = \frac{2n+1}{2}\pi\,;$$

that is whenever

$$\delta = \frac{(2n+1)\lambda\sec\phi}{4}.$$

Before discussing these results further, let us try to see the physical meaning of the first of the equations used to determine e and cf; it is $b + e = 0$, or $b = -e$.

When light travels from glass to air, the amplitude of the incident vibration being a, that of the reflected is $a\,b$; while if the light travels from air to glass the reflected vibration is $a\,e$. But $a\,e = -a\,b$.

Thus the alteration produced by the two reflections is the same in amount but opposite in sign, and this is the fact we have used above in our first investigation into the colours produced.

Again, a wave of amplitude a going from glass to air gives rise to two of amplitudes $a\,b$ and $a\,c$ respectively. The energy of a wave is proportional to the square of its amplitude, so that the amounts of energy in these three waves vary as a^2, $a^2 b^2$, and $a^2 c^2$ respectively, but if we suppose

that no energy is absorbed by the reflection, the energy in the reflected and refracted waves must be just equal to that in the incident. Thus we get

$$a^2 = a^2 b^2 + a^2 c^2, \text{ or } 1 = b^2 + c^2.$$

Again, if we consider in the same way light going from air to glass we should obtain

$$1 = e^2 + f^2,$$

but we have seen that $b + e = 0$,

$$\therefore c^2 = 1 - b^2 = 1 - e^2 = f^2,$$

or $$c = \pm f = \pm \sqrt{(1 - b^2)},$$

and $$cf = \pm (1 - b^2).$$

At present we have nothing to tell us whether we should take the positive or negative sign; our previous illustration with the balls may help us. In both cases the motion of the ball struck after impact must be in the same direction as that before impact of the striking ball.

Thus, reasoning from this analogy, c and f must have the same sign, and therefore

$$cf = 1 - b^2,$$

and this is the second equation we were discussing.

Let us now turn to the conditions at which we have arrived for the points of maximum and minimum illumination.

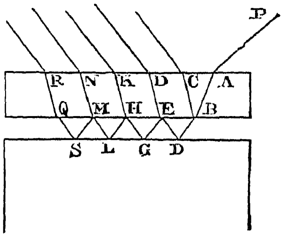
There will be darkness whenever the thickness of the film is $\frac{n\lambda}{2}$ sec ϕ, and maximum brightness when it is $\frac{(2n+1)\lambda \sec \phi}{4}$, ϕ being the angle which the light in the film makes with the normal to the surface.

Let us now suppose that our thin plate is the film of air between two plates of transparent material, as glass, and that a stream of white light from some distant source falls on the glass. ϕ is then the angle of incidence on the glass.

In considering the conditions for interference, we sup-

posed, for simplicity, the point at which the illumination was required to be in the glass. This, however, is not necessary. Let B A (fig. 71) be a ray in the glass refracted out along A P. Then if the illumination at A is nothing, there will be no ray travelling in the direction B A, and therefore no refracted ray along A P. The conditions for interference at P will be the same as those for interference at A. The eye of the observer may be anywhere in either of the lines B A or A P.

FIG. 71.



Let δ be the thickness of the film, and suppose at first it is uniform and that the observer is at some distance from it.

Then if

$$\delta = \frac{n}{2} \lambda \sec \phi$$

the film will appear black, if

$$\delta = \frac{2n + 1}{4} \lambda \sec \phi$$

it will seem bright.

But we have supposed white light to fall on the film, and we have already seen that white light consists of innumerable waves of different lengths. Thus we are not at liberty to suppose that λ has one definite value, and we must consider the effect of each value separately. For one value of λ—that is, for all the waves of one certain period—the first condition may hold, and they will be wanting in the emergent light. We have seen that the colour of a ray of light corresponds to the periods of the waves of which it is composed, so that if waves of a certain length are absent, the light of the corresponding colour will be absent too.

It may then happen that δ and ϕ, or the thickness of the film and the angle of incidence, are such that all light of

some definite colour, say red, is absent from the emergent beam, and the film would thus appear of a bluish-green hue.

By altering the angle of incidence of the light we could cause the absence of waves of a different length, and then the film would appear of a different colour.

These phenomena are easily seen in the vivid colours emitted by a thin film of oil or turpentine floating on water, or again in some of the ornamental iridescent glass which has become common recently.

Again, the colours may be made to vary by altering the thickness of the film. This is easily done with the thin film of air between a plate of glass and a thin slip of microscope glass which is pressed on to it by the fingers. If monochromatic light be used the plate will appear to be crossed by a series of bright and dark bands corresponding to the lines of equal thickness of the air film. On increasing the pressure so as to make the film thinner, the bands will broaden out, and since the thickness requisite to produce interference varies as the wave length, the bands will occupy different positions for different colours, being closer together for violet than for red light, so that if the incident light be white, the film will appear to be crossed by a series of rainbow-coloured bands. The experiment is easily performed, and for a source of monochromatic light a spirit lamp with a salted wick is sufficient.

Another method of obtaining these colours of thin plates was employed by Newton, who investigated them carefully and explained the appearances according to the emission theory. He used, instead of the thin plates of glass pressed between the fingers, a very flat convex lens. This rested on another flat convex lens, with a film of air of variable thickness between the two. The lenses touch at one point, and there the thickness of the air is of course nothing. The surface of the lens being spherical, the thickness of the air increases at the same rate along all lines on the plate drawn through this point, so that for all points in the plate

at the same distance from the point of contact the thickness is the same. The lines of equal thickness, therefore, are circles with the point of contact as centre, and since the colour of the emergent light at each point depends on the thickness of the film there, these circles of equal thickness will appear of the same colour, so that the lines of uniform colour, or isochromatic curves as they are called, will be circles with the point of contact as centre. This point being one of darkness for all wave lengths will appear black, and will thus be surrounded by a series of coloured rings. At any one point light of a definite wave length is quenched, the colour at that point depends on the other colours present in the incident beam.

These phenomena are described by Newton, 'Optics,' Book II. The lenses used were a plano-convex, the object-glass of a fourteen-foot telescope, and a double convex from a fifty-foot. The order in which the colours were observed by him was, reckoning from the centre outwards :—

Black, blue, white, yellow, red, violet, blue, green, yellow, red ; purple, blue, green, yellow, red ; green, red ; greenish-blue, red ; greenish-blue, pale red ; greenish-blue, reddish-white. This is generally referred to as Newton's scale of colours.

We can easily find a relation between the wave length of the light which is absent from the emergent beam at any point, and the distance of that point from the centre.

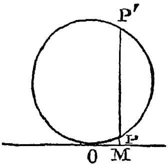
Let us suppose that the lower surface is plane, and consider a section by a plane passing through the point of contact of the lens and plane surface and perpendicular to that surface.

Let O (fig. 72) be the point of contact. The section of the lens is an arc of a circle. Let P be any point on it, and P M perpendicular to the plane surface. Let r be the radius of curvature of the lens, and let M P meet the circle of which the section of the lens forms part in P′. Then we know that $MO^2 = MP \cdot MP'$, and if M is not far away from O′ com-

pared with the radius of the lens, M P′ is very nearly equal to $2r$.

Thus $MP = \frac{MO^2}{2r}$ very nearly. But we have already

FIG. 72.



seen that if λ is the wave length of the light which at that point is quenched, and ϕ the angle of emergence from the lens into the film of air, then the point in question appears black if $PM = \frac{n\lambda}{2}\sec\phi$. That is, the radius of a black ring, for waves of length λ, is O M where

$$OM = \sqrt{n r \lambda \sec\phi}.$$

For a bright ring we shall have

$$OM = \sqrt{\frac{2n+1}{2} r\lambda \sec\phi}.$$

Thus the squares of the radii of the bright rings are in the ratio of the odd numbers 1, 3, 5, 7, &c., while the squares of the radii of the bright rays are in the ratio of 2 : 4 : 6, &c. This law was proved by Newton, 'Optics,' II., Obs. 5.

He found that the thicknesses of the air at the centre of his dark rings were $\frac{2}{178{,}000}$, $\frac{4}{178{,}000}$, &c., inches, the angle of incidence being about 4°, and from this we get for the wave length of the light quenched

$$\lambda = \frac{4}{178{,}000}\cos 4^\circ = \frac{1}{44{,}400}$$

inches, and this is equal to 5,720 tenth metres.

We shall see further on that this is the wave length of light in the most luminous part of the spectrum, that between the green and the yellow.

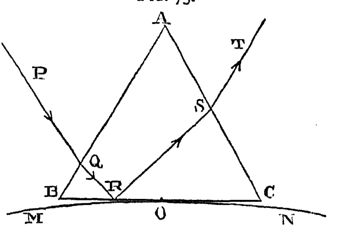
These measurements of Newton's were the first from which the wave length of light might have been deduced, though he himself did not use them for that purpose. We

see now how small the wave length is, at any rate in one part of the spectrum.

The difference in the thickness of the film for points on two consecutive dark bands is $\frac{\lambda}{2}\sec\phi$, and this increases as $\sec\phi$ increases. Thus by increasing the angle of emergence into the film we increase also the distance between the bands. The rings may be seen distinctly and their radii measured by reflecting homogeneous light normally on to the lenses, and viewing their appearance through a microscope fitted with a micrometer eye-piece.

If we use a lens with almost parallel faces the angle of emergence into the film is nearly equal to the angle of incidence on the first face of the lens. In order, then, that this angle of emergence may be large, it is necessary that the angle of incidence on the first face be large also, but in this case very little light would enter the lens, nearly all would be reflected from the first surface. We may avoid this by using for our upper medium a prism with one of its faces slightly curved, or if we prefer it a prism with flat faces, the lower medium having at the same time a curved surface, for in this case the angle of incidence on the second face of the prism can be made large, and with it the angle of emergence on the air film, without making the angle of incidence on the first face large also. A figure will render this clear. A B C (fig. 73) is the prism, B C being the face in contact with the lens M O N. Light is incident on the face A B at a small angle in the direction P Q say, it is then refracted along Q R and falls on the second surface B C at R, at a sufficiently large angle. Some of it is reflected along R S, and emerges from the third face of the prism A C. The rest is refracted

FIG. 73.



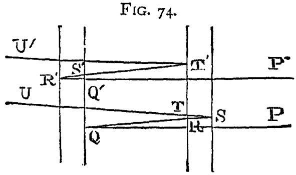
into the air film, and after reflection at the second surface of the film, and a second refraction at the surface B C, emerges again from A C. This arrangement of apparatus was also used by Newton.

Newton believed that he could explain by means of these colours of thin plates the colours of natural objects. He thought that the first layer or particle in a body behaved like a thin plate, some of the incident light penetrated them was reflected from their under surface, and on emergence interfered with the light reflected from the upper surface, thus producing colour. Brewster proved that this explanation failed by analysing with a prism the coloured light coming from various natural objects, and light of the same colour produced by a thin plate.

We proceed now to discuss two other phenomena connected with these coloured rings.

Brewster showed that in certain cases interference bands might be produced with a thick plate of air between two plates of a transparent refracting substance. For take two plates of such a substance as nearly as possible of equal thickness, and placed almost parallel to each other, and consider light coming from a luminous source, so as to fall nearly perpendicularly on the plates; the light reaching the eye placed behind the plates may have been transmitted directly through both, or it may have been reflected at two or more of the four surfaces of the plates, and a number of images of the source will be formed. Consider two rays whose paths are P Q R S T U and P′ Q′ R′ S′ T′ U′ (fig. 74). The first suffers reflexion at Q at the first surface of the second plate and again at S at the first surface of the first plate. The other is reflected at R′ on the second surface of the second plate, and again at T at the second surface of the second. These two rays will help to form an image of the

FIG. 74.



source on one side of and distinct from the principal image formed by light which passes straight through each plate, or is reflected at the two surfaces of the same plate. If the two plates were of equal thickness and exactly parallel, the length of path traversed by each ray would be the same; and since in each ray there has been one reflexion when the light was travelling from glass to air, and one when it was travelling from air to glass, the change of phase produced by reflexion has been the same for both. Thus the two emergent rays would be in the same phase, but since the plates are not exactly parallel the path of one ray will differ slightly from that of the other, and the phase on emergent will be slightly different in the two. They will therefore be in a condition to interfere and produce coloured bands. Brewster in his experiments used a tube blackened on the inside; one end was closed and a small hole made for the incident light in it, the plates of glass were inserted at the other end nearly perpendicular to the length of the tube and inclined at a small angle to each other. The bands formed are parallel to the line of intersection of the plates.

The plates used by Brewster were about ·25 centimetres thick, and when inclined at an angle of $1° 11'$ to each other, the breadth of each fringe was $26' 50''$. The breadth of a fringe was found to vary inversely as the angle between the plates.

These phenomena are known as the interferences of thick plates. They are distinct from the coloured rings of thick plates discovered by Newton ('Optics,' Book II., part IV.), and were described by him as follows:—

'Obs. I. The sun shining into my darkened chamber through a hole one-third of an inch wide, I let the intromitted beam of light fall perpendicularly upon a glass speculum (mirror), ground concave on one side and convex on the other to a sphere of five feet and eleven inches radius and quicksilvered over on the convex side, and holding a white opaque chart or quire of paper at the centre of

the spheres to which the speculum was ground, that is at a distance of five feet and eleven inches from the speculum, in such a manner that the beam of light might pass through a little hole in the middle of the chart to the speculum, and be thence reflected back to the same hole. I observed upon the chart four or five concentric irises or rings of colours like rainbows encompassing the hole, much after the manner that those which in the fourth and following observations of this second book appeared between the object-glasses encompassing the black spot, but yet larger and fainter than those.'

If the distance of the chart from the speculum was much greater or much less than six feet, the rings became dilute and vanished.

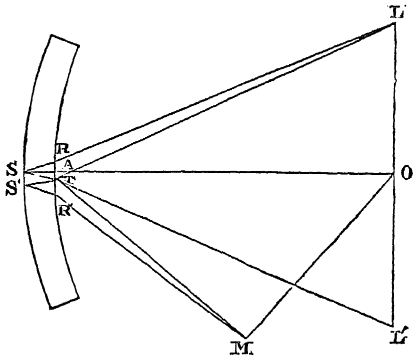
The squares of the radii of the bright rings were found to be in the ratios 0, 2, 4, 6, &c., while those of the dark rings were in the ratios 1 : 3 : 5 : 7, &c.

The squares of the radii of the various rings was found to vary inversely as the thickness of the mirror used. As in the case of thin plates, the radii of the rings were less for violet light than for red, and the ratio of the radius of a violet ring to the radius of a corresponding red ring was found to be the same as for the thin plates. Newton endeavoured to explain these phenomena on the emission theory. It is to Thomas Young again to whom we have to turn for an explanation on the undulatory theory, and his work was extended by Sir J. Herschel and completed by Stokes and Schäfli.

No surface, however well polished, reflects or refracts regularly all the light which falls on it; some small portion of the incident light is 'scattered' by the surface, that is, is diffused generally in all directions; the amount of light so scattered depends on the polish of the surface, and it is to this scattering by the surface that the rings in question are due. They may always be made more distinct by slightly dimming the surface of the mirror. This is most easily effected by brushing it over with a very weak mixture of milk and water, and leaving it to dry.

In Newton's experiments quoted above the incident light fell normally on the mirror, the origin of light being in the axis. He investigated the phenomena when this was no longer the case, and in our discussion, therefore, we shall suppose that the light comes from a point L not very far from the axis, and lying in a plane passing through the centre of the mirror and perpendicular to the axis. Let O (fig. 75) be the centre and O A the axis. Join O L and produce it to L′,

FIG. 75.



making O L′ = L O; then an image of L will be formed by reflexion in the mirror at L′.

Consider a plane passing through L L′ perpendicular to O A, and let M be any point in it.

Let L R S T L′ be a ray of light refracted at R, reflected at S, and refracted out at T. Owing to the want of perfect polish of the surface, part of the light is scattered on emergence at T, and of this scattered light some portion reaches M.

Consider now another ray incident at this point T; some portion of this is scattered on incidence and part is refracted into the glass of the mirror along T S′. This part is reflected regularly at S′ along S′ R′, and after regular refraction at R′ reaches M. Thus light scattered at the same point T reaches

M by two different paths. One portion of this light was scattered on emergence, the other on incidence.

These two paths L R S T M, L T S′ R′ M, will in general differ slightly, and so these two portions of light will be in a condition to interfere, and coloured bands will be produced on the screen through O.

Of course light scattered at other points of the surface besides T may reach the point M. Professor Stokes, however, has pointed out that two rays of scattered light cannot interfere unless the scattering has taken place at the same point of the reflecting surface. For by the act of reflexion a difference of phase is produced, and this depending, as it probably does, on the state of the polish of the surface, is different for different points.

We proceed now to calculate the difference in path of the two rays, supposing that the angle of incidence is so small that we may neglect its cube and higher power, so that if ϕ be the angle of incidence we have $\sin\phi = \phi = \tan\phi$ and $\cos\phi = 1 - \frac{\phi^2}{2}$.

Let $\mathrm{OL} = u$; $\mathrm{OM} = v$; $\mathrm{OA} = \mathrm{OR} = \mathrm{OT} = \mathrm{OR}' = r$, and let μ be the refractive index; the angle L O A is a right angle, and since R is not far from A, the angle L O R is very nearly a right angle. Also $\mathrm{LRO} = \phi = \frac{\mathrm{LO}}{\mathrm{OR}}$ (approximately) $= \frac{u}{r}$,

$$\therefore \mathrm{RL} = \mathrm{RO}\sec\phi = r\left(1 + \frac{\phi^2}{2}\right) = r\left(1 + \frac{u^2}{2r^2}\right).$$

Again, since

$$\sin\phi = \mu\sin\phi'; \quad \phi' = \frac{\phi}{\mu} = \frac{\mathrm{LO}}{\mu\,\mathrm{LA}} = \frac{u}{\mu r}.$$

Let t be the thickness of the mirror, S is clearly on O A produced, and $\mathrm{AS} = t$, also S A R is a right angle very nearly, and $\mathrm{ASR} = \phi'$,

$$\therefore \mathrm{SR} = \mathrm{SA}\sec\phi' = t\left(1 + \frac{\phi'^2}{2}\right) = t\left(1 + \frac{u^2}{2\mu^2 r^2}\right) = \mathrm{ST}.$$

Similarly T O M is very nearly a right angle,

$$\therefore \text{T M} = \sqrt{(\text{T O}^2 + \text{O M}^2)} \text{ (approximately)}$$

$$= \sqrt{r^2 + v^2} = r\left(1 + \frac{v^2}{2\,r^2}\right),$$

if we neglect $\frac{v^4}{r^4}$ and higher powers.

Now the length of path of the ray scattered at emergence when measured in air is

$$\text{L R} + \mu\,(\text{S R} + \text{S T}) + \text{T M},$$

and, substituting, this becomes

$$r + \frac{u^2}{2\,r} + 2\,\mu\,t + \frac{t\,u^2}{\mu\,r^2} + r + \frac{v^2}{2\,r}$$

$$= 2\,r + 2\,\mu\,t + \frac{v^2 + u^2}{2\,r} + \frac{t\,u^2}{\mu\,r^2}.$$

We turn now to the ray scattered at incidence. We shall find the length of its path most easily from the consideration that if M be taken as the source of light and L as the point at which we require the illumination, M R′ S′ T L becomes a ray reaching L after scattering at emergence at T.

To find the length of its path, then, we have only to interchange L and M in the above investigation, that is to interchange u and v in the above expression.

We get then equivalent path of L T S′ R′ M in air.

$$= 2\,r + 2\,\mu\,t + \frac{v^2 + u^2}{2\,r} + \frac{t\,v^2}{\mu\,r^2}.$$

The difference in length of path is

$$\pm\, t\,\frac{(v^2 - u^2)}{\mu\,r^2}.$$

There will, therefore, be darkness on the screen if

$$\frac{t\,(v^2 - u^2)}{\mu\,r^2} = \pm\,\frac{2\,n + 1}{2}\,\lambda,$$

and maximum brightness if

$$\frac{t(v^2 - u^2)}{\mu r^2} = \pm n\lambda,$$

λ being the wave length of the light considered.

In the case considered first by Newton quoted above, L and O coincide, and therefore $u = 0$.

Thus for darkness

$$v^2 = \frac{(2n + 1)\mu r^2 \lambda}{2t},$$

and for brightness

$$v^2 = n\frac{\mu r^2 \lambda}{t}.$$

Thus the isochromatic curves are circles round O as centre. The squares of the radii of the bright rings are in the ratio 0 : 1 : 2 : 3, &c., or 0 : 2 : 4 : 6, &c.

Those of the dark rings are in the ratio

$$\frac{1}{2} : \frac{3}{2} : \frac{5}{2} : \text{ or } 1 : 3 : 5,$$

&c. ; also the squares of the radii of the various rings vary inversely as t, the thickness of the mirror; and finally the squares of the radii vary as λ, and this we found was the case in the coloured rings of thin plates; these agree with Newton's observations.

Turning now to the case where u is not zero, we have

$$v^2 = u^2 \pm \frac{(2n + 1)\mu r^2 \lambda}{2t}$$

for darkness, and

$$v^2 = u^2 \pm \frac{n\mu r^2 \lambda}{t}$$

for brightness. Thus the isochromatic curves in this case also are circles with O as centre; moreover, putting $u = v$, we see that $v = u$ is a bright circle for all wave lengths, so that if white light be used, this circle, which passes through the luminous point L and its image L′, is white. Suppose we wish

to determine the illumination at o, the centre of the mirror, we must put $v = 0$. If we use monochromatic light, the central point will be dark if

$$u^2 \pm \frac{(2n+1)\mu r^2 \lambda}{2t} = 0,$$

that is if $\frac{2u^2 t}{\mu r^2 \lambda}$ is an odd whole number, and bright if

$$u^2 \pm \frac{n \mu r^2 \lambda}{t} = 0.$$

that is if $\frac{2u^2 t}{\mu r^2 \lambda}$ is an even whole number.

Thus in general, white light being used, the central point is coloured, and its colour changes as the distance of the luminous point from the centre of the mirror is varied. (Newton, 'Optics,' II., Obs. 10.) This central point is surrounded by a series of coloured circles. A white circle passes through the luminous point, and at the end of the diameter of this circle opposite to the luminous point there is an image of that point formed by regular reflexion in the mirror. This white circle is surrounded by another series of coloured rings. The rings get closer and closer together the further we go from the centre, for we can show that the area between any two consecutive rings is constant. Thus at a short distance from the centre the appearances become all confused.

The rings can be seen with a plane or convex mirror, though they are less bright. If the mirror be convex the incident pencil of rays must be a convergent one in order that the reflected rays may form a real image of the source.

If the face of the mirror is dimmed in any manner it is necessary to take care that the small particles deposited for that purpose are not regular in form, as lycopodium seeds, for in this case the phenomena are masked by the rings formed by diffraction by a great number of equal small particles.

The Duke de Chaulnes obtained these rings by using

a concave metal mirror in front of which he placed a thin plate of glass or mica dimmed with milk and water. The surface of the glass acted like the front surface of the glass mirror described previously. The air between the glass and the metallic mirror takes the place of the glass of the mirror, and the metallic reflector plays the same part as the back surface of the mirror.

Before leaving this part of our subject we may refer to the colours of mixed plates discovered by Dr. Young, 'when looking at a candle through two pieces of plate glass with a little moisture between them.' According to Dr. Brewster, in order 'to see the phenomena to advantage it is only necessary to rub up a little froth of soap and water almost dry between two plane glasses and hold them between the eye and a source of light,' or to spread a little white of egg between the glasses, drying it before the fire to remove the excess of moisture.

According to Young, the colours are due to the interference of rays which in the space between the glasses traverse different media, and this explanation has been developed by Verdet ('Traité d'Optique Physique,' Vol. I. Section 44).

CHAPTER VII.

DIFFRACTION.

WE turn now to a remarkable series of effects produced when a wave of light passes through a small aperture. The phenomena in question are known generally by the name of diffraction. We have already seen that sound and light resemble each other in being forms of energy propagated by wave motion. The analogy between the two appeared to fail when light and sound are allowed to pass through an opening. We hear the thunder even though we cannot see the lightning by which it is produced. Light, apparently

travels in straight lines, sound does not. Now we have already seen that the rectilinear propagation of light depends on the extreme smallness of the waves which produce it, compared with the other dimensions involved. An obstacle placed between our eye and a source of light prevents us from seeing the light because its size is great compared with the wave-length of light. Light travels in straight lines through a hole because that hole is large compared with the wave-length of light. If, then, we can arrange for sound to pass through an aperture large compared with the wave-length of the sound, we may expect that it too will travel in straight lines; and conversely, if we can arrange for light to pass through an aperture which is not large compared with the wave-length of light, we may expect that it will cease to appear to travel in straight lines. We can obtain a sound shadow and we can make light bend round a corner as sound in general seems to do. For the former we require an aperture large compared with the wave-length of sound, and no obstacle near to reflect the sound back after it has passed the aperture, and so mar the effect. For the latter we need an aperture not large compared with the wave-length of light. Some few years since a powder hulk exploded on the river Mersey. Just opposite the spot there is an opening of some size in the high ground which forms the watershed between the Mersey and the Dee. The noise of the explosion was heard through this opening for many miles, and great damage was done. Places quite close to the hulk, but behind the low hills through which the opening passes, were completely protected, the noise was hardly heard, and no damage to glass and such like happened. The opening was large compared with the wave-length of the sound.

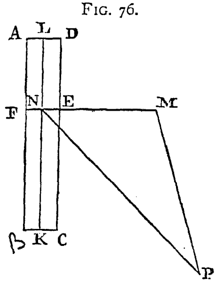
A very well marked sound shadow is often formed by a mountain ridge. If, for example, there is a stream running at the foot of the ridge the rush of the water is heard so long as you can see the stream. On descending sometimes

only a few inches below the ridge to a position from which the stream cannot be seen, the noise, too, will appear to stop abruptly. The change from the full rush of the waters to almost complete silence is most marked. A perfect sound shadow is formed.

This is very marked along a ridge leading from High White Sides to Grisdale Pike in the Lake District. There is a stream running between this ridge and Grassmoor from which a most perfect sound shadow can be obtained.

At present, however, we are concerned with the passage of light through a small opening. We shall suppose for simplicity's sake that the source of light is at a great distance compared with the size of the opening. We consider the effects on a screen placed behind the opening in a plane parallel to it. Let us suppose that our aperture is in the form of a rectangle A B C D (fig. 76), which is very long compared with its breadth. Let P be a point in the screen at which we wish to consider the effect. Draw P M perpendicular to the plane of the aperture, and M E F in that plane perpendicular to A B and C D. We shall suppose the distance of P so great compared with the size of the aperture that we may treat lines from P to any point in E F as if they were parallel. We shall see afterwards how this condition may be realised in practice. Draw L K parallel to A B in the plane of the aperture, and let M N meet E F in N. Join P N, P N is at right angles to L K.

FIG. 76.



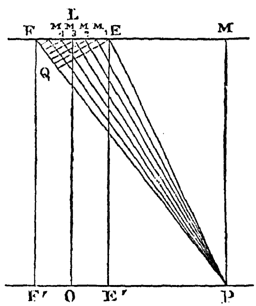
We have seen already that if N be not too near the end of the line L K, the effect of a linear wave such as L K in illuminating P will be confined to a small element of that wave near N. The portion of our plane wave which passes the aperture is made up of a number of these linear waves like L K, and the effective part of each of these linear waves

in illuminating P is confined to a small element in the neighbourhood of the point in which the wave considered cuts F E. Thus the part of the aperture which is effective in illuminating P is a small portion near to the line F E.

We have, therefore, to consider the effect at P of a narrow strip of our aperture along the line F E.

Let the plane of the paper (fig. 77) represent the plane F E M P of fig. 76. We have to consider the effect at P of a wave front F E. Join P E, P F; F E is by assumption so short that we may consider P E and P F as parallel. With P as centre and P E, $\text{P E} + \frac{\lambda}{2}$, $\text{P E} + \lambda$, $\text{P E} + \frac{3\lambda}{2}$, &c., as radii describe a series of circles cutting F E in E, M_1, M_2, M_3, &c., and let the point of intersection nearest F be M_n. Let O be the point in which a line from the middle point of E F perpendicular to the screen meets it. Let E E′, F F′, be perpendicular to E F, and meet the screen in E′ and F′, and let P be further from O than E′ or F′. Then since P E and P F may be considered parallel, $\text{E M}_1 = \text{M}_1\text{M}_2 = \text{M}_2\text{M}_3 =$, &c.

FIG. 77.



The amount of light sent to P by each of the elements E M_1, M_1M_2, &c., is the same, but since $\text{P M}_1 - \text{P E} = \frac{\lambda}{2}$, &c., the light from any element is opposite in phase to that from the preceding one. The effect, therefore, of alternate elements is equal and opposite.

If, then, we suppose n to be even and M_n to coincide with F, there will be an even number of these equal elements in F E, and the total effect at P will be zero, and in this case $\text{P F} - \text{P E} = m\lambda$ where $2m = n$. If n is odd and M_n coincides with F, there will be an odd number of these equal elements, and the effect at P will be that due to one of

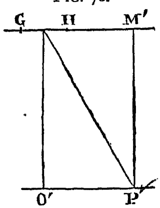
them alone, and in this case $PF - PE = \frac{2m+1}{2}\lambda$, where $2m+1 = n$. The effect, however, is not a maximum at this point, but will be so, as further investigation shows, somewhere not far off.

If M_n does not coincide with F we must consider separately the cases when n is even and n is odd.

If n be even, the effect of the portion EM_n is zero, the effect at P is that due to the element FM_n. If n be odd, the effect of EM_{n-1} is zero, the effect at P is that due to FM_{n-1}, and this is the same as the effect of $M_{n-1}M_n$ minus the effect of M_nF. Thus there are a series of points along the line OP at which the intensity is zero; these are given by $PF - PE = m\lambda$.

Between two consecutive such points the intensity gradually increases, reaching a maximum somewhere near the points for which $PF - PE = \frac{2m+1}{2}\lambda$, and then decreases again to zero. If we consider any other section of our aperture as GH (fig. 78), also perpendicular to its length, and let a plane through GH perpendicular to the aperture meet the screen in the line O′P′, GH is effective in illuminating this line on the screen. There will be a series of bright and dark points on this line at the same distance from O′ as the bright and dark points on the line OP are from O, so that considering the whole aperture we see that our screen will be crossed by a series of light and dark bands parallel to the length of the aperture.

FIG. 78.



We proceed to find the distance of one of these dark bands from the point O. Let $2b$ be the breadth of the slit, and let the circle through E cut PF in Q, fig. 77. Then $FQ = PF - PE = m\lambda$. Since FE is small, FQE is very approximately a right angled triangle with a right angle at Q. Let L be the middle point of FE and let the angle $OLP = \theta$.

Since lines from P to any point in F E may be considered as parallel,

$$\text{PLE} = \text{PFE} = 90^\circ - \text{QEF}.$$

$$\therefore \text{ angle QEF} = \theta \text{ and FQ} = \text{FE} \sin \theta.$$

$$\therefore m\lambda = 2b \sin \theta\,; \text{ so that} \sin \theta = \frac{m\lambda}{2b}.$$

Since we have supposed P to be without E′ F′, the value $m = 0$ must be excluded from this equation. Let c be the distance of the screen from the aperture, c is very great compared with b, let O P $= x$. Then

$$\sin \theta = \frac{\text{OP}}{\text{PL}} = \frac{x}{\sqrt{x^2 + c^2}} = \frac{x}{c}\left(1 - \frac{x^2}{2c^2}\right)$$

neglecting higher powers of $\frac{x}{c}$.

Hence, if we suppose that the distance x is so small compared with c that we may neglect $\frac{x^3}{c^3}$ and such terms, we have $\sin \theta = \frac{x}{c}$, and therefore $x = \frac{c m \lambda}{2b}$.

Thus to this approximation the dark bands are at equal distances, $\frac{c\lambda}{2b}$ apart, and the bright bands are nearly half way between them.

With a given slit and given distance between it and the screen, the distance between O and the centre of a given dark band varies as λ the wave length. Thus with white light the positions of the lines of greatest darkness are different for the various waves of which it is composed. The central band, the principal image of the slit, will be white. The positions of minimum brightness for violet light being nearer the centre than those for red light, the next bright band on either side will be violet coloured on the edge turned towards the centre, and red on the outside, then there will be a dark space and again another rainbow

coloured band. After a few such recurrences the red of one band will overlap the violet of the next, and the appearances will be confused and fade away.

The distance between two consecutive dark bands varies inversely as the breadth of the slit, so that the appearances on the screen are rendered more marked by decreasing the breadth of the slit. The interval between the bands also varies directly as the distance between the screen and the slit. By increasing this distance we increase the interval, but at the same time we diminish the intensity of the light, and that in a greater ratio.

The formulæ obtained above are sufficient to give us the points of minimum brightness. They do not, however, tell us how the intensity of the light varies between those points, nor where exactly the points of maximum brightness lie. We will therefore consider the question again more fully. At any moment the displacement of an ether particle is the same for all points of the aperture. We require, then, to find the effect at P of a given displacement at all points along the line EF. Now it is quite clear that if P, Q are any two points, the effect at Q produced by a displacement at P is the same as the effect which would be produced at P by the same displacement at the point Q.

Let us suppose, then, a certain displacement ξ is produced at P, and find its effect at a given moment at any point Q in the aperture. This will be exactly the same as the effect at P of the same displacement ξ produced at Q. If, then, we add together the effects produced at all points of the line EF by the displacement ξ at P, we shall get the effect at P due to the same displacement ξ at all points of the line EF.

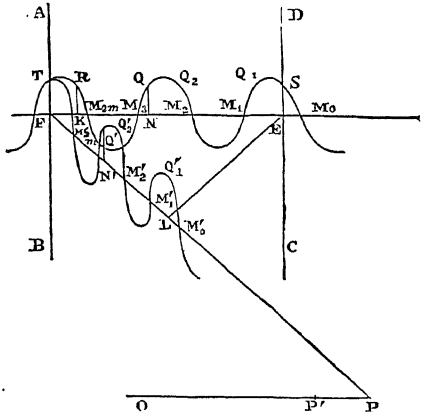
Let us suppose that the displacement at P takes place in the direction of the length of the slit, and draw the displacement curve for the moment considered along the line PF, fig. 79. This curve lies in a plane through PF and the length of the slit.

From each point of the curve draw a perpendicular to

this plane to meet the plane of the slit. The points of intersection will lie on a curve line $Q_1\,Q_2\,Q_3$, &c.

Let $Q_1'\,M_1'\,Q_2'\,M_2'$, &c., be the displacement curve, $Q_1\,Q_1'$, $M_1\,M_1'$, &c., being perpendicular to the plane P F A. Let Q N be perpendicular to E F, $Q'\,N'$ to P F, $N\,N'$ being perpendicular to the plane P F A. Then $Q\,N = Q'\,N'$. $Q'\,N'$ is the displacement produced at N' at the moment considered by a displacement ξ at P. Since P is by hypothesis so distant that P N is parallel to P F and the angle $P\,N'\,N$

FIG. 79.



is a right angle $P\,N = P\,N'$, and the same displacement is produced at each moment at N and N'. Thus Q N is the displacement produced at N by ξ at P, and this again is the effect produced at P by a displacement ξ at N. Now let us suppose we consider a very small bit κ of the line E F just in the neighbourhood of N so small that we may consider the disturbance produced at P by each point of this small line κ to be the same and to be therefore Q N. The effect at P due to this element κ then is $\kappa \times Q\,N$, and the total effect due to the line E F is the sum of all these separate effects like $\kappa .\, Q\,N$. This we may denote by $\Sigma\{\kappa .\, Q\,N\}$. But

evidently $\Sigma\{\kappa . \text{QN}\}$ is equal to the whole area of the curve $Q_1 M_1 Q_2 M_2$, &c., between the points in which it cuts the sides AB and CD of the aperture, considering the parts of the area which lie above and below the line EF as of opposite sign.

In general neither E nor F will coincide with any of the points $M_1 M_2$, &c., in which the curve cuts EF. Starting from E let M_{2m} be the last even point of intersection before we come to F, and let M_0 be the first point in which the curve cuts FE produced beyond E. In M_{2m} F take a point K such that $KM_{2m} = M_0 E$, and let KR perpendicular to EF meet the curve in R and let CD meet it in S and AB in T. Since between M_0 and M_{2m} there is as much of the curve below as above EF, the whole area $M_0 Q_1 M_1 \dots M_{2m} = 0$, but the area $S E M_0 =$ area $K R M_{2m}$.

Thus area $E S Q_1 M_1 \dots M_{2m} R K = 0$, and the whole displacement at P is proportional to the area KRTF. If KRTF should vanish there will be no displacement at P.

Now from the definition of the displacement curve we have $M_0' M_{2m}' = m\lambda$, and since the angles $M_0 M_0' P$, $M_{2m} M'_{2m} P$ are both right angles, $M_0' M_{2m}' = M_0 M_{2m} \cos PFE$.

Let $PFE = 90° - \theta$ so that θ is the angle which a line drawn from P to any point of EF makes with the direction of the incident light $M_0 M_{2m} = \dfrac{m\lambda}{\sin\theta} =$ But $M_0 M_{2m} = EK$.

$\therefore EK = \dfrac{m\lambda}{\sin\theta}$, and the condition for darkness at P becomes as before, $2b$ being the breadth of the slit.

$$2b = \frac{m\lambda}{\sin\theta}.$$

We can by means of a theorem in trigonometry find the displacement at P when it is not zero. We require the area KRTF. Now we know that

$$Q' N' = \rho \sin\frac{2\pi}{\lambda}(vt - PN'),$$

v being the velocity of the light and $\rho \sin \frac{2\pi}{\lambda} vt$ the displacement at P at the moment considered. Let EL be perpendicular on PF and let PL $= d$. Then

$$\text{PN}' = \text{PL} + \text{LN}' = d + \text{ME} \sin \theta,$$

and

$$\text{QN} = \text{Q}'\text{N}' = \rho \sin \frac{2\pi}{\lambda} (vt - d - \text{NE} \sin \theta).$$

Now take a number of points $\text{N}_1 \text{N}_2 \text{N}_3$, &c., at distances κ, 2κ, 3κ, &c., from E, and suppose that we may treat QN as constant for each of the pieces κ between these points; let there be n such elements in the space FE so that $n\kappa = 2b$, if N lie in the rth such element $\text{NE} = r\kappa$.

We have seen that we require to find the sum of a series like $\Sigma\{\kappa . \text{QN}\}$, and this sum becomes

$$\text{S} = \Sigma_n \left\{ \kappa \rho \sin \frac{2\pi}{\lambda} (vt - d - r\kappa \sin \theta) \right\}$$

where Σ_n means that r is to have all integral values from 1 to n. This series is of the form

$$\sin \alpha + \sin (\alpha - \beta) + \sin (\alpha - 2\beta) + \dots + \sin (\alpha - \overline{n-1}\,\beta),$$

where

$$\alpha = \frac{2\pi}{\lambda} (vt - d - \kappa \sin \theta), \quad \beta = \frac{2\pi}{\lambda} \kappa \sin \theta.$$

But we know from trigonometry that

$$\sin \alpha + \sin \overline{\alpha - \beta} + \dots \sin (\alpha - \overline{n-1}\,\beta) = \frac{\sin \left(\alpha - \frac{\overline{n-1}\,\beta}{2} \right) \sin \frac{n\beta}{2}}{\sin \frac{\beta}{2}}$$

Thus the sum required is

$$\text{S} = \frac{\rho \kappa \sin \frac{2\pi}{\lambda} \left(vt - d - \frac{n+1}{2} \kappa \sin \theta \right) \sin \left(\frac{\pi}{\lambda} n \kappa \sin \theta \right)}{\sin \left(\frac{\pi}{\lambda} \kappa \sin \theta \right)}.$$

Remembering that $n\kappa = 2b$, and that κ is ultimately very small, n being very large, we have

$$s = \frac{\rho\lambda}{\pi \sin\theta} \cdot \frac{\dfrac{\pi\kappa\sin\theta}{\lambda}}{\sin\left(\dfrac{\pi\kappa\sin\theta}{\lambda}\right)} \cdot \sin\frac{2\pi}{\lambda}(vt - d - b\sin\theta).$$

$$\sin\frac{2\pi b\sin\theta}{\lambda},$$

and since κ is very small

$$\frac{\dfrac{\pi\kappa\sin}{\lambda}}{\sin\dfrac{\pi\kappa\sin\theta}{\lambda}} = 1,$$

so that

$$s = \frac{\rho\lambda}{\pi\sin\theta} \ldots \sin\frac{2\pi b\sin\theta}{\lambda} \sin\frac{2\pi}{\lambda}(vt - d - b\sin\theta).$$

Now $d + b\sin\theta$ is the distance between P and the middle point of EF. Let us call this distance f. Then since the intensity of the light at a distance f from the source varies as $\frac{1}{f^2}$, and since also the intensity varies as the square of the amplitude, we have for the amplitude of the vibration at P

$$\frac{\rho\lambda}{f\pi\sin\theta}\sin\frac{2\pi b\sin\theta}{\lambda},$$

or as it may be written

$$\frac{2b\rho}{f} \times \frac{\sin\dfrac{2\pi b\sin\theta}{\lambda}}{\dfrac{2\pi b\sin\theta}{\lambda}}$$

Moreover f will never differ much from c; we may, therefore, put c for f in the denominator, and remembering that the intensity is given by the square of the amplitude, we have if I be the intensity

$$I = \frac{4\,b^2\,\rho^2}{c^2}\left\{\frac{\sin\left(\frac{2\,\pi\,b\,\sin\theta}{\lambda}\right)}{\frac{2\,\pi\,b\,\sin\theta}{\lambda}}\right\}^2$$

Thus if $\frac{2\,\pi\,b\,\sin\,\theta}{\lambda}$ be put equal to ϕ, the intensity depends on the value of $\left\{\frac{\sin\phi}{\phi}\right\}^2$.

This is zero whenever $\phi = m\,\pi$; that is whenever

$$\sin\theta = \frac{m\,\lambda}{2\,b}$$

where m has any integral value excluding zero. When $\phi = 0$ $\frac{\sin\phi}{\phi} = 1$, and this is its greatest value. There will, therefore, be a point of maximum brightness when

$$\frac{2\,\pi\,b\sin\theta}{\lambda} = 0.$$

so that $\theta = 0$, that is directly opposite the hole. The other points of maximum brightness are given by the maximum values of $\frac{\sin\phi}{\phi}$, and these we may show depend on the values of ϕ which satisfy the equation $\tan\phi = \phi$.

These have been calculated by Schwerd, and are given in the following table, calling the values $\phi_0\,\phi_1\,\phi_2$, &c.

$$\phi_0 = 0.$$

$$\frac{\phi_1}{\pi} = 1{\cdot}4303. \qquad \frac{\phi_3}{\pi} = 3{\cdot}4709.$$

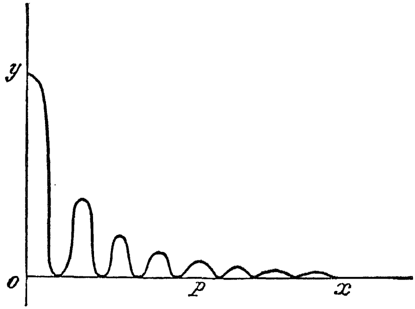
$$\frac{\phi_2}{\pi} = 2{\cdot}4590. \qquad \frac{\phi_4}{\pi} = 4{\cdot}4774.$$

If x be the distance of the point P from O, then we have as previously $\sin\theta = \frac{x}{c}$, and if at every point along the line O P we draw an ordinate equal to

$$\frac{4\,b^2\,\rho^2}{c^2}\left\{\frac{\sin\dfrac{2\,\pi\,b\,x}{c\,\lambda}}{\dfrac{2\,\pi\,b\,x}{c\,\lambda}}\right\}^2$$

the extremities of the ordinates will lie on a curve like that given in fig. 80, the ordinates themselves representing the intensity of the light at each point. The ordinates of course are zero at the points given by $x = \dfrac{m\,c\,\lambda}{2\,b}$, m having any integral value not zero, while the consecutive

FIG. 80.



maxima values lying between these minima decrease very rapidly, so that after two or three alternations any difference in the intensity of the light from point to point ceases to be appreciable.

Write the equation again in the form

$$\mathrm{I} = \frac{4\,b^2\rho^2}{c^2}\left(\frac{\sin\phi}{\phi}\right)^2$$

and remember that the maxima values of I are those for which the values of ϕ are given by the table. Then we see these values omitting the first do not differ greatly from the odd multiples of $\dfrac{\pi}{2}$, so that the value of $\sin\phi$ in each case

is nearly unity, and the consecutive maxima are approximately in the ratio of the numbers

$$1, \frac{4}{9\pi^2}, \frac{4}{25\pi^2}, \frac{4}{49\pi^2}, \text{\&c.}$$

We can now find the positions of the points of maximum brightness in terms of their distances from E and F. For they are given by

$$\frac{2\pi b \sin\theta}{\lambda} = \phi_1$$

or ϕ_2, or ϕ_3, &c., but

$$2b\sin\theta = \text{PF} - \text{PE},$$

and

$$\phi_1 = \pi \times 1{\cdot}430.$$
$$\phi_2 = \pi \times 2{\cdot}459, \text{\&c.}$$

Hence the points required are given by

$$\text{PF} - \text{PE} = \lambda \times 1{\cdot}430.$$
$$\text{or } \lambda \times 2{\cdot}459.$$
$$\text{or } \lambda \times 3{\cdot}470, \text{\&c.},$$

and these approximate to the consecutive odd multiples of $\frac{\lambda}{2}$, omitting $\frac{\lambda}{2}$ itself. Thus the effect is a maximum at points not far from those given by

$$\text{PF} - \text{PE} = \frac{2m+1}{2}\lambda,$$

where m has any integral value not zero.

A little consideration will now show us why the effect is not a maximum exactly at the points for which

$$\text{PF} - \text{PE} = \frac{2m+1}{2}\lambda.$$

At such a point the effect is that due to an element of FE equal in length to $\frac{\lambda}{2\sin\theta}$, at a point P′ for which the value of θ is θ' the effect is that due to some fractional part of an

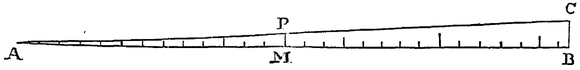
element $\frac{\lambda}{2 \sin \theta'}$. If P′ is not far from P but somewhat nearer D, this fraction will be only slightly less than unity, and since $\frac{\lambda}{2 \sin \theta'}$ will be greater than $\frac{\lambda}{2 \sin \theta}$, θ' being less than θ, it is possible for the effect of part of the element $\frac{\lambda}{2 \sin \theta'}$ to be greater than that of the whole element $\frac{\lambda}{2 \sin \theta}$, so that we might expect the points of maximum brightness to be somewhat nearer O than points given by the equation

$$\text{PF} - \text{PE} = \frac{2m + 1}{2} \lambda,$$

would be. This, of course, agrees with the result of exact calculation.

These diffraction images may be seen by viewing a gas flame edgeways or a candle flame through a narrow

FIG. 81.

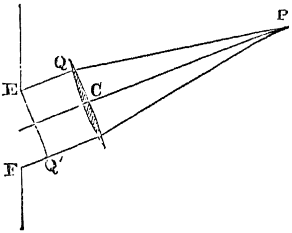


aperture. In this case the retina of the eye is the screen on which the bands are formed. We may make use of them to determine the wave-length of light if we can measure the breadth of the slit, the distance between the dark bands and the distance between the slit, and the plane in which the bands are formed. The optical bench already described can be used for the two latter measurements; to measure the breadth of the slit, a wedge of small angle is used. Let ABC (fig. 81) be the wedge; the edge AB is graduated and the distance BC is known, or it can be measured, the angle B being a right angle, the wedge is inserted in the slit and gently pushed in, BC being kept parallel to the breadth of the slit, until it will enter no further. The graduation M at which the slit rests when this is the case is noticed, and

the breadth of the slit is P M parallel to B C, and is given by the proportion P M : B C = A M : A B.

We have assumed in the preceding discussion that the incident light is a parallel pencil perpendicular to the slit, and that the screen is at a considerable distance from the slit. The first condition is secured by taking as our source of light a second slit at some distance in front of the first. If this distance is not very small this is in general sufficient, we may if necessary insert between the two slits a convex lens at a distance from the second slit equal to its own focal length. We know then that the light which reaches the slit used to produce the diffraction bands will be strictly parallel. To ensure the second condition place between the diffraction slit and the screen a second convex lens, and adjust this so that the distance between it and the screen is equal to its focal length.

FIG. 82.



Let E Q, F Q′ be two rays which after refraction through the lens meet on the screen at P. Draw E Q′ perpendicular to F Q′ and let C be the centre of the lens. Then since P is in the focal plane, and the rays E Q, F Q′ meet at P, they must be parallel, and since F Q′ is perpendicular to E Q′, E Q′ is the front of the incident wave which illuminates the point P, so that time from E to P = time from Q′ to P. Thus the difference in the length of the path of the two rays which reach P coming from E and F respectively is F Q′, and this is exactly what the difference would have been had the screen on which the effect is produced been so far off the slit that we might have treated P F and P E as parallel. In practice instead of observing the bands on a screen we receive the incident light on a lens or eye-piece, the screen on which the bands are formed will then be the focal plane of the eye-

piece, and this eye-piece and the convex lens together constitute an astronomical telescope. Thus to observe the bands under the conditions in which they have been described above we have simply to focus an astronomical telescope for parallel rays, and placing the slit before the object glass view a small distant source of light. This may be another slit seen through a collimating lens or some distant point as a star, or the image of the sun reflected in a small bright bead. The bands described above will then be seen in focus in the telescope.

We have hitherto supposed the sides of the slit to be parallel, and our lines of maximum and minimum intensity in this case are bands parallel to the slit. If we take the case in which the edges of the slit are inclined to each other at a small angle so that the breadth of the slit continually increases, the bands will no longer be parallel, and since the distance of any given band from the central line varies inversely as the breadth of the slit, they will be closest together where the slit is widest. We can show that the form of each band is that of a rectangular hyperbola with the line bisecting the angle of the slit and a line at right angles to it as asymptotes. This case is thus described by Newton ('Optics,' Book III. Obs. 8 and 10): 'I caused the edges of two knives to be ground truly straight, and pricking their points into a board so that the edges might look towards one another and meeting near their points contain a rectilinear angle, I fastened their handles together with pitch to make this angle invariable. The distance of the edges of the knives from one another at the distance of four inches from the angular point where the edges of the knives met was the eighth part of an inch, and therefore the angle contained by the edges was about 1° 54′. The knives thus fixed together I placed in a beam of the sun's light let into my darkened chamber through a hole, the forty-second part of an inch wide at the distance of 10 or 15 feet from the hole (Obs. 10). When the fringes of

the shadows of the knives fell perpendicularly upon a paper at a great distance from the knives, they were in the form of hyperbolas.'

Fraunhofer, of Munich, was the first to examine carefully the phenomena of diffraction through a single slit, and he arrived experimentally at the laws stated above. He also considered the case in which the aperture placed before the object glass was a small circular hole or a circular annulus. The most important parts of his researches are those which relate to the appearances presented by a large number of equal parallel rectangular slits—gratings he called them. To these we shall refer again shortly.

Sir J. W. Herschel has described in the 'Encyclopædia Metropolitana' (Art. *Light*) the phenomena produced by apertures or diaphragms of various shapes applied to mirrors and object glasses, and illustrations are there given of the appearances presented. But diffraction bands are formed at the boundary of the shadow of any object cast by a beam of light coming from one point. These were first observed by Father Grimaldi in 1665, and afterwards by Newton ('Optics' Book III.). Young was the first to attempt to explain these bands on the undulatory theory, attributing the fringes to the interference of the direct rays and rays reflected from the obstacle at grazing incidence.

The true theory of diffraction, however, was first given by Augustin Fresnel. He, to quote again from Verdet's Introduction to the works of Fresnel, 'like Young, recognised at once that the phenomena of shadows, which were generally considered to be the most serious difficulty in the wave theory, presented in the accessory phenomena of diffraction peculiarities which could not be explained on the emission theory, and he grasped the importance of an exact knowledge of these peculiarities.' In his country home at Mathieu he possessed no micrometer with which to measure the fringes; one was made out of thread and pieces of cardboard, some of his apparatus was the work of the village blacksmith, and with

this rough material he was able by care and patience to obtain results sufficiently accurate to establish some of the most remarkable laws of the phenomena.

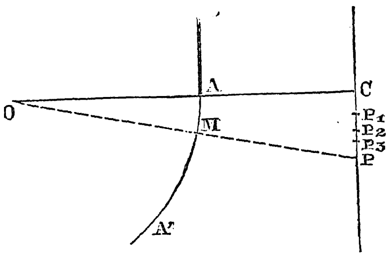
In his first two papers he studied the bands produced by placing a fine wire or thread in the path of the light, adopting the same hypothesis as Young as to the theoretical cause of the phenomena. 'In 1816 he presented to the French Academy a supplement to these first communications in which, for the first time, diffraction is referred to the effects of the interference of the vibrations sent from the different parts of a wave which is limited by the opaque obstacle.' At this time many influential members of the French Academy, among them Biot and Laplace, were warm supporters of the emission theory, and they proposed as the subject of the 'Grand Prix des Sciences Mathématiques de l'année 1819' the question of diffraction in the following terms:

(1) 'To determine by exact experiment all the effects of the diffraction of rays of light direct or reflected when they pass separately or simultaneously near the boundaries of one or more bodies of either limited or indefinite extent, having regard to the distance between these bodies as well as to the distance of the focus from which the rays proceed. To deduce from these experiments by mathematical induction the motion of the rays in the neighbourhood of the bodies.' Fresnel, at the urgent request of Arago and Ampère, was a candidate, and his memoir, presented in August 1818, was crowned by the Academy in the following year. In this paper, Fresnel, for the first time, obtained in the form of an integral an expression for the intensity of the light produced at a given point by a wave of definite form. If, for example, we wish to examine the effect of the passage of light through a small aperture at any point, Fresnel has shown us how to calculate the effect produced by each little element of the aperture, and then how to integrate or add together all these separate effects, so as to find the result at the point in ques-

tion. This result depends on the form and dimensions of the aperture, and on the position of the point with respect to it and the source of light.

At present, however, we must content ourselves with a general explanation of these fringes. Let us consider, then, a wave of light coming from a luminous point O. Let us suppose that in the path of the light from this point we place an obstacle indefinitely extended in one direction and bounded by a straight edge. Let A B, fig. 83, be a section of this by the plane of the paper, the straight edge in question

FIG. 83.



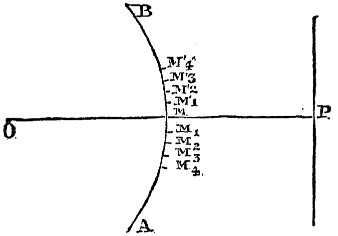
being perpendicular to A B, and the plane of the obstacle perpendicular to that of the paper.

Let O A cut the screen on which the bands are observed in C, C will be the edge of the geometrical shadow of A B, and if the light were simply propagated in straight lines, all the screen on one side of C would be dark, all on the other would be bright, the brightness decreasing gradually the further we travel from the luminous point O. Now observation had shown that this was not the case. If the source of light be simply a small bright point, the image of the sun suppose formed by a convex lens of short focus, the boundary of the geometrical shadow on the screen is not marked by a rapid transition from darkness to light. Within the shadow the light fades away rapidly but gradually, beyond it there are several transitions from light to darkness, so that

the edge of the shadow is bordered by a series of fringes. In homogeneous light these fringes are alternately light and dark; with sunlight, however, they are coloured.

A reference to the considerations by which we established in general the fact of the rectilinear propagation of light will help us to understand these various phenomena. Consider a spherical wave diverging from a point O, fig. 84. To find

FIG. 84.



its effect at P we join O P cutting the wave in M, and on the wave take points M_1 M_2, &c., such that

$$PM_1 - PM = PM_2 - PM_1 = \&c., = \frac{\lambda}{2}.$$

And the same for points M_1' M_2', &c., on the other side at M. Calling the effects produced by these separate elements m_1, m_2, m_3, &c., we saw that the whole effect at P was given by

$$2\,(m_1 - m_2 + m_3 - m_4 + \ldots .)$$

We found, moreover, that the terms in this series at first diminished rapidly in value and then became very nearly equal to each other, so that it is the first few terms of the series which determine its value. Let us call the effect at P when no obstacle is in the way of the light I, then

$$I = 2(m_1 - m_2 + m_3 - \ldots .)$$

Now let us suppose that owing to the interposition of the obstacle all the wave above M is stopped, so that P is on the edge of the geometrical shadow of the obstacle, and the intensity at P being I_0 suppose, we must have

I

Now let us suppose the edge of the obstacle moved towards M_1'. The light which reaches P from the first element $M M_1'$ being in the same phase as that from the half wave $M \ldots . M_n$ tends to increase the illumination at P, which will become, when the edge of the obstacle has reached M_1', equal to $I_0 + m_1'$. Moreover, since $I_0 = m_1' - m_2' + m_3'$, and each term is less than the preceding, I_0 is less than m_1', so that the intensity at P in this case is greater than $2 I_0$, or the value it would have if the obstacle were removed. As the obstacle is moved still further away, the light from the second element $M_1' M_2'$ is allowed to reach P, this being in the opposite phase to that from $M M_1'$, the effect at P decreases, again becoming a minimum when the obstacle arrives at M_2', when it will be $I_0 + m_1' - m_2'$, and this is less than $2 I_0$. As the obstacle is moved still further away the intensity at P again increases, and after reaching a maximum diminishes again; but since the differences between two consecutive terms of our series such as m_n and m_{n-1}, continually diminish, the differences between these consecutive maxima and minima diminish also, and after the obstacle has been moved to no very great distance from its original position the effect at P will have become constant. Thus the illumination at any point of a screen in the neighbourhood of the edge of the geometrical shadow of an obstacle placed in the way of the light depends on the position of the point with reference to the obstacle, and can be made to vary by varying slightly the position of the obstacle. But these slight variations in the relative position of the two can be produced equally by varying that of the point considered. Thus consider the point C in fig. 83. It is situated with reference to the obstacle A B exactly as P was in fig. 84 when the edge of the obstacle was at M. The intensity then of the light which it receives is $\frac{I}{2}$.

Let P be any point in the screen beyond the geometrical shadow. Join P O and with O as centre, and O A as radius

describe a sphere cutting P O in M and extending beyond M to A′. The points P M O are situated with reference to each other exactly in the same way as P M O are in fig. 84.

As before, the light received from the wave M A′ is $\frac{I}{2}$. The light received from M A depends on the number of the half period elements which it contains. As P moves away from C the amount of light it receives from the portion M A increases at first until P has reached a position such that $PA - PM = \frac{\lambda}{2}$, so that as we go from the edge of the geometrical shadow outwards, the brightness at first increases, and at the point P_1 for which $P_1A - P_1M = \frac{\lambda}{2}$, the intensity is $\frac{I}{2} + m_1$; m_1 being the intensity due to the first half-period element, and this is greater than I. If P be just beyond P_1 the part M A contains more than one half-period element, and the effects of two consecutive such elements being contrary, the intensity at P is less than that at P_1, and decreases as we move onwards along the line C P. This continues until M A contains two half-period elements, so that the intensity in this case is $\frac{I}{2} + m_1 - m_2$, which is less than I, and if P_2 be the position of P then we shall have

$$P_2A - P_2M_2 = \lambda.$$

From this point onwards the intensity increases again until a point P_3 is reached, for which $P_3A - P_3M_3 = \frac{3\lambda}{2}$, when it is

$$\frac{I}{2} + m_1 - m_2 + m_3.$$

It then diminishes; and so on in succession. Thus we should expect a series of points of maximum and minimum intensity in the line on the screen, the intensity at these points being alternately greater and less than it would be if the obstacle were not there. Thus our screen in the neighbourhood of

the geometrical shadow will be crossed by a series of bands alternately brighter and darker than the rest of the field.

If we wish to see what happens within the geometrical shadow of the obstacle, we must turn again to fig. 84. The intensity of P when the edge of the obstacle is at M is, as we have seen, $\frac{I}{2}$ and the phase of the disturbance there is the same as that of the effect sent by the first half-period element MM_1.

Let us now suppose the obstacle moved downwards, so as to intercept part of the light from this first element. We cannot in this case apply the reasoning we have just used, and argue that the disturbance first decreases as the light from the first element is intercepted, to increase again as the obstacle moves over the second element, and so on alternately, for when the edge of the obstacle is at M the phase at P is that due to the first half-period element; as the obstacle moves towards M_1 that portion of the wave which determined the phase is obstructed; not merely is the amplitude of the disturbance at P changed, but its phase is altered. When, as above, the obstacle is raised, the phase at P remains the same, the amplitude only is altered.

The part of the wave which is most effective at P is that which passes close to the edge of the obstacle and for which the time of passage is a minimum, and it is this part of the wave on which the resultant phase mainly depends. To estimate the effect at P we must divide the front into half-period elements beginning from the edge of the obstacle; when P is a little way within the shadow the sizes of these elements rapidly decrease, and, as before, the resultant effect is approximately half that due to the first element. As P moves further into the shadow the size of this first element gets less and less, and therefore the effect at P as it moves inwards gradually diminishes, there are no fringes formed within the shadow, and when P is a short distance within the intensity is zero.

Thus, turning again to fig. 83, as P moves to the right from C within the geometrical shadow, the intensity gradually decreases from the value $\frac{I}{2}$ to zero.

We have seen that the points of maximum brightness, P_1, P_3, P_5, &c., are given by the formula

$$PA - PM = \frac{(2n + 1)\lambda}{2},$$

M being the point in which OP cuts the spherical wave through A. Those of minimum brightness are given by $PA - PM = n\lambda$.

We proceed to find the distance between the edge of the geometrical shadow, C, and any band. Let this distance, CP, be x, let $OM = a$,

$$AC = b$$

$$AP = \sqrt{x^2 + b^2} = b + \frac{x^2}{2b} \text{ approximately,}$$

since x is very small compared with b;

$$PO = \sqrt{(a + b)^2 + x^2}$$
$$= a + b + \frac{x^2}{2(a + b)}.$$

to the same approximation.

But $OM = a$,

$$\therefore MP = b + \frac{x^2}{2(a + b)}$$

Thus

$$PA - PM = \frac{x^2}{2}\left(\frac{1}{b} - \frac{1}{a + b}\right) = \frac{a x^2}{2b(a + b)}.$$

Thus the distances from the edge of the shadow of the bands of maximum brightness are given by

$$x = \sqrt{\frac{b(a + b)}{a}(2n + 1)\lambda},$$

while the positions of the bands of minimum brightness are given by

$$x = \sqrt{\frac{b\,(a+b)}{a}\,2\,n\,\lambda}.$$

The positions of the points of maximum and minimum brightness in this and the succeeding problems are given more accurately by Dr. Schuster's construction, according to which we are to put for maximum brightness

$$\text{PA} - \text{PM} = \frac{2\,n + 1}{2}\,\lambda - \frac{\lambda}{8},$$

and for minimum brightness

$$\text{PA} - \text{PM} = n\,\lambda - \frac{\lambda}{8}.$$

We thus get for the light bands

$$x = \sqrt{\frac{b\,(a+b)}{a}\,(2\,n + 1 - \tfrac{1}{4})\,\lambda},$$

and for the dark ones

$$x = \sqrt{\frac{b\,(a+b)}{a}\,(2\,n - \tfrac{1}{4})\,\lambda}.$$

When b is varied while a remains constant, so that the position of the screen, but not of the source of light or the obstacle, is altered, the value of x is that of the ordinate of an hyperbola whose abscissa is b. Thus the successive points on the same fringe belong to an hyperbola whose foci are the edge of the obstacle and the luminous point. This was one of the earliest laws verified by Fresnel in his first 'Memoir on Diffraction.' This law follows, of course, directly from the equation

$$\text{PM} - \text{PA} = \frac{2\,n + 1}{2}\,\lambda.$$

For, add to each side OM, then

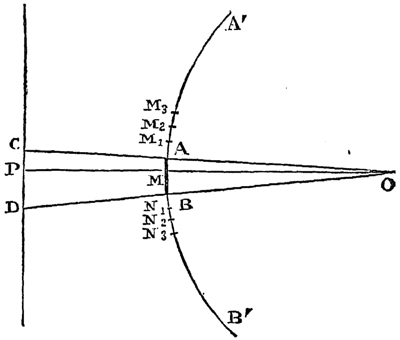
$$\begin{aligned}\text{PO} - \text{PA} &= \frac{2\,n + 1}{2}\,\lambda + \text{OM} \\ &= a \text{ constant},\end{aligned}$$

and P lies on an hyperbola, with O and A as foci:

Let us now consider the case in which the obstacle is a narrow opaque body, such as a fine wire or hair. Let A B (fig. 85) be a section of it, O being, as before, the luminous point, and let C D be the geometrical shadow of A B. Let P be any point in C D, join O P and let it cut A B in M, and suppose that A B is so small that parts of only a few half-period elements lie within M A or M B.

Join P A and describe through A with O as centre a circle, A A′. Divide this in M_1, M_2, M_3, &c., into half-period

FIG. 85.



elements, beginning from A, and let m_1, m_2, &c., be the effects due to each of these elements at P.

The total effect due to A A′ is $m_1 - m_2 + m_3 \quad . \quad . \quad .$

But since we suppose that A M only contains two or three half-period elements, we should only need to put two or three terms to this series to obtain the effect of the wave A′ M. But we know in this case that the terms of the complete series decrease rapidly at first, and since our series begins at the third or fourth term of the complete series, it thus must decrease rapidly at first, so that just as we proved the effect of a given wave at a given point to be due to the disturbance which the pole of the point would transmit to it, so here the effect sent by the wave A A′ to P depends

mainly on the disturbance coming from A M_1, the first element of this wave. Turning now to the effect of the wave B B′ from the other side of the obstacle, we see, in the same manner, that the phase of the effect which it sends to P will be the same as that of the first element, B N_1, and if we may suppose that the amplitude of the disturbance arising from the wave A A′ does not differ much from that which arises from B B′, the resultant effect at P will depend on the difference between A P and B P. The appearances on the screen will be exactly those which would arise from two equal and similar sources of light at A and B respectively. These have been already discussed at length. There will be, therefore, within the shadow of the screen, a series of bands, and as the position of a band depends on the wave-length of the light to which it is due, the bands will be coloured when white light is used. The central band, however, will be white, and if c be the breadth of the obstacle and a the distance between it and the screen, the distance of the bright bands from the centre of the shadow will be given by

$$x = \frac{a\, n\, \lambda}{c},$$

while that of the dark bands is given by

$$x = \frac{a\,(2\, n + 1)\,\lambda}{2\, c}.$$

These formulæ have been obtained and discussed in Chapter V. They were obtained by Fresnel and verified by numerous experiments described in his earlier papers; and it was this case of diffraction which led him to the arrangements for the fundamental interference experiment described in Chapter V.

We see, too, from the above reasoning why it is important that the obstacle should be narrow. It must, as viewed from the point P, contain only a few half period elements. We have seen that the space occupied by r such elements

is equal to $\sqrt{r a \lambda}$, provided r is sufficiently small for us to neglect terms like r^2, λ^2. In order, then, that we may have diffraction effects within the shadow, the breadth of the obstacle must be of the order $\sqrt{a \lambda}$. If this breadth is exceeded, then the consecutive half-period elements A M_1, $\text{M}_1 \text{M}_2$, &c., become very nearly equal to each other, the effect sent by anyone of them to P is almost equal and opposite to that sent by the next succeeding one; and so the total effect of both waves A A′, B B′ is zero.

Instead of removing only one or two terms from the series expressing the effect of the complete wave M A′, we have removed a considerable number of the first terms, which are, as we know, those on which the value of the series depends.

To give some idea of the magnitude of the quantities involved, let us suppose that the distance a between the screen and obstacle is 100 centimètres, and find the size of the first half-period elements for light of wave-length 49×10^{-6} centimètres. This corresponds to the green-yellow light in the spectrum.

The length of the first half-period element peing $\sqrt{a \lambda}$, is in this case $\sqrt{10^2 \times 49 \times 10^{-6}}$, or $\frac{7}{100}$ centimètres. The length of the first two half-period elements is

$$\sqrt{2 \times 10^2 \times 49 \times 10^{-6}}, \text{ or } \frac{\sqrt{98}}{100} \text{ cm.}$$

This is just under one millimètre. Thus, a wire of one millimètre thickness seen at this distance would intercept rather more than two half-period elements.

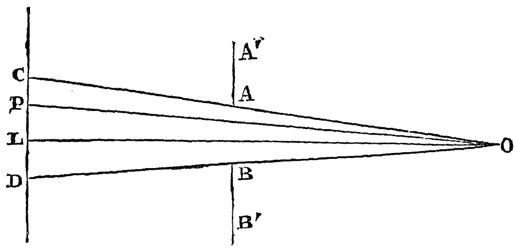
In addition to these fringes produced within the geometrical shadow, there are others beyond the limit of that shadow whose positions will be determined by the same formula.

We have already considered the appearances produced by light passing through a small aperture on a screen at some

distance from it. The case of a screen near the aperture is somewhat different. Let A B (fig. 86) be the aperture, and let C D be its projection on the screen, and P a point in C D. The distance between A B and C D is to be such that the difference A D – B D may contain a small number of half wave-lengths. In the case considered previously, this difference was less than half a wave-length.

Divide the aperture into half-period elements with reference to P. The effect at P will depend on the number and size of these elements; since the effects of alternate elements are of contrary sign, the disturbance at P will be

FIG. 86.



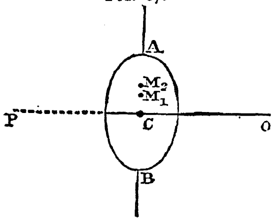
less when the number of elements is even than it is when it is odd. The disturbance, however, will never vanish, for the effect of the elements about the pole of P is so much greater than that of the others. Thus, as we go inwards from D, the light will vary in intensity and fringes will be visible. These are the interior fringes found in the image of a small aperture, and described by Newton. Of course, just as in the case when the distance between the screen and aperture was great, exterior fringes are formed too.

The optical bench already described and shown in fig. 62 may be used to perform experiments on diffraction.

In the case of rectangular apertures or of narrow obstacles the source of light should be a slit with its edges parallel to the obstacle, as shown in the figure. The aperture or obstacle is attached to the second upright, and the bands formed are viewed by the eye-piece.

The complete discussion of the phenomena produced by an aperture of any form requires mathematics for its treatment; we can, however, consider the case of a circular disc and arrive at some interesting results. Let O (fig. 87) be the source of light, A B the circular disc centre C, P any point on O C produced, and suppose we wish to examine the effect at a point in a screen through P perpendicular to O P. Let C P $= a$, and let c be the radius of the aperture, divide C A into half-period elements with regard to P in M_1, M_2, &c., and suppose the aperture to be so small that it does not contain a large number of these elements.

FIG. 87.



In this case the positions of M_1, M_2, &c., are determined by

$$\text{O}\,M_1 + M_1\,\text{P} - \text{O P} = \frac{\lambda}{2}.$$

$$\text{O}\,M_2 + M_2\,\text{P} - \text{O P} = \lambda, \text{ \&c.}$$

Let C $M_1 = r_1$, C $M_2 = r_2$, &c., and let O C $= b$.

Then
$$\text{O}\,M_1 = \sqrt{b^2 + r_1^{\,2}} = b + \frac{r_1^{\,2}}{2\,b},$$

$$\text{P}\,M_1 = \sqrt{a^2 + r_1^{\,2}} = a + \frac{r_1^{\,2}}{2\,a}, \text{ approximately.}$$

$$\text{O P} = a + b.$$

Thus
$$\frac{r_1^{\,2}}{2}\left(\frac{1}{a} + \frac{1}{b}\right) = \frac{\lambda}{2},$$

and
$$r_1^{\,2} = \frac{a\,b\,\lambda}{a + b}.$$

Similarly
$$r_2^{\,2} = \frac{2\,a\,b\,\lambda}{a + b}, \text{ \&c.}$$

With C as centre and r_1, r_2, r_3, &c., as radii, describe a series of circles, dividing the circular aperture thus into a

series of annuli. The areas of the consecutive circles thus formed will be πr_1^2, πr_2^2, &c., and these are equal to

$$\frac{\pi a b \lambda}{a b}, \; \frac{2 \pi a b \lambda}{a + b},$$

&c., and the areas of the consecutive annuli are all equal to

$$\frac{\pi a b \lambda}{a + b}.$$

The displacements sent by alternate annuli to P are in opposite phase, and, since the annuli are of equal area, of equal amount. Thus the effect of one annulus is entirely destroyed by that of the next, and the total effect at P depends on the number of annuli which can be described in our circle. If this is even, the effect at P will be zero; if it is odd, the effect at P will be that arising from one annulus. In arriving at this result we have neglected both the effect of the alteration in distance between P and the successive annuli, and also that of the variation of the angle between the line joining P to a point of the wave, and the wave normal there. But since the circle is small compared with C P, both these effects will be small.

Thus there will be points of maximum brightness and of darkness along C P, the points of darkness being given by the consideration that C A then contains an even number of half-period elements, so that $\text{PA} + \text{AO} - \text{OP} = n \lambda$, and if r be the radius of the aperture and x the distance of P from C for a point of darkness, we have

$$r^2 \left(\frac{1}{x} + \frac{1}{b}\right) = 2 n \lambda.$$

The points of maximum brightness cannot be found without further consideration. They are not, as is sometimes stated, just half-way between those of darkness, for just the same reason which we applied to show that the bright exterior bands formed by diffraction through a narrow slit are not half-way between the dark bands formed by the same slit.

In fact an investigation like that used then shows that the intensity of illumination is given by

$$\frac{4\lambda^2 a^2 b^2}{(a+b)^2} \sin^2 \left(\frac{\pi}{\lambda} \frac{a+b}{2ab} r^2\right).$$

If we put

$$\frac{\pi}{\lambda} \frac{a+b}{2ab} r^2 = \phi,$$

this may be written

$$I = \pi^2 r^4 \left\{\frac{\sin \phi}{\phi}\right\}^2$$

and the discussion of Chapter V. will apply here.

If the light used be white, since the condition for darkness depends on the wave-length, for no position of the screen will the central point be quite black, but as we move the screen parallel to itself along O P, the central point will present a series of colours, and these colours will follow each other very nearly in the same order as in Newton's scale.

For any one position of the screen the central point will be surrounded by a series of rings of colours, but the complete discussion of this case is beyond our limits.

Now let us suppose that light coming from some point, O, falls on and illuminates a screen. Let O P be perpendicular to the screen, and suppose that A B is a plate of transparent material, perpendicular also to C P, and parallel, therefore, to the screen. Let O P cut this plate in C, and, as in the case of the circular aperture above, divide C A in M_1, M_2, M_3, &c., into a number of half-period elements with regard to P, so that

$$O M_1 + M_1 P - O P = \frac{\lambda}{2},$$

$$O M_2 + M_2 P - O P = \lambda, \text{ \&c.}$$

With C as centre, and C M_1, C M_2, &c., as radii, describe a series of circles. Then we have seen that the areas of the annuli between any two consecutive circles are the same, and the phases of the disturbances which they send to P are

opposite. Let us now suppose that the central circular space and the alternate annuli cease to be transparent, so that no light can pass the space C M_1, or the annuli $M_2 M_3$, $M_4 M_5$, and so on. The phase of the disturbance at P sent by the remaining annuli is the same for all, so that the effects arising from each annulus separately are added together, and the resulting disturbance at P is very much greater than it was before the central space and the rings $M_1 M_2$, &c., ceased to transmit the light. Thus, by preventing selected parts of the disturbance from reaching P, the effect there has been greatly intensified. The same would of course have been the case had the rings $M_1 M_2$, $M_3 M_4$ ceased to transmit the light while the other series remained transparent.

Such a series of rings may be easily obtained by photography. We require to draw a series of circles on paper of such radii that the areas between consecutive circles are equal. These may be drawn large in size to secure sufficient accuracy. The alternate rings are then to be blackened, and a photograph taken on glass of the target-like picture thus formed. If this photograph be placed in the path of a beam of light, it will produce the effect of the plate described above. Let b be the distance O C, and r the radius of the first circle in our photograph, and let C P $= x$. The position of P, for which the displacement arising from a wave of given length λ will be greatest, is given by

$$r^2\left(\frac{1}{x}+\frac{1}{b}\right)=\lambda,$$

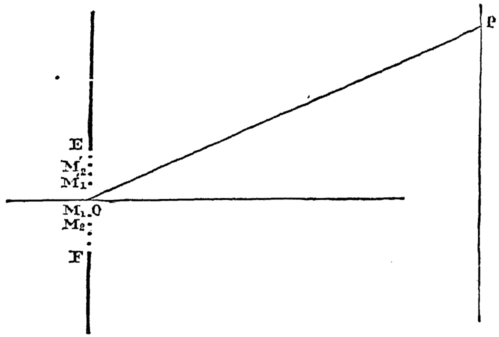
for in this case the rings correspond exactly to half-period elements on the plate. Thus, x will be greatest when λ is least; so that if white light is used the point of maximum brightness for red light is nearer the disc than that for violet light. If the source of light is very distant, as, for example, in the case of the sun, we have $b = \infty$, and $x = \frac{r^2}{\lambda}$.

The action of such a plate resembles that of a lens, though the two are not identical. If P be the geometrical focus of a point O formed by a convex lens A C B, P is a point of maximum brightness, because the time taken by light to travel from O to P is the same for all paths, which it can pursue, so that all the disturbances arrive in the same phase. If, however, the maximum effect at P is produced by a plate of the nature described above, the times taken by the light in travelling by the two paths OM_1P, OM_3P are not the same. The difference, however, is a complete period, so that the phase of the resultant disturbance produced at P is the same in the two cases, just as with the lens. We may, however, notice, that while with the plate the focus for violet light is farther off than that for red, with the lens the reverse is the case. Rings formed in this manner are frequently known as Huyghens' zones.

We have considered now at some length the phenomena presented by diffraction through a single aperture of various forms. We turn, in conclusion, to the appearances presented when a distant source of light is viewed through a large number of equal and equidistant rectangular apertures. We shall suppose that the light falling on this system, known as a diffraction grating, is a parallel pencil of rays, either coming from a distant source or rendered parallel by a collimating lens. After passing through the system of apertures, the light falls on another lens, and the screen on which the effects are produced is placed in the focus of this second lens. We have seen previously that the conditions in this case are those which would hold if the screen were a long way behind the aperture, so that the rays travelling from the aperture to a point P might be treated as parallel. Let us turn again to the case of the rectangular aperture. Let E F (fig. 88), as before, be a section of it, perpendicular to its length, and let P be a point of minimum brightness. Let O be the middle point of E F, and let the angle between O P and the direction of the incident light be θ. Then we know

that since P is a point of minimum brightness, we can divide E F into an even number of equal half-wave elements with regard to P in points such as M_2, M_1, O, M_1', &c. The effect at P is zero, because the disturbance due to any element, such as M_1' O, is exactly equal and opposite to that due to the next O M_1, and so on. Let us now suppose that the alternate elements, M_1' O, M_1 M_2, M_3 M_4, &c., cease to be transparent. The effects arriving at P from the other system of elements, M_2' M_1', O M_1, M_2 M_3, &c., being all in the same

FIG. 88.

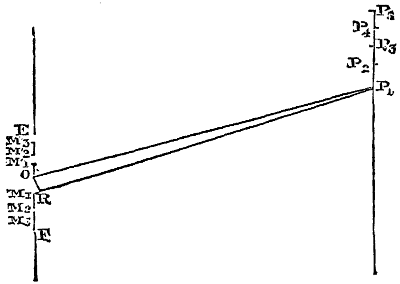


phase, help each other, and the intensity at P, instead of being zero, becomes a maximum. Instead of a series of circular zones, we have now a series of parallel strips, alternately opaque and transparent. We have, in fact, the case of diffraction through a large number of equal and equidistant rectangles.

We may approach the subject in a somewhat different manner, perhaps with advantage. Let E F (fig. 89) be a section of the grating perpendicular to the length of the rectangles, and let us suppose that the breadth of a rectangle is just equal to the distance between it and the next, so that E F is composed of a number of equal parts, M_2 M_1, M_1O, O M_1', &c., which are alternately transparent and opaque. Let O P_1 and

$M_1 P_1$ be the paths of the two rays travelling from O and M_1 respectively to the point P_1. These two are by hypothesis parallel, for P_1 is either distant or in the focal plane of a lens through which all the light passes. Let O R be perpendicular from O on $M_1 P_1$. Then $M_1 R$ is the difference of path between O and P_1 and M_1 and P_1 respectively. Let us suppose that $M_1 R = \frac{\lambda}{2}$. Then the light coming from the element $O M_1$ is exactly opposite in phase to that which would reach P_1 from the next element $M_1 M_2$. But, by hypothesis,

FIG. 89.



this element is opaque. The light reaching P_1 from $M_2 M_3$ is in the same phase as that from $O M_1$, and so on. Thus, all the light reaching P_1 is in the same phase, and P_1 is a point of maximum illumination for that wave length.

Let e be the breadth of the elements, $O M_1$ a section of one of the transparent rectangles, and, as before, let θ be the angle between O P and the incident light, then

$$R O M_1 = \theta$$

$$\sin \theta = \frac{R M_1}{M_1 O} = \frac{\lambda}{2 e}.$$

Again, let us suppose that the direction of $O P_3$ is such that the difference of path $R M_1 = \frac{3 \lambda}{2}$. Then the space $O M_1$ contains three equal half-period elements with regard

to P_3. The effects of two of these destroy each other, and the resultant effect at P is that due to one of these elements. The effect again of the next transparent space, $M_2 M_3$, is that of one half-period element, and this is in the same phase as that due to $O M_1$. Thus, in this case also, the effects at P_3 of the transparent elements are of the same phase, and the resulting displacement is again a maximum.

We have in this case, if the angle between the incident light and $O P_3$ be θ_3, $\sin \theta_3 = \frac{3 \lambda}{2 e}$; and if we find another position of P, say P_5, for which the difference of path $R M_1 = \frac{5 \lambda}{2}$ the disturbances sent to P from all the elements will be again in the same phase, and the effect will be a maximum. We shall then have $\sin \theta_5 = \frac{5 \lambda}{2 e}$; and, in general, whenever θ is such that $\sin \theta = \frac{2n + 1}{2} . \lambda$, the point in which the line O P drawn in this direction meets the screen will be one of maximum brightness.

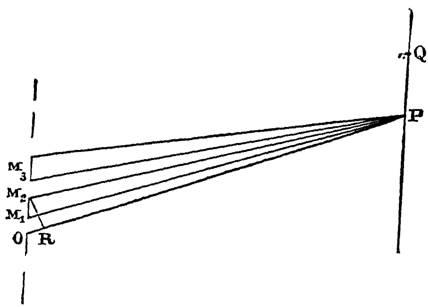
There are another series of points, $P_2 P_4$, &c., lying between these, at which the light sent from all the transparent spaces is in the same phase. For, suppose that the direction of O P is such that the difference of path, $R M_1$, is equal to λ. Then the difference of path pursued by the light reaching P from O and M_2 respectively is 2λ, so that these two waves are in complete accordance. And the light sent in this direction, $O P_2$, by all the transparent spaces, arrives in the same phase at P_2; but since each of these spaces, as $O M_1$, contains two equal half-period elements, with regard to P_2, the effect of one of these elements exactly cancels that arising from the other, so that the effect produced at P_2 by each of the transparent spaces is zero, and the whole effect, therefore, zero also. There are, in fact, two conditions to be satisfied in order that any point P may be one of maximum intensity. In the first place, the

effects of all the transparent spaces must arrive at P in the same phase; in the second place, the effect at P due to each of these spaces separately must not be zero.

The first condition is satisfied by all the points P_1 P_2 P_3, &c., for which $\sin\theta = \frac{m\lambda}{2e}$.

The second, however, is not satisfied by points such as P_2 P_4, &c.—that is, for points corresponding to even values of m. Thus, in the case in which the opaque spaces in our grating are equal to the transparent, we have bright points

FIG. 90.



on the screen only at the points for which the value of m in the above formula is odd—that is to say, in the directions given by $\sin\theta = \frac{2n+1}{2} \cdot \frac{\lambda}{e}$; but this all turns on the supposition that the opaque spaces are equal to the transparent. In general this is not the case.

Let us now, therefore, consider a grating in which the opaque spaces are not equal to the transparent. Let e, as before, be the breadth of the transparent spaces O M_1, M_2 M_3, &c., and g that of the opaque spaces M_1 M_2, M_3 M_4, &c. Consider the rays going from the transparent elements to the point P (fig. 90). Let M_2 R be perpendicular to O P. Then the difference in path for the light reaching P from O M_1 and

$M_2 M_3$ is OR. Thus, the difference in phase for these two portions of light depends on OR. If OR is equal to λ or any multiple of λ, these two portions of light will be in the same phase, and the same will be the case for all the light that can reach P from the transparent spaces, and under these circumstances the effect at P from each transparent element will be of the same sign, and the whole effect at P will be that due to any one element multiplied by the number of such elements.

Again, if θ be the angle between OP and the direction of the incident light, as before, we have

$$OR = OM_2 \sin \theta,$$

or

$$\lambda = (e + g) \sin \theta.$$

The waves which reach P from the various elements will be in complete accordance not only when OR is equal to λ, but when it is any multiple of the wave-length, and therefore we shall have a series of points P_1, P_2, P_3, &c., on the screen at which the effect is a maximum, these points being such that the difference $P_n M_2 - P_n O = n\lambda$, and if θ_n is the angle between OP_n and the direction of the incident light, and we draw $M_2 R_n$ perpendicular to OP_n, then OR_n is equal to $n\lambda$, and we have the formula

$$n\lambda = (e + g) \sin \theta_n,$$

so that

$$\lambda = \frac{(e + g)}{n} \sin \theta_n,$$

and n may have any integral value 1, 2, 3, &c.; $e + g$ is the length of a bright and dark space together, that is, it is the distance between the centre of one bright space and the centre of the next, or equally the distance between the centre of one dark space and the centre of the next. Thus if, for example, the grating be formed by ruling a number of parallel lines on glass, and if, as in many of Nobert's gratings, there be 3,001 lines in a Paris inch, the distance between the centre of one line and the centre of the next is $\frac{1}{3,000}$th of a Paris inch, so

that in this case we have $e + g = \frac{1}{3,000}$th of a Paris inch, and the value of λ is given by $\frac{\sin \theta_n}{3,000 \times n}$ Paris inches.[1]

We have thus seen that along our screen there will be a series of bright points, and that the lines joining these points to the centre of the grating make certain definite angles with the direction of the incident light. It remains to consider what is the effect at points on the screen lying between these; we shall see easily that the intensity on any such point will be so small that we may neglect it compared with the maxima. For taking the above grating for example, let us suppose that we consider a point Q on the screen very close to P, but such that the difference $Q M_2 - Q O_n$ is equal to $\lambda + \frac{1}{3,000} \lambda$ instead of λ; for the next bright element $M_4 M_5$ the retardation will be $2 \lambda + \frac{2 \lambda}{3,000}$, and so on, so that when we arrive at the 1,501st bright element, that is just halfway across the grating, the retardation is $1,500 \lambda + \frac{1,500 \lambda}{3,000}$, that is $1,500 \lambda + \frac{\lambda}{2}$; thus the light from the 1,501st bright element is just opposite in phase to that from the first, and the combined effect of the two is zero, similarly the light from the second bright element is quenched by that from the 1,502nd, and so on. Thus one-half of the grating sends to the point in question light of exactly opposite phase to that from the other, and in consequence there is darkness at the point considered.

Thus we see that under the circumstances considered, when we are using light of one definite wave-length, the appearance on the screen consists of a number of bright bands parallel to the length of the lines on the grating.

The position of these bands depends on the distance

[1] 1 Paris inch = 2·707 centimètres.

between two consecutive transparent portions of the grating, and on the wave-length of the light used. If θ_n be the angle between a line joining O to the nth bright band, and the direction of the incident light produced, and $e + g$ the distance from centre to centre between two consecutive transparent spaces in the grating, and λ the wave-length of the light, then $\lambda = \frac{(e + g)}{n} \sin \theta_n$.

Let us now take a more general case and suppose we are using white light. This we have learnt already consists of a number of waves of different wave-lengths superposed and travelling in the same direction.

The points of maximum brightness on the screen will be different for the different waves of which our white wave consists. We have seen already that two waves differing in length appear to the eye to differ in colour. The white wave consists of a number of waves of different colours, and for each colour there are points of maximum brightness on the screen. Using the light from a lamp or candle we should see on the screen or in the focal plane of the telescope a coloured band. If we call the angle between the direction in which light travels from the grating to any given point on the screen, and the direction of the incident light the deviation for light of that colour, we have seen that the sine of the deviation varies as the wave-length. Now the wave-length is greatest for red light, and least for violet. The deviation, then, is less for violet light than for red, and the coloured band seen is violet at the end nearest to the direction of the incident light and passes through the colours of the spectrum to red. A similar band will, of course, be formed on the other side of the centre of the screen.

Again, there is more than one point of maximum brightness for light of any colour, so that each of these bands of colours is succeeded by a second, and that again by a third, and so on.

It may happen that the deviation for the second violet

maximum is less than that for the first red, so that the violet end of the second band could overlap the red end of the first, and the colours there would be mixed. If this is not the case there will be a dark interval between the first and second spectra.

The order in which the colours succeed each other is the same as in the band produced by passing white light through a prism, namely, violet, indigo, blue, green, yellow, orange, red. With the diffraction grating, as described above, any point on screen is illuminated with light of one definite wave-length—unless, indeed, two consecutive spectra happen to overlap ; the spectrum thus formed is said to be pure. A pure spectrum, then, is one in which the illumination at each point consists of light of one definite wave-length. In the spectrum formed by refraction through a prism this is not generally the case ; we shall see shortly how, by a proper arrangement of lenses and prisms, a pure spectrum may be obtained.

The diffraction spectrum formed from the light of a lamp or candle consists, we have seen, of continuous bands of colours. To obtain by diffraction a pure spectrum let the light pass through a narrow slit placed in front of the candle, with its length parallel to the lines of the grating, so that we may treat this slit as our source of light. Place between the slit and the grating a convex lens at a distance from the slit equal to its own focal length. The rays of light coming from the slit will emerge parallel and the wave which falls on the grating will be plane. An arrangement of this kind, consisting of a slit and a lens which can be adjusted so as to make rays of light coming from the slit parallel on emergence, is called a collimator.

If now the diffracted beam be received on a telescope focussed for parallel rays, or on a screen at a long distance behind the grating, the conditions assumed above will be satisfied, and the diffracted spectrum will be pure. If we consider any line in it parallel to the grating it is just an

image of the slit formed by light of one definite wave-length after diffraction, and these bright images of different colours placed side by side form the whole spectrum.

Now, however, let us suppose that our slit is illuminated with sunlight, the other adjustments remaining the same. A spectrum is formed as before, but we shall find that it is crossed by a large number of dark lines or bands parallel to the length of the slit. These dark lines are visible whenever a pure solar spectrum is obtained, whether by diffraction or refraction. They were first observed by Fraunhofer in 1814, and are frequently called after him—Fraunhofer's lines.

Thus in the pure solar spectrum a certain number of the images of the slit, which placed side by side make up the whole spectrum, are wanting; all the rays of certain definite wave-lengths in air, or better still, if certain definite periods of vibration are absent. Each line in the spectrum parallel to the slit is a line of brightness for light of some definite period; certain lines are dark because there are no rays of the proper periods in the incident light. In a future chapter we shall return to many important consequences of this discovery. At present we intend to use the dark lines merely as definite marks in the spectrum which we can recognise.

We may speak of the wave-length of red light, but the phrase is indefinite, for red has many shades, and we could never feel certain that the shade of red we were observing at any given moment was exactly the same as that we had observed at some past time; but there is in the red part of the solar spectrum a distinctly marked dark line which a little practice enables us to recognise at once when we see it, and we know that this line corresponds to light of a certain period which is absent from the solar spectrum. If by any means we can measure either the wave-length in air or the period of vibration of the light corresponding to this line, we have a physical constant for use through all time, and one which we can recover at any moment by repeating our observation. The principal lines in the solar spectrum are

denoted by the letters A, B, C, D, E, F, G, H. The dark line in the red referred to above is the line C. D is in the orange, E in the yellow end of the green, the most brilliant part of the spectrum, F in the blue end of the green, G in the indigo, and H in the violet. Again, we know that the velocity of light of a given colour depends on the medium in which it is travelling. If, then, we wish to compare the velocity of light in two different media we must make sure that the light is of the same period in the two cases. This we can do by observing in each case the velocity of the light corresponding to the same dark line in the solar spectrum.

Fraunhofer was the first to apply the theory of the diffraction grating given above to the measurement of wave-lengths. His first gratings were made by winding a fine wire round two screws of the same thread placed parallel to each other. Much finer gratings have since been obtained by ruling parallel lines on the surface of glass or metal with the point of a diamond. Mr. Rutherfurd, of New York, has ruled some containing 17,280 lines to the inch, and of these copies have been made by photography by Lord Rayleigh and others, while more recently Professor Rowland, of Baltimore, has constructed a machine by the aid of which he can rule gratings of six inches in breadth containing 28,876 lines to the inch.

The most accurate measurements of the wave-lengths of the principal Fraunhofer lines are those of Rowland ('Phil. Mag.' July 1893), and given in the following table in tenth mètres, a tenth mètre being $\frac{1}{10^{10}}$ mètres.

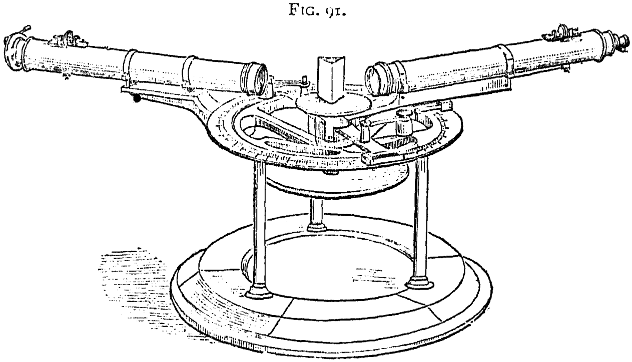
Table of Wave Lengths.

A	7594·06	E	5270·50
B	6870·19	F	4861·50
C	6563·05	G	4307·91
D_1	5896·15	H	3968·62
D_2	5890·18	K	3933·81

To obtain the wave-length of light from observations with a diffraction grating we require to know the distance between two consecutive transparent spaces measured from centre to centre, and the deviation of the light whose wave-length is sought. We proceed, then, to describe the apparatus and method of making the observation. The grating is mounted on the table of a spectrometer. This instrument (fig. 91) consists of a graduated circle, which is generally fixed in an horizontal position.

A collimator is rigidly attached to the circle, the axis of

FIG. 91.



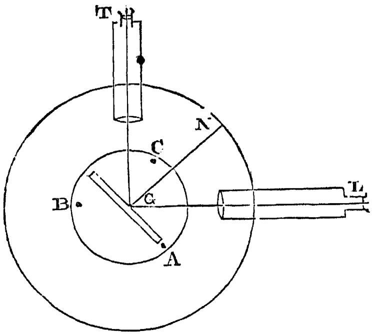
the collimator is in a plane parallel to that of the circle, and is directed to a point above the centre of the circle. An astronomical telescope, generally fitted with cross wires and a Ramsden's eye-piece, is fixed to an arm which turns in a plane parallel to that of the circle about its centre. The axes of the telescope and the collimator should both be parallel to the plane of the circle, and the two should meet above its centre. The position of the telescope with reference to the graduated circle is read by means of a vernier. Above the centre of the circle there is an horizontal table, which is generally capable of rotation about the vertical axis of the circle, and attached to this table there is a second vernier, so that its position can be determined.

The whole instrument rests on three levelling screws, and the collimator and telescope are held in their positions by moveable screws, so that their axes can be adjusted until they are parallel to the circle. To perform this adjustment it is generally sufficient to place a spirit level on the circle and level the instrument by means of its feet. Then place the level on the tube of the telescope with its length parallel to the tube, and level it by altering the screws which fix it to the moveable arm. Then do the same for the collimator. If the optical axes of the telescope and collimator coincide with the axes of their respective tubes, the required adjustment is now made, and in practice this will generally be sufficiently accurate. The instrument now requires to be focused. Turn the telescope towards some very distant object, which should be clearly marked and distinct, and adjust the eye-piece until the cross wires are seen distinctly. Then adjust the focusing screw until the distant object as seen through the telescope appears to be in the same plane as the cross wires. This will be the case when a small motion of the eye before the eye-lens produces no change in the relative positions of the cross wires and the image of the object. The object is supposed to be so distant that the rays which fall from it on the telescope may be treated as parallel. The telescope then is focused for parallel rays, and whenever an object is seen distinctly through it the rays falling on the object-glass of the telescope must be parallel. Now turn the telescope to look directly into the collimator, and place a light behind the slit, which we shall suppose is vertical; it will generally appear as a bright blurred line in the field of view of the telescope. Adjust the focusing screw of the collimator until the slit is seen clearly and distinctly. When this is the case the rays which fall on the object-glass of the telescope must, we know, be parallel; thus the rays which emerge from the collimator are parallel and the instrument is ready for use. The focusing screws of the collimator and telescope should not

again be touched. It may, however, be necessary to alter the position of the eye-piece to suit the eyesight of a second observer.

The grating is now fixed on a small stand on three levelling screws and placed on the table of the instrument with its lines as nearly as may be vertical.[1] It is best to place the grating on this stand so that its plane is perpendicular to the line joining two of the levelling screws, for in this position it can most easily be brought into the proper adjustment. We require that the lines of the grating should be parallel to the slit, and that this again should be parallel to the axis round which the telescope turns. This axis we have supposed is vertical. Thus the lines of the grating are to be vertical. We must therefore place the face of the grating in a vertical plane, and then level it until the lines are vertical. Let us suppose that A B C are the three levelling screws, and that the plane of the grating is perpendicular to the line joining B C (fig. 92).

FIG. 92.



Turn the telescope so as to view the slit directly, and fix with a piece of wax across the slit a hair, in such a position that it appears to coincide with the horizontal cross wire; or, if more easy to do, cover up half the slit, making the junctions of the covered and uncovered portions coincide with the horizontal cross-wire. Some of the light from the slit will fall on the grating and be reflected there, and by turning the telescope the reflected image can be brought into the field. In

[1] In figure 91 a prism is shown in the position which the grating would occupy.

general the image of the wire placed across the slit will no longer coincide with the horizontal cross-wire, but the two can be brought into coincidence by altering either of the screws, C or B; and it is quite evident that when this is the case the plane of the grating is vertical, and, moreover, it will remain vertical, even if we alter the levelling screw A. By adjusting this screw we can alter the angle between the lines of the grating and the vertical, and bring the two into coincidence. This will be the case when the diffracted images of the slit are as clear as possible. Illuminate the slit with homogeneous light and turn the grating so that the light falls on it approximately normally. Turn the telescope to view one of the diffracted images, and adjust the levelling screw C until this appears as clear as possible. The lines of the grating will then be sufficiently nearly vertical.

In obtaining the formula $\lambda = \frac{e+g}{n} \sin \theta_n$ we supposed that the incident light was travelling at right angles to the plane of the grating. We have therefore to place the grating at right angles to the axis of the collimator. This can be done with considerable accuracy by eye; the following method is applicable when the table of the spectrometer is moveable, and has a vernier. Turn the telescope to view the slit directly and bring the slit into coincidence with the vertical cross-wire. The telescope is generally fitted with a clamp by means of which it can be rigidly connected with the circle and a tangent screw, which will give it a small slow motion; with the help of this it is easy to bring the cross-wire over the image of the slit. Now read the vernier attached to the telescope. Unclamp the telescope and turn it through 90°. This of course is easily done by means of the graduations. Thus if when viewing the direct light the vernier reading was 171° 21′ 15″, we should have to turn the telescope until the vernier read 81° 21′ 15″ or 261° 21′ 15″. The axis of the telescope is now inclined at 90° to that of the collimator. Now turn

the table carrying the grating until the image of the slit reflected from the grating appears to coincide with the cross-wire. Then it is clear that light which before it fell on the grating was travelling parallel to the axis of the collimator has been reflected so as to travel parallel to the axis of the telescope, that is, it has been turned through 90°.

Let L G (fig. 92) be the axis of the collimator, T G that of the telescope, then the path of the ray is L G T, and the angle L G T is 90°; and if G N be normal to the grating, G N bisects the angle L G T, so that the angle L G N is 45°. If now we turn the table of the spectroscope carrying the grating through 45° in the proper direction, we shall bring G N into coincidence with P L, and so set the plane of the grating at right angles to the incident light. To do this we have only to read the vernier of the table when the reflected image coincides with the cross-wire, and then turn the table in the right direction until the vernier reading differs from this by 45°.

The grating is now at right angles to the axis of the collimator. Of course, the above adjustment may be used to put any plane reflecting surface at a given angle to the axis of the collimator.

Now turn the telescope to view the slit through the grating. A brilliant image will be seen, and on moving the telescope in either direction from this position, if the light used be white, a series of spectra will come into the field. If the sun be the source of light, these spectra will be crossed by narrow dark lines, provided the grating be fairly good and the slit sufficiently narrow. If the light be homogeneous, instead of the spectra we shall have a series of images of the slit. Such a source of homogeneous light may be obtained from a spirit-lamp with a salted wick, or from a Bunsen gas-burner, in the flame of which a platinum or asbestos wick, moistened with some salt of sodium—common salt, for example—has been placed. The number of images visible depends on the brightness of the source of light and on the closeness with which the lines are ruled on

the grating. The spectrum of sodium vapour consists of two brilliant yellow lines very close together, so close that, except with powerful instruments, they appear as one, and some other faint lines, so faint that they are rarely observed in a diffraction spectrum. Thus, we should have a bright central yellow image, and on either side of this a number of yellow images growing fainter and fainter. Each of these diffracted images really consists of two very close together, but with ordinary instruments they would appear as one. Let us suppose that our grating is one of Nobert's, with 3,001 lines to the Paris inch. Then with telescopes of aperture of about $1\frac{1}{4}$ inch we may see four or five images on each side of the central one. Turn the telescope to view the central image, and bring this into coincidence with the cross wire. Then read the vernier. Turn the telescope to the right till the first diffracted image coincides with the slit, and read again. The difference between the two readings gives a value for θ_1, the deviation of the first image. Now turn the telescope to the first image to the left, and read again. The difference between this and the direct reading gives us a value θ_1' for the deviation of the first image to the left. If we have made our adjustments with care, these two—θ_1, θ_1'—will be nearly the same. If we could have done everything with absolute accuracy, they would be exactly the same. The mean of the two will give us a value for the deviation more accurate than either. Thus, in the case considered above, we have $e + g = \frac{1}{3000}$ Paris inches $=$ ·000009023 mètres, and we might find

$$\theta_1 = 3^\circ\ 44'\ 30''$$
$$\theta_1' = 3^\circ\ 44'\ 45''.$$

Thus, we take for the deviation, $3^\circ\ 44'\ 37''{\cdot}5$, and substituting in the formula

$$\lambda = (e + g) \sin\theta, \text{ find}$$
$$\lambda = {\cdot}00000058913 \text{ mètres},$$
$$= 5891{\cdot}3 \text{ tenth mètres}.$$

On referring to our table of wave-lengths, we find that this is very nearly the wave length thus given for the solar dark line D. A more accurate measurement would have told us that the two agreed absolutely, and that the solar line D is really two lines very close together, coinciding exactly in position with the two bright sodium lines. This important fact we shall return to in considering spectrum analysis. If we observe in a similar manner the deviations corresponding to the second and third diffracted images, we get other equations to determine λ, and the mean value deduced from the results of all the observations will be more accurate than the result of any single measurement.

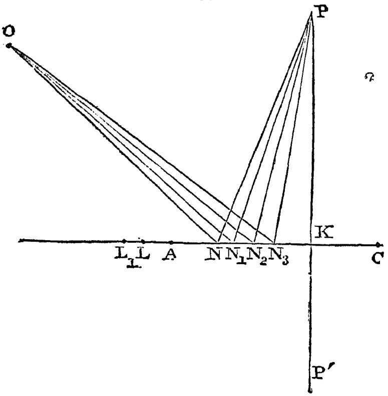
Moreover, the observation of each single deviation ought to be repeated to eliminate errors made in setting and reading the vernier. The accurate reading of a vernier is greatly facilitated by a proper placing of the light by which it is illuminated. The line joining the vernier to the light should pass through the axis of the circle, and the eye should be placed so as to catch the reflection of the light from the scale. The eye will thus look directly down on the graduations of the scale, being in the same vertical plane with them, and the error of parallax, which may arise from the scale and vernier division not being in the same plane, will be avoided.

We may require to measure the distance between the lines of the grating, if this is not already known from the maker. To do this we require a good microscope fitted with a micrometer eye-piece, or else some well-defined object whose breadth we can measure accurately, and which we can bring into the field of view at the same time as the grating. We then have to place the grating so that its lines are parallel to the edge of this object, and measure the number of lines which occupy the same breadth as the object. From these measurements we can easily calculate the distance between the lines.

Many of the best gratings used are not transparent, but

consist of parallel lines ruled on a plane polished metallic surface. The incident light falls on the surface and is there reflected; but, since the surface is not regularly polished, the appearances presented by the reflected beam differ from those described above in Chapter III. Let A C (fig. 93) be the reflecting surface, and suppose the incident wave to be plane. Let L be the point of the surface from which light regularly reflected would reach P. Divide the surface exactly as in Chapter III., in L_1, L_2, L_3, &c. Then we have

FIG. 93.



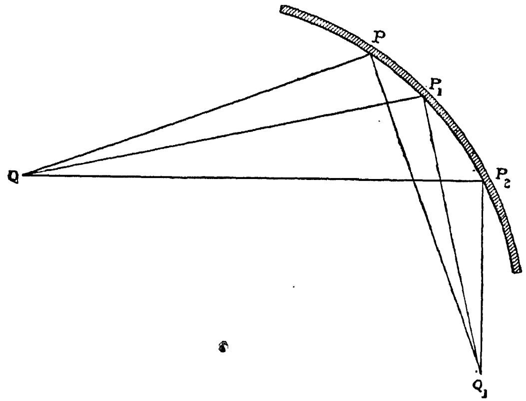
seen that the disturbances from these consecutive elements, $L L_1$, $L_1 L_2$, &c., are in opposite phase when they arrive at P. Moreover, at but a short distance from L the consecutive elements are of equal length. Let $N N_1$, $N_1 N_2$, &c., be some of these elements, and suppose that the reflecting surface does not extend as far as L. Then, since the effects of the elements $N N_1$, &c., are equal in amount opposite in phase, they destroy each other, and there is darkness at P; but if we suppose that the alternate elements, $N_1 N_2$, $N_3 N_4$, &c., are blackened so as to cease to reflect light, the disturbances coming from $N N_1$, $N_2 N_3$, &c., are in the same phase, and

augment each other's effects, so that there is brightness at the point P.

The case is exactly analogous to that of the refraction grating just considered, in which the alternate elements ceased to transmit light. In fact, if we draw P K perpendicular to the surface and produce it to P′, making K P′ = P K, then the diffraction effects produced at P by the reflexion grating will be similar to those which would be produced at P′ by a refraction grating with the same interval between the lines occupying the position of the given grating. The two spectra would differ in brightness.

Moreover, as Professor Rowland has pointed out, a reflexion grating need not be ruled on a plane surface, for let P, P_1, P_2 (fig. 93*a*) represent any surface, and let Q be a source

FIG. 93*a*.



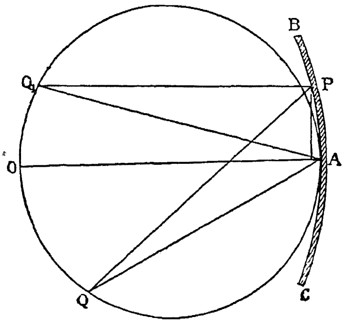
of light, and Q_1 any point at which the effects are required; the waves diverging from Q reach the surface, each element becomes a centre of disturbance, and the effect at Q_1 is the resultant of these. Now let us divide the surface up into a series of half-period elements with regard to Q and Q_1. This is done by taking points P, P_1, P_2, &c., such that

$$Q P_1 + Q_1 P_1 = Q P + Q_1 P + \tfrac{1}{2}\lambda,$$
$$Q P_2 + Q_1 P_2 = Q P_1 + Q_1 P_1 + \tfrac{1}{2}\lambda, \text{ \&c.}$$

The disturbances sent to Q_1 by each pair of consecutive elements are opposite in phase and approximately equal in amount, hence, in general, the effect at Q_1 is zero. But now suppose that by some means the nature of the surface of one series of alternate elements is changed so that they no longer regularly return the waves to Q_1 but scatter them in an irregular manner, while that of the other series of alternate elements is left unaltered. Then all the waves which arrived at Q_1 are in the same phase, and a real image of Q is formed there, the action being the same as that described on p. 183. The end can be attained theoretically by ruling a series of fine lines or scratches on the surface at proper distances apart, and in some cases it appears, from further consideration, that the effect can be produced by a number of parallel equidistant straight lines, which can be produced by a diamond point moving along a geometrical slide and made to move equal distances between each scratch by means of an accurate screw.

Professor Rowland shewed that a spherical surface ruled with a series of parallel and equidistant lines would thus produce a real image of an illuminated slit placed parallel to the lines.

FIG. 93*b*.



Thus, in fig. 93*b* let O be the centre of the sphere on which the grating BAC is ruled, A the centre of the face, P any point on the grating situated say n lines from A. Then waves from A and P will reach Q_1 in the same phase if

$$QP + PQ_1 = QA + AQ_1 \pm n\lambda.$$

Let $QAO = \phi$, $Q_1AO = \phi_1$, $AOP = \omega$, $QA = u$, $Q_1A = u_1$, $OA = a$.

Then the angle $QAP = \phi + \frac{\pi}{2} - \frac{\omega}{2}$, and $AP = 2a \sin\frac{\omega}{2}$.

Hence,

$$QP^2 = u^2 + 4a^2 \sin^2\frac{\omega}{2} - 4au \sin\frac{\omega}{2}\sin\left(\frac{\omega}{2} - \phi\right)$$

Now ω is a small angle in all practical cases, the radius of the surface being large compared with its breadth, we may therefore write

$$\sin\tfrac{1}{2}\omega = \tfrac{1}{2}\sin\omega,$$

$$\cos\tfrac{1}{2}\omega = 1 - \tfrac{1}{2}\frac{\omega^2}{4} = 1 - \tfrac{1}{8}\sin^2\omega,$$

and expand the expression for QP in powers of $\sin\omega$ as far as the second. We thus find

$$QP = u + a\sin\phi\sin\omega - \frac{a\sin^2\omega}{2}\left(\cos\phi - \frac{a}{u}\cos^2\phi\right)$$

$$+ \text{ terms involving higher powers.}$$

Similarly,

$$Q_1P = u_1 - a\sin\phi_1\sin\omega - \frac{a\sin^2\omega}{2}\left(\cos\phi_1 - \frac{a}{u_1}\cos^2\phi_1\right) + \&c.$$

Hence we get

$$a\sin\omega(\sin\phi - \sin\phi_1)$$

$$-\tfrac{1}{2}a\sin^2\omega\left\{\cos\phi + \cos\phi_1 - a\left(\frac{\cos^2\phi}{u} + \frac{\cos^2\phi_1}{u_1}\right)\right\} = \pm n\lambda.$$

Now suppose that the distance which the diamond point is moved between each line is d. Draw PN perpendicular to AO, then, since there are n lines between A and P and PN is the distance which the point must be moved to pass from the first to the n^{th} of these, then we have $nd = PN = a\sin\omega$.

Thus
$$nd(\sin\phi - \sin\phi_1)$$

$$-\tfrac{1}{2}\frac{n^2d^2}{a}\left\{\cos\phi + \cos\phi_1 - a\left(\frac{\cos^2\phi}{u} + \frac{\cos^2\phi_1}{u_1}\right)\right\} = \pm n\lambda.$$

The values of ϕ and u are fixed; we can determine ϕ_1 and u_1 so that this equation is satisfied for all values of n, *i.e.* for every line on the grating. This will be the case if

$$\sin\phi - \sin\phi_1 = \pm\frac{\lambda}{d}$$

and

$$\cos \phi + \cos \phi_1 - a \left(\frac{\cos^2 \phi}{u} + \frac{\cos^2 \phi_1}{u_1} \right) = 0.$$

The first equation gives ϕ_1, and hence fixes the direction of the line $Q_1 A$, while the second equation gives u_1 and fixes the distance of Q_1 from A. Thus light diverging from Q will be brought to a focus at Q_1 by all parts of the grating. Moreover, since the value of ϕ_1 depends for a given value of ϕ on λ, light of different wave-lengths will be focussed in different directions, and a real pure spectrum will be formed.

Again, we may write the second equation, which gives u_1, in the form

$$\cos \phi \left(1 - \frac{a}{u} \cos \phi \right) + \cos \phi_1 \left(1 - \frac{a}{u_1} \cos \phi_1 \right) = 0.$$

Consider now a circle on AO as diameter, and let Q lie on this circle; then AQO is a right angle and $QA/OA = \cos \phi$, or $u = a \cos \phi$; hence $1 - a \cos \phi / u$ is zero, and therefore, from the above equation, $1 - a \cos \phi_1 / u_1 = 0$, or $u_1 = a \cos \phi_1$; thus Q_1 lies on the same circle. Moreover, this result is true for all values of λ, though, of course, the value of ϕ_1, and therefore the position of Q_1 on the circle, depends on λ. Thus, if the slit lie anywhere on the circle and be illuminated with white light, a pure spectrum is produced forming an arc of the same circle. This pure spectrum can either be viewed directly or magnified by the aid of an eye-piece, or it can be thrown on a sensitive plate and photographed.

Just as with the transmission grating already described, so also in this case there will be formed a series of spectra corresponding to the positions for which the difference of path is two, three, or any integral number of wave-lengths. All these spectra lie on the same circle, their positions being given by the equations

$$\sin \phi_1 + \sin \phi = \frac{\lambda}{d},$$

$$\sin \phi_2 + \sin \phi = \frac{2\lambda}{d},$$

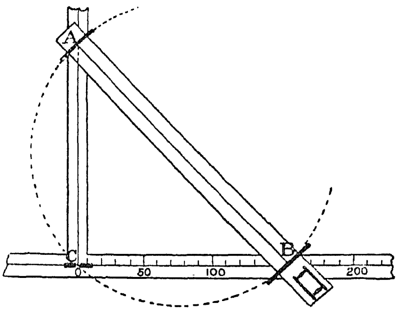
$$\sin \phi_3 + \sin \phi = \frac{3\lambda}{d}, \text{ \&c.}$$

The simplest case is obtained when either the origin of light or the spectrum coincides with the centre of the grating so that either ϕ or ϕ_1 is zero.

When this happens the sine of the angle of diffraction is directly proportional to the wave-length.

This is secured in Professor Rowland's arrangement. The grating is mounted at one end, A, of a beam A B (fig. 93*c*), the length of the beam being equal to the radius of the grating and its direction normal to the grating; thus the centre is at B. The photographic camera or the observing eye-piece are fixed to the same beam at the end B. C A and C B are two rigid guides secured at right angles; one end of the beam A B slides in each of these guides so that the centre of the grating moves along C A while the photographic plate or the focus of the observing eye-piece moves along B C.

FIG. 93*c*.



Hence the point C always lies on a circle with A B as diameter. If, then, a slit be placed at C and the arm A B moved in the guides till the angle B A C, or ϕ, has the right value, a real spectrum will be seen in focus at B. Moreover, since in this case for the first spectrum ϕ_1 is zero, while $\sin \phi = \text{BC} / \text{BA} = x / a$ say, where a is the radius of the grating and x the distance of B from C measured along a scale attached to the guide B C,

$$\therefore \; x = a \sin \phi = \frac{a\lambda}{d}, \text{ or } \lambda = \frac{dx}{a}.$$

From this the wave-length of the line observed may be determined. For convenience the scale may be graduated so as to read wave-lengths directly.

In this manner Professor Rowland has produced a large photographic map of the solar spectrum.

A similar map has been constructed by Mr. Higgs; in his arrangement, however, the grating is mounted at one end of a diameter of a large circle, the slit is at the other end of the same diameter, while the photographic plate is made to move round the same circle, being always tangential to it.

The brilliant iridescent colours shown by mother-of-pearl and other striated substances are due to diffraction effects produced by them on the light falling on them.

In the diffraction gratings considered above, we have supposed that the breadths of the transparent intervals are all equal, and also those of the opaque. If our apertures be equal but not equidistant, it is possible to show that the effect produced at any point is that due to a single aperture multiplied by a number of apertures; or if, again, the apertures be equidistant but not equal, so that the diffraction effects are produced by a number of equal and parallel but not equidistant fine fibres, we may show that the effect at any point is that due to a narrow transparent aperture equal in breadth to any one of the fibres, multiplied by the number of spaces between the fibres.

The diffraction appearances in these two cases remain unaltered if we suppose the transparent portions to become opaque and the opaque transparent; and this, in fact, as we have seen, is always the case.

Again, we may show that the intensity produced by a number of equal circular apertures placed in an opaque screen at irregular intervals is just that due to a single aperture multiplied by the number of apertures, while, as before, the appearances are unchanged if we suppose the apertures to become opaque discs and the screen transparent. Instead

of the opaque discs we may equally well have small opaque bodies of a regular form, such as the small globules of condensed vapour in a cloud.

It is this diffraction by a number of opaque particles which gives rise to the luminous rings seen in contact with the surface of the sun or moon when viewed through a thin cloud. Fraunhofer imitated these halos by looking at a luminous object through a glass covered with fine grains, such as those of lycopodium powder. It is necessary for the success of the experiment that the grains should be nearly equal in size. He also experimented with a number of small equal circular discs placed between two parallel plates of glass, and allowed to arrange themselves side by side in any manner under their weight. He measured the diameters of the rings formed, and found that, conformably with theory, they varied directly as the wave-length, and inversely as the diameter of the discs.

These rings had been previously observed by Young, and the fact that their radii are inversely proportional to the diameter of the small opaque particles to which they are due, had been applied by him in an instrument invented to measure the diameter of very minute objects, which he called the eriometer. A plate of metal has a hole of about ·5 mm. diameter perforated in it, and a series of smaller holes in a circle of between 10 or 15 mm. radius round it. This is placed in front of the lamp and viewed through the substance to be examined. A ring or halo is seen round the image of the central hole, and the distance between the plate and the substance is varied until this ring appears to coincide with the ring of small holes in the plate. The distance between the plate and the substance is then measured, and the experiment repeated with a substance containing particles of known size. The diameters of the particles vary inversely as the angular diameters of the rings produced, and since these rings were of the same size, their angular diameters vary inversely as the corresponding instances between the

plate and the substances. Thus the diameters of the particles vary directly as these distances, and since the diameter is known for the second set of particles, that for the first can easily be obtained.

The luminous rings (couronnes) referred to above are those which are seen close to the surface of the sun or moon, and must not be confused with the halos frequently formed at some apparent distance away.

The theory of the rainbow belongs to this part of our subject, and though this can hardly be given completely without the introduction of mathematics, a general explanation may perhaps be of use.

The rainbow is produced by the reflexion and refraction of the sun's rays at the surfaces of the drops of water forming the cloud against which it is seen.

Let us then consider one such drop and suppose we have a parallel pencil of rays falling on it. We shall treat the drop as a sphere of water. Take any one ray P Q of the pencil incident on the drop at Q, it is then refracted into the drop in the direction Q R, falls on the inner surface at R, is there reflected along R S, and after refraction again at S emerges along S T. A neighbouring ray $P' Q'$ follows the course $P' Q' R' S' T'$. Both these rays have been deviated or turned out of their course by the drop. The deviation is measured by the angle between their initial and final directions, and in general it is clearly different for these two rays. Let us suppose P Q and T S meet in U,[1] $P' Q'$, and, $T' S'$ in U' the deviation in the one case is $180° - PUT$, in the other $180° - P'U'T'$. The two refracted rays Q R, $Q' R'$, will in general intersect in some point. Let it be K. If we draw another neighbouring ray $P'' Q'' R'' S'' T''$, $Q'' R''$ will intersect $Q' R'$ in L say, and so on, and we can get a number of points K L M, &c., which are the points of intersection of consecutive refracted rays. These points all lie on a curve called the caustic curve, and this caustic can be drawn, and we can show that its form will be L M N F as in the figure.

[1] U and U' are not shown in the figure.

Each point on it is the point of intersection of two consecutive rays after refraction at the first surface, and all these

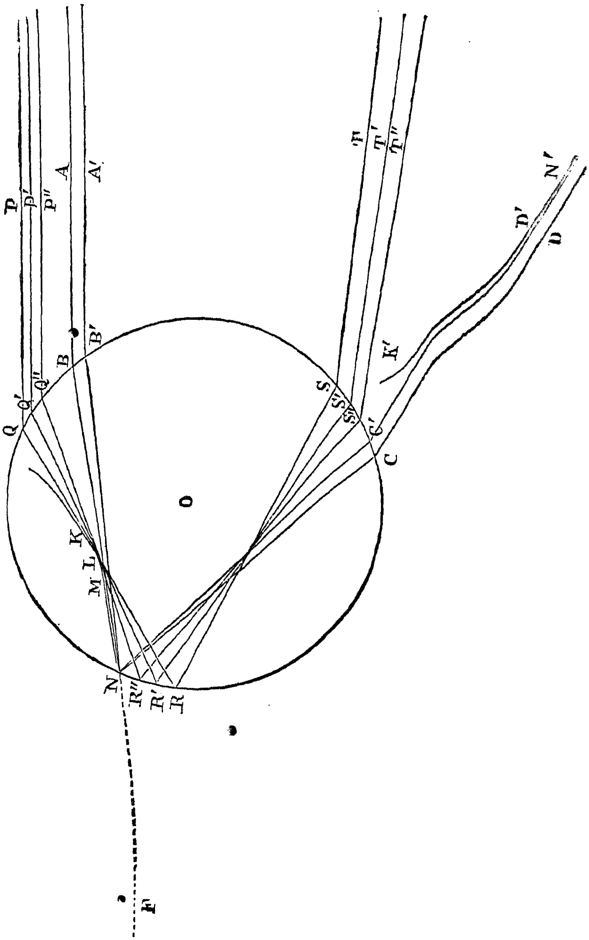


FIG 94

rays touch the caustic. Now this caustic curve will cut the drop somewhere. Let N be the point of intersection. Two

rays B N, B′ N refracted at B and B′ respectively fall on the inner surface of the drop at N. B and B′ are very close together, and the incident rays A B, A′ B′ are consecutive rays. These two rays are reflected at N in the directions N C, N C′ respectively, and refracted out along C D and C′ D′, and we may show without difficulty that these rays C D, C′ D′ are parallel.

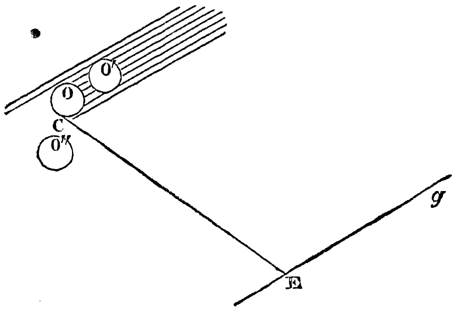
Just as a caustic curve is formed by the intersection of two consecutive rays after the first refraction, so too is one formed by the intersection of the consecutive emergent rays, and these rays are all tangents to the caustic. But the two consecutive rays C D, C′ D′, do not intersect till we reach an infinite distance from the drop, since they are parallel. Thus the form of the caustic made by the emergent rays will be as in the curve K′ N′ in the figure; N′ being supposed to be the point of intersection of the two parallel rays C D, C′ D′ which touch the caustic at an infinite distance from the drop. The ray C D is said to be an asymptote to the caustic. Now it is quite clear from the figure that there will be a very large number of tangents to this caustic which do not differ greatly in direction from C D, and these tangents are rays of light. Thus there are a great number of emergent rays travelling from the drop in directions which do not differ much from C D, and an eye placed in the line C D and looking towards the drop would receive more rays of light than if it were placed in any other position. That direction would appear brightest to the eye if we admit that the effect of two rays is twice that of one, and do not consider the differences of phase which may exist between the rays travelling in directions nearly coincident with C D. For the present let us make this assumption.

Now the two rays C D, C′ D′ are parallel, and they were parallel before incidence. They have, therefore, been deviated through the same angle. Thus, in going from A B to the consecutive ray A′ B′, the deviation has not altered. The deviation, then, is either a maximum or a minimum for the

ray A B. If we consider the ray which falls directly on the drop and passes through the centre, we see that its deviation is two right angles, and is a maximum. But in all cases maxima and minima values of a quantity occur alternately, so that the deviation of the ray A B, which we have shown must be either a maximum or minimum, will be a minimum, and no light can emerge from the drop in a direction to reach an eye placed below C D.

Thus in the direction of the ray which undergoes minimum deviation the brightness will be very great; below that

FIG. 95.



ray there will be no light at all, and above that ray there will be but a faint illumination from the drop. We shall call the ray A B N C D the effective ray. In the neighbourhood of the effective ray the emergent rays are nearly parallel and thus many of them fall into the eye, below it there are no emergent rays from the drop, and above it the rays are inclined to each other at considerable angles, and but few from the neighbourhood of any one point on the drop reach the eye, so that there is but a faint illumination.

Thus an eye situated anywhere on the line C D would perceive a bright spot in that direction. Now let E (fig. 95) be the eye of the observer, and E g parallel to the direction

of the incident light, so that Eg is the line joining the observer's eye to the sun.

Let us suppose the figure made to turn about gE as an axis, so that the drop describes a circle in a plane perpendicular to this line. All the drops in this circle would be situated in exactly the same manner with reference to the observer, and the rays reaching him from each would be effective rays. He would thus see a bright circular arc in the sky whose centre would be on the line gE produced, and whose angular radius would be 180° minus the deviation of effective rays.

If we consider a drop centre O′ somewhere above this circle, it is clear that the deviation of light reaching the eye from it must be less than that for the drop centre O. But this drop sends its effective rays to the eye, and the deviation of the effective rays is less than that of any other. Thus no light comes to the eye from drops above this bright circle. For a drop centre O′ below the circle the deviation will be greater than that for the drop centre O; some small amount of light can reach the eye from the space below this circle, which thus forms a bright upper boundary to a faintly illuminated space.

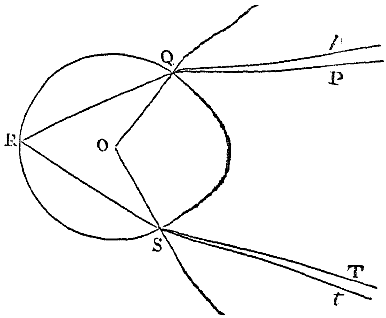
The radius of this bright boundary depends on the minimum deviation of the light falling on the drop. This again depends on the refractive index of the drop, and this refractive index is different for rays of different colours. Thus the radius of the bright boundary is different for rays of different colours. Thus since solar light is compounded of a number of rays of different colours, the bright arc seen will not be white, but coloured like a spectrum made by a prism, and this bright-coloured band is the primary rainbow.

We might without difficulty find an expression for the deviation of light passing through the drop in this way, and determine the condition that it should be a minimum. The expression would involve the refractive index, and on substituting the values for red and violet light in water

respectively, we should find that the angular radius of the red bow was about 42° 2′, while that of the violet was 40° 17′. Thus the violet bow is the lower.

We can show this, however, without actually finding expressions for the deviation. For let P Q R S T (fig. 96) be the effective violet ray, and consider a red ray *p* Q R S *t* having the same path in the drop.

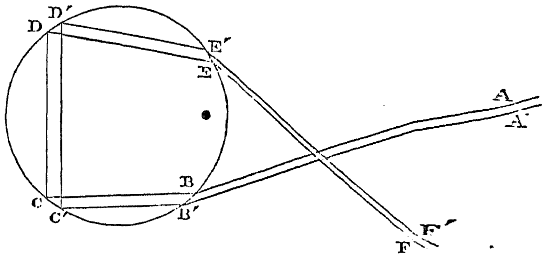
FIG. 96.



It is less deviated both at incidence and on emergence; thus the deviation of this red ray is less than the minimum violet deviation, and so *à fortiori* the minimum deviation for red light is less than that for violet. But the angular radius is 180° − minimum deviation. Thus the angular radius of the red bow is greater than that of the violet, or the red bow is the upper.

We have thus accounted on the supposition of the

FIG. 97.



arithmetical addition of the effects of rays of light for the formation and colours of the primary bow. Again, frequently, when a bright rainbow is seen, a second and fainter bow of greater angular radius is visible. This is called the second-

ary bow. It is formed by rays which emerge nearly parallel after two reflexions at the interior surface of the drop, such as the rays A B C D E F, A′ B′ C′ D′ E′ F′, in fig. 97, and its order of colours and angular radius can be found in a manner similar to that indicated for the primary bow.

The angular radius is about 50° 58′ for red light, and 54° 9′ for violet, so that the red bow is the lower in the secondary bow.

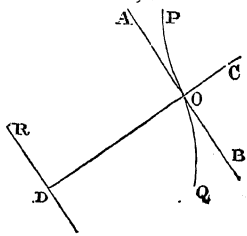
Rays which emerge parallel after three, four, and five reflexions form the third, fourth, and fifth rainbows; of these the third and fourth are formed in that part of the sky which lies between the observer and the sun, and are rendered invisible by the brightness of the sun's light. The fifth bow, like the first and second, is formed on the side of the observer away from the sun, but the light is so much weakened by the five reflexions that it is rarely, if ever, visible.

According to the explanation developed above, the space directly below the violet of the primary bow should be dark. In fact, however, several alternations of colour are frequently seen below the first violet. The primary bow is accompanied by a number of additional or supernumerary bows, and of these our investigations have given us no information, and for a complete explanation we must turn to the wave theory. In the direction of the effective ray a large number of rays reach the eye. We have hitherto entirely neglected any differences of phase which may exist between these rays and the interference effects these differences of phase may produce. Now we have seen previously that the method to adopt in considering the effect of a wave of light at any point is to take the wave front at any instant, divide it up into half-period elements, determine the effect due to each element separately, and add together algebraically all these individual effects. Thus the result will depend partly on the form of the wave front, partly on the position of the point considered with reference to the wave front. We

require, then, to find the form of the wave emerging from the drop of water.

Let C D (fig. 98) be the direction of the effective ray as defined above, and let A O B cut C D at right angles in O, then it can be shown that the form of the emergent wave is that of the curve P O Q, which touches A B at O and above C D lies to the right of A B, while below C D it is to the left. O is called a point of inflection. We have then to enquire what will be the illumination produced by such a wave on a screen drawn through D perpendicular to C D. Now this is a complicated problem, and we must be content with stating the results. Let R be the point on the screen at which we require the intensity. Then Sir George Airy has shown that as we travel from D downwards the intensity of the light decreases very rapidly, so that the illumination is soon insensible for points below the effective ray. As we travel upwards from D the intensity increases very rapidly at first and soon reaches a maximum. This corresponds to the primary rainbow.

FIG. 98.

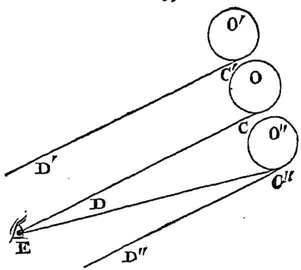


The direction then of the light forming the primary bow does not coincide exactly with that of the effective ray, but is slightly above it. The deviation for the primary bow is therefore very slightly greater than the minimum, and its angular radius slightly less than that given above. After leaving this point R, the intensity rapidly diminishes to a minimum and then increases again to a maximum, passing through a series of maxima and minima as we travel on for a short distance. The maxima, however, decrease after the first, which is much greater than any of the others. To these maxima correspond the supernumerary bows, and since the deviation for these maxima increases as we go from the

primary bow, the radii of the supernumerary bows are less than that of the primary.

Now let E (fig. 99) be an eye placed to see the primary bow formed by a drop centre O; that is to say at a certain small distance above the effective ray C D. Consider a drop centre O′ above O. Its effective ray is C′ D′ parallel to C D, and since E is below C′ D′, but very little light from this drop reaches the eye. The space above the primary bow is all dark.

FIG. 99.



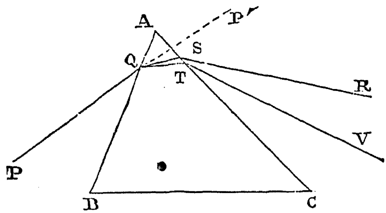
Take now a drop O″ below D; its effective ray is C″ D″ parallel to C D. E is above this ray, and may be the position of one of the additional maxima for the drop O″. At any rate a position can be found for O″ below O such that E shall be a point of maximum illumination for light from O″, though, of course, E cannot be the first maximum. In this case, then, the eye would see one of the supernumerary bows in the direction E O″. The same point E can clearly be a first maximum for O, a second for O″, a third for O‴, &c., O″, O‴, &c., being below O, and in these directions respectively an eye at E will see the primary bow and the successive supernumerary bows. These theoretical conclusions of Sir George Airy were verified by the late Professor Miller, of Cambridge, who measured the angular radii of the primary and supernumerary bows formed by a stream of water drops illuminated by a small bright light.

CHAPTER VIII.

DISPERSION AND ACHROMATISM.

WE have seen already that when a ray of white light falls on a prism of glass or other refracting substance its direction is altered, and the emergent beam is no longer white but coloured. If the light be a ray of sunlight, which, as in Newton's experiment, is admitted to a darkened room through a hole in the shutter, and is allowed to fall on a prism with its edge horizontal and its refracting angle turned downwards, a coloured band will be seen on the opposite wall, red at its lower end, and passing through orange, yellow, green, to blue, indigo, and violet at the upper. This band is, we have learnt, called the solar spectrum, and the fact that white light can thus be split up into differently coloured rays is known as the dispersion of light. We have seen, too, that this dispersion is a consequence of the fact that, in refracting media such as glass or water, the velocity of a wave of light depends on its wave-length, so that red

FIG. 100.



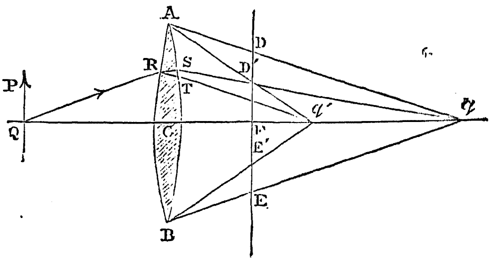
and violet waves travel in the prism with different velocities; they are, therefore, differently refracted by the glass, and emerge in different directions. The angle which any ray on emergence makes with the direction in which the light was originally travelling is called the deviation of that ray, and the angle between the directions of any two emergent rays coming from the same source is known as the disper-

sion of those two rays. Thus, if B A C (fig. 100) be a prism of refracting substance, P Q an incident ray, Q S R, Q T V the refracted red and violet rays, and P Q be produced to P′, the angle between S R and Q P′ measures the deviation of the red ray. That between V T and Q P′ is the deviation of the violet ray, while the dispersion of the prism for the red and violet rays is the angle between S R and T V.

We proceed to consider the effects of dispersion in modifying the action of lenses or other optical instruments.

Let us take, first, a convex lens A B C (fig. 101); let P Q

FIG. 101.



be a luminous object in front of the lens emitting white light, and consider any ray Q R falling on the lens at R. This ray is refracted, but its red and violet components are refracted at different angles along R S, R T respectively; it is dispersed, and this dispersion is further increased by the refraction at the second surface at the points S and T. The red light emerges from S along S q, the violet from T along T q'. A red image of Q will be formed at q, a violet image at q'. In fact, as we have seen before, the part of the lens about the points R, S, T acts just like a prism whose refracting angle is turned towards A. The red and violet rays are both bent downwards. but there is dispersion, the violet ray being deviated more than the red. If we placed between the source of light P Q and the lens a violet glass a violet image would be formed at q', if we placed in the way instead a red glass we

should have a red image formed at q. Q, q are conjugate foci for red light, Q, q' for violet, while the foci for rays between the red and violet lie between q and q'.

Thus, our lens will not form us a white image of the white object. Let us suppose the light falls on a screen perpendicular to the axis of the lens between it and q', and that Q is simply a luminous point. The violet rays will form a cone with q' as vertex, the red rays another cone with q as vertex, and the intermediate rays cones with points between q and q' as vertices. These cones will cut the screen in circles whose centres lie on the axis of the lens. Let us suppose we have only red and violet light, and let D D′ F E′ E be a position of the screen cutting q A, q B in D and E, q' A, q' B in D′ and E′. Throughout the space D′ E′ we have red and violet light mixed, but the annulus D D′ E′ E is only illuminated by the red rays. And if our light be white, the rays of all colours will pass through the central space D′ F E′, which will, therefore, be whitish—not white, because the light will not be mixed in the right proportion, there will be too much violet; as we travel outwards towards D and E there will be less and less of the violet end of the spectrum; finally, when we get to D and E we shall have nothing but red light. Thus our whitish centre will have a red border. If the screen were placed beyond q, the central part of the image formed would be reddish and the border violet.

For some positions of the screen between q and q' we should have a violet border, for others a reddish, and in no position would the image formed be absolutely colourless, though, except with very bright light and a lens of short focal length, the coloured edge, when the object is best defined, is not much noticed.

We can easily show this effect of dispersion by forming an image of a candle-flame on a sheet of white paper with an ordinary magnifying-glass. If the paper be moved towards the lens the image becomes blurred and its border

is coloured red; if, on the other hand, the paper be moved away, the image again becomes blurred, but the border is blue or violet.

Whenever for any reason a lens or system of lenses does not bring all the rays diverging from one point to a focus at another point, there is said to be aberration. When, as in the present case, the aberration is due to the dispersion of the light by the lens, it is called chromatic.

We may consider this chromatic aberration with advantage from another point of view. When we say that two points Q, q are conjugate foci for red rays, we imply that the time taken by red light to travel from Q to q is the same for all paths possible. But the violet rays travel through the lens more slowly than the red, and, therefore, the time taken by violet light in travelling from Q to q will not necessarily be the same for all paths which might be possible. Q, q will not be conjugate foci for violet light; in fact, the only violet ray which will reach q is that which travels along the axis.

Or, again, we know that, if v and u are the distances of the conjugate foci from a convex lens of focal length, f, v, u, and f are connected by the equation

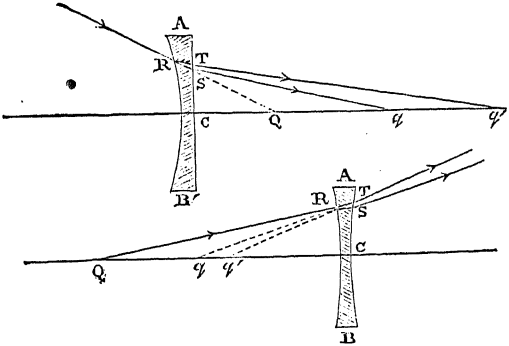
$$\frac{1}{v} - \frac{1}{u} = -\frac{1}{f}.$$

Now the value of f depends on the form of the lens and the refractive index of the material of which it is made. This refractive index is different for light of different colours, and, therefore, f is different for light of different colours, being least for violet, greatest for red light. Then, if the distance of the source of light from the lens—that is, the value of u—be given, the value of v depends on the colour of the light used. A convex lens forming a real image of a distant object brings the violet rays to a focus nearer to the lens than the red. If, on the other hand, the image formed be virtual, the violet focus will be the more

distant of the two. This is clear if we remember that a violet ray falling on the lens at any point is more deviated than the red ray which falls on it at the same point. If our lens be concave the same reasoning shows us that, in the case of a divergent incident pencil, the violet focus is nearer to the lens than the red, while, if the incident light be converging to a point beyond the lens, the violet focus will be the most distant. Fig. 102 gives these two cases, q being the focus for red rays, q' for violet.

This fact, the chromatic aberration of light, is of the utmost

FIG. 102.



importance in the construction of telescopes and other optical instruments. In a telescope a large convex lens or concave mirror collects a great number of the rays from a distant object, and forms at its focus an image of that object. This image is viewed with a lens or lenses, convex or concave, as the case may be, which magnify it. In the case of refracting telescopes, the image formed by an object-glass consisting of a single lens will not be colourless, but surrounded by a coloured ring, which appears red or violet, according to the position of the eye-piece, and all clear definition is rendered impossible. Newton, who, we have seen, discovered the dispersion of light, was of the opinion that this difficulty was insuperable.

A lens forms an image of an object because it deviates or turns from their course the rays of light which fall on it. At the same time, it disperses those rays. Newton, misled by an experiment he made, believed that any means which could be used to prevent the dispersion would also prevent the deviation, and so render the lens useless. The light on emergence, he thought, would be travelling parallel to its original direction, and thus the lens might as well be removed; and this opinion of Newton's became prevalent, and for many years hindered the development of refracting telescopes.

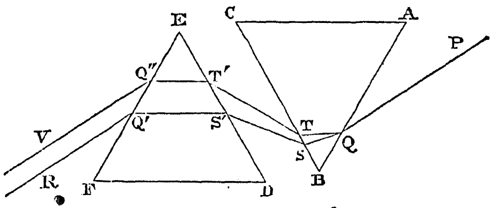
It was this belief which led Newton to construct the reflecting telescope known by his name; for in a reflector the image is formed by reflexion at a polished surface, and, rays of all colours being reflected according to the same law, chromatic aberration does not come in. Of course it does affect the image formed by the eye-lens, but this is equally the case with refractors. This same belief led Huyghens to construct giant refracting telescopes with object-glasses of 60 to 100 feet in focal length, for the effect of chromatic aberration is less marked in lenses of long than in those of short focal length.

We shall now show how it is possible to combine two or more lenses to produce deviation without dispersion—to form a combination which shall be achromatic, that is, without chromatic aberration. This was first discovered by Mr. Hall, of Worcestershire, and rediscovered by Dollond, a London optician.

Let us take the case of a prism first, and suppose we have a ray P Q (fig. 103) falling on a prism A B C. Consider the red and violet refracted rays, which emerge in the directions S S′ and T T′ respectively. Can we by any means combine them so as to make them again parallel? One method is very simple. Place behind the prism A B C a second prism D E F of the same material and with the same angle, but in such a position that the refracting edges of the two

are turned in opposite directions, and turn the second prism until D E becomes parallel to C B, B and E being the refracting angles. Then clearly the rays falling on this second prism are deviated by it through the same angle as by the first, but in the opposite direction. The red ray

FIG. 103.



falling on the second prism at S′ emerges along Q′ R, the violet ray at T′ emerges along Q″ V, and these two lines are parallel; the two prisms act just like a plate of glass. The dispersion has been destroyed, but so too has the deviation, for R Q′ V Q″ are both parallel to P Q.

The ratio of the dispersion to the deviation is the same in the second prism as in the first. The two being of the same material, whenever then we make the two dispersions equal in amount but opposite in direction, so that the emergent beam is achromatic, we do the same for the two deviations, and the emergent beam is parallel to the incident. Newton's mistake lay in supposing that this ratio of the dispersion of two rays to the deviation of one of them was the same for all media, so that in all cases in which we produced two equal and opposite dispersions we produced two equal and opposite deviations.

Hall and Dollond showed that this was not the case. The ratio—dispersion to deviation—is very different for different media, so that it is possible to produce two equal and opposite dispersions without producing two equal and opposite deviations. If I have given a prism A B C, I can construct a second D E F, which, when placed to receive the

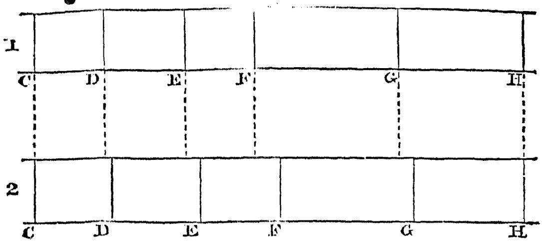
light from A B C, shall render the two emergent rays Q′ R Q″ V parallel to one another, but not to the incident ray P Q. The refracting edge E of the second prism will be turned in the opposite direction to that of the first. The dispersion which it produces is the same as that produced by the first, though opposite in direction; the deviation, too, is opposite in direction but not of the same amount. The emergent beam is achromatic, but it has been turned through an angle equal to the difference of the deviations of the two prisms separately.

Thus, for example, the ratio dispersion to deviation is very much greater in flint glass than in crown glass. If we have a prism of an angle of 60° of crown glass we can produce the same dispersion between the red and violet rays with a flint glass prism of about 37°. The minimum deviation for the yellow-green rays for the crown-glass prism will be about 40°, that for the flint glass only 25° 48′, so that by combining these two we shall get our red and violet light combined and have a deviation of 14° 12′, the difference between 40° and 25° 48′. The combination is achromatic for the red and violet rays. In order to combine by means of two prisms two given colours, we must arrange the prisms so that the distance between these colours in the spectrum each would form on the screen if used separately shall be the same. Then, using the two together, with their edges in opposite directions, these two colours are combined. If, for example, we wish to combine the red and violet light seen near the dark lines C and H respectively in the solar spectrum, we must take two prisms which would give spectra in which the distance from C to H was the same; then, using the two together, this distance would be reduced to nothing, or the light from the lines C and H would be combined.

But the spectra formed by our two prisms separately would by no means be identical; we should find that although the whole distance C H was the same in both; so

that if, as in fig. 104, the two spectra were placed one above the other, the lines C and H in the two would appear continuous: this would not be the case for the other dark lines such as D, E, F. In the lower spectrum, that from the crown-glass prism, they appear pushed to the right so that the distances C D, C E and C F are not the same for the two. When the two are combined, then, though the lines C and H coincide, the same will not be true for the other lines D, E, F, &c. The compound image formed will not be absolutely colourless, though much more nearly so than that formed by either prism separately, the dispersion between two rays such as C and E being the difference of the dispersions in the two

FIG. 104.



prisms separately. In short, just as the ratio dispersion to deviation is different for different media, so also is the ratio of the dispersion of two colours to that of two others. In consequence of the first law we are able to construct a combination of two prisms which shall be achromatic for two given colours. In consequence of the second that combination is not achromatic for all colours. We can, however, combine three colours by using three prisms, and so on. The spectra formed when two prisms are combined in this manner are called secondary spectra.

The fact that the ratio of the dispersion of two colours to that of the extreme colours is not constant for different media is known as the irrationality of dispersion.

We have thus seen how to form an achromatic combi-

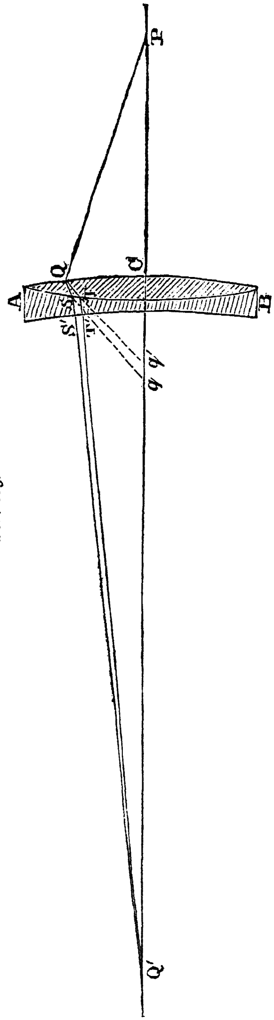


FIG. 105.

nation of two colours by means of two prisms. Let us turn now to the case of a lens. The rays refracted through the convex lens in the neighbourhood of any point Q (fig. 105) travel after refraction as if they had passed through a prism with faces parallel to the tangent planes to the lens near Q. They are therefore dispersed, but this dispersion can be corrected without destroying entirely the deviation by allowing the rays to pass through another prism of a different material with its edge turned towards the axis of the lens. The same is true for all points of the first lens, only, of course, the angle of this correcting prism would have to vary for different positions of the point Q. Now suppose we place behind the convex lens a second concave lens of different material. The red and violet rays into which the ray P Q is dispersed by the first lens fall on this second lens at S and T, which are not far apart. This second lens affects these rays as a prism bounded by the tangent planes at S or T, and the opposite face of the lens; and by inclining these tangent planes at a proper

angle, that is, by properly adjusting the form of the second lens, the dispersion produced in the ray P Q by the first lens can be destroyed, so that the emergent rays S′ Q′, T′ Q′ will cut the axis of the two lenses in the same point Q′, and the red and violet images produced by the combination be brought into coincidence.

In other words, considering only the first lens, the violet ray incident along P Q is refracted to q', the red ray to q; these two rays fall on the second concave lens. Both are refracted upwards by it, the violet more than the red, and it is possible to adjust the forms of the lenses and the distance between them, so that the focus for the red rays on emergence from the second lens coincides with that for the violet. A combined red and violet image is produced at Q′.

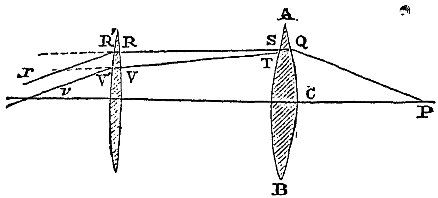
In practice the first face of the concave lens is usually made of the same curvature as the second face of the convex, and the two are brought into contact. We have then at our disposal the curvature of the second face of the concave lens, and that can be made such as to combine two of the colours in the image formed by the combination. Owing to irrationality of dispersion the image formed will not be perfectly colourless, but much more nearly so than if the convex lens had been uncorrected. The combination generally adopted in the object-glass of an achromatic telescope is a convex lens of crown glass followed by a concave lens of flint glass.

The two colours chosen for combination are not the extreme red and violet of the spectrum, but two rays from the more luminous part, a ray from the yellow-orange being combined with one from the green-blue. Thus suppose we had a convex lens of crown glass of 20 centimètres focal length. To correct this with a flint-glass concave lens so as to combine the rays C and F of the solar spectrum we should require a lens of 33 centimètres focal length, and the focal length of the combination would be

about 51 centimètres. The focal length of the concave lens required to correct for these two rays a given convex lens will always be to the focal length of the convex lens in the ratio of 33 to 20, while the focal length of the combination will be about $\frac{51}{20}$ of that of the convex lens. The combination thus formed will be achromatic only for rays coming from a very distant point, so that the incident wave may be considered as plane.

The combination of lenses described above will form for us an achromatic image of a distant object. Now in a telescope this image is viewed with an eye-piece, and this, at any rate if it consists of a single lens, will again produce

FIG. 106.



chromatic aberration. Now we have seen that when an eye-piece is in adjustment for giving distinct vision, the rays of one definite refrangibility from each point of the object viewed, emerge parallel. If the violet rays coming from a point P of the object are parallel among themselves, the red rays from P are also very nearly parallel but not parallel to the violet. A virtual violet image is seen by the eye in the direction of the violet light; a virtual red image in the direction of the red light; and these two only partially overlap.

The white ray P Q (fig. 106) is dispersed into S R and T V, a red and violet ray with the others between them. Now the effect of two parallel rays in producing vision is the same as if the rays were coincident. If then by any means we can render the violet ray T V parallel to the red ray S R before they reach the eye, they will

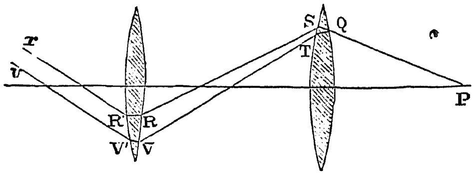
appear to the eye to come from the same point. The virtual red and violet images of the point P which the eye sees will coincide, and the combination will be achromatic for these red and violet rays. Let us then consider the effect of placing behind the first lens another convex lens, and let us suppose that S R and T V fall on this lens at R and V before they cross the common axis of the two lenses. V will be nearer to the centre of this lens than R, and the faces of the lens at the points V, V′ in which the violet ray cuts them will be more nearly parallel than they are at the points R, R′ in which the red ray cuts them.

Let R′ *r* be the direction of the emergent red ray, and let V′ *v* be parallel to R′ *r*. We wish to determine if it is possible for V′ *v* to be the direction of the emergent violet ray. Now clearly the angle between R′ *r* and S R produced is greater than that between V′ *v* and T V produced, and the first of these angles is the deviation of the red ray, while the second would be that of the violet ray. Thus, in order that V′ *v* may be the violet ray, the deviation produced by the lens at R, R′ in the red ray must be greater than that produced at V, V′ in the violet ray. If the angle between the faces of the lens at R, R′ were equal or less than that at V, V′ this would clearly be impossible, but since the angle at R, R′ is greater than that at V, V′, it is possible for the deviation of the red light at R to be greater than that of the violet light at V; that is to say, it is possible for the violet ray to emerge in the direction V′ *v* parallel to the red ray R′ *r*. We can then combine two convex lenses to form an achromatic eye-piece.

The dispersion produced by the first lens is compensated in the second from the fact that the red light falls further from the centre than the violet, at a point, therefore, where the refracting angle of the prism which would produce the same deviation as the lens is greater than at the point of incidence of the violet light. A similar argument will apply to the case in which the light cuts the axis of the lenses

between the two. In this case (fig. 107) V will be further away from the centre of the second lens than R, and in order that the two colours may emerge parallel, the violet light must be more deviated than the red. This it will be for two reasons. Firstly, because it is violet light and is therefore more deviated than red incident in the direction T V would be; and secondly, because the faces of the lens at V, V′ are inclined to each other at a greater angle than they are at R, R′, so that red light travelling along T V would be more deviated than red light along S R; thus *à fortiori* the violet ray T V is more deviated by the second lens than the

FIG. 107.



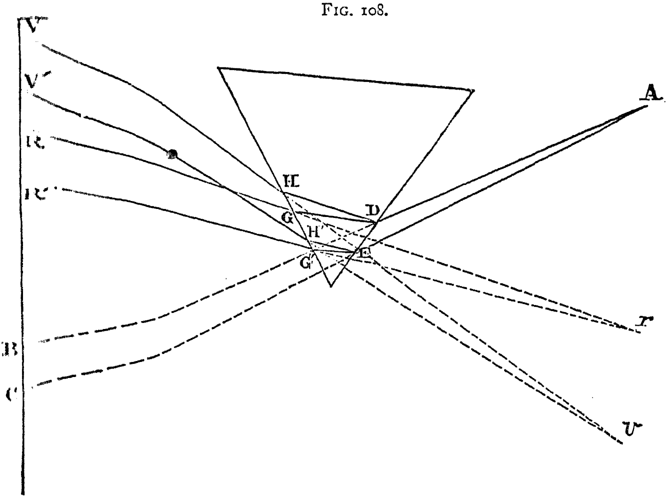
red ray S R, and the two may emerge parallel. Thus a second convex lens placed behind the first lens of an eye-piece tends to correct the chromatic aberration produced by the first lens. It may, of course, over-correct it, but in general if we have two convex lenses of given focal length we can put them at such a distance apart that they form an achromatic combination.

The complete investigation shows that if f_1 f_2 are the focal lengths of two lenses of the same material, and the light incident on the first be inclined at a small angle to the axis, the combination will be achromatic when the distance between the two lenses is $\frac{f_1 + f_2}{2}$. This condition is exactly satisfied in Huyghens's eye-piece, though it was designed by its inventor for quite different reasons, and without any knowledge of achromatic combinations.

We have already described Newton's experiment with

he prism; we must now, however, consider it a little more ully. Let us suppose the sunlight comes through a narrow lit, A (fig. 108), whose length is perpendicular to the paper. We shall have a sort of wedge of light diverging from the lit and falling on our screen at B C. A prism with its edge arallel to the slit is placed in the path of this wedge of ight, and a spectrum is formed on the screen. Turn the rism about until the lower end, R′, of the spectrum is as

FIG. 108.

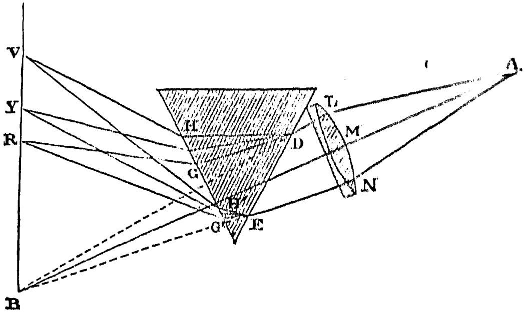


ear as possible to B C, so that the deviation is a minimum. Had we been using red light we should have found on our creen, near R, a red patch of light about the same width as C. In fact, the red light all comes as if it diverged from a irtual image, *r*, of the slit, *r* being at the same distance rom the prism as the slit. The violet light again comes rom a virtual image, *v*, and the intermediate rays from mages between *v* and *r*. Thus, corresponding to rays of ach colour, we have coloured patches of light. Let A B, A C neet the prism in D and E. Let D G, E G′ be the paths of

the red rays, A D, A E, through the prism, D H, E H′ those of the violet rays. Join r G, r G′ and produce them to meet the screen in R, R′. R R′ will be the breadth of the red patch of light which would be formed were the incident light all red.

Join V H, V H′, and let them meet the screen in V, V′. V V′ will be the violet patch. Now, it is clear that if the screen is sufficiently near to the prism, these red and violet patches will overlap, and the colours there be mixed, and for all positions of the screen, if the sun be the source of light, so that we have rays coming from all points between

FIG. 109.



v and r, the rays coming from one point will overlap those from the next, and the colours will be mixed. The spectrum will not be pure. We must introduce some additional apparatus. This is most simply done (fig. 109) by putting a convex lens, L M N, between the slit and the prism, and so arranging the screen that, before the prism is brought into position, the lens forms an image of the slit on the screen, as at B, so that we have a narrow line of light at B instead of a broad patch. The light, then, which falls on the prism placed behind the lens is converging to B instead of diverging from A, and if the prism be placed in a position of minimum deviation, the rays of any definite refrangibility will converge so as to form a real image of B after refraction

through the prism, instead of diverging, as before, from

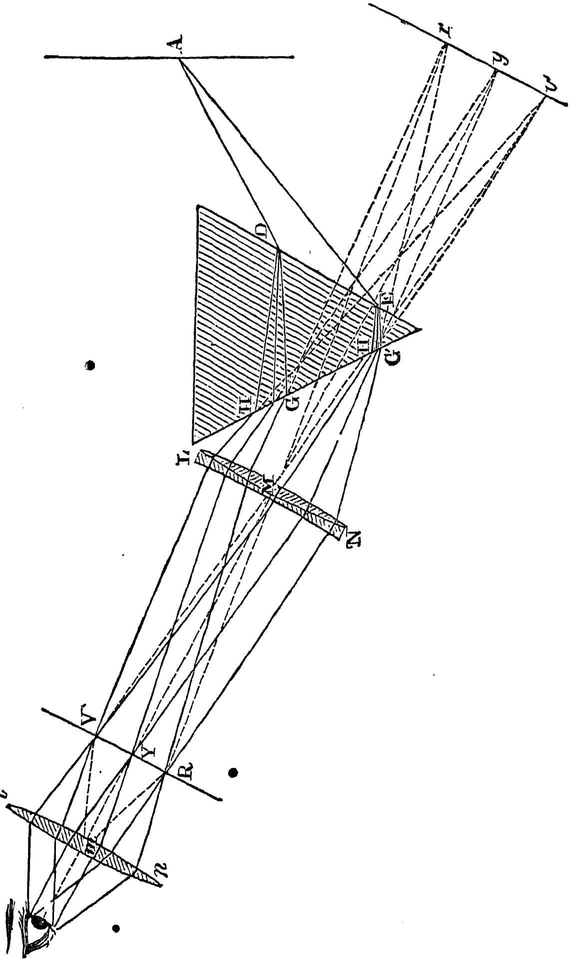


FIG. 110.

virtual images of the slit. Thus, we shall get behind the prism a series of real images, V, Y, R, of B, each formed by

light of one definite colour. These images, arranged side by side, form the pure spectrum. Of course, B being an image of the slit A, the images which thus constitute the spectrum are images of the slit. All the yellow light of a certain refrangibility has been made by the lens and prism to converge to Y, and form there a yellow image of the slit, and so on. The spectrum thus formed may be viewed on the screen, or it may be looked at from behind with a lens or eye-piece.

But we can arrange our lens and prism to give us the pure spectrum somewhat differently. Let us suppose that the light, after passing through the prism, is received on a convex lens, L M N (fig. 110). The violet rays, diverging as they do from the virtual focus *v*, will be brought to a focus at some point V on *v* M produced, M being the centre of the lens, and a real image of the slit will be formed at V by these rays. The red light will come to a focus at R, a point on *r* M produced, and we shall have at R a red image of the slit. Rays of intermediate refrangibilities will form images in positions intermediate between V and R, and we shall thus get a pure spectrum on a screen placed in the position V R to receive the light. But this spectrum will, in general, be small, though very bright. If, then, we wish to see it distinctly, we can place another convex lens, *l m n*, behind it, adjusting the distance between *l m n* and the screen, which we must suppose to be transparent, so that the spectrum is distinctly seen. But these two lenses—a convex lens, L M N, focused on the virtual images *v r*, and a convex eye-piece, *l m n*—constitute an astronomical telescope adjusted to view these virtual images. If, then, we receive the light from the prism on a telescope, we shall be able, by focusing the telescope, to see a pure spectrum.

But the lenses are not necessary if we merely wish to see the spectrum, for our eye itself is a convex lens, and by placing the slit at such a distance that we can see it distinctly, and then interposing a prism, adjusted so that the

deviation of the light passing through it is a minimum, the rays, on emerging from the prism, will appear, as before, to come from a series of virtual images, *v r*, and the lenses of our eye will form on the retina a real image of these points—that is, a pure spectrum. Since the distance between the prism and the slit is the same as that between the prism and the images *v r*, and the slit has been placed so that we can see it distinctly, the virtual images *v r* are in the position for producing distinct vision on our eye, situated just behind the prism.

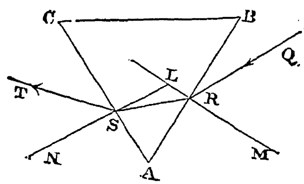
It remains now to point out why the prism in all these experiments is placed in one particular position—viz., that of minimum deviation. Let us consider any pencil of rays of a definite refrangibility diverging from a point and falling obliquely on the prism. Since the incidence is oblique the refracted pencils will not diverge from, or converge to, one given point. There will be no focus conjugate to the origin of light Q, but, as we have seen (Chapter IV.), there will be two focal lines, lying respectively in, and perpendicular to, the plane of incidence, and through these all the refracted rays will pass. This refracted pencil falls on the second surface of the prism, and is there again refracted. The emergent pencil will pass through two focal lines at right angles, and the nearest approach to an image of Q will be the circle of least confusion somewhere between them.

When, then, a divergent pencil of rays of one refrangibility falls on a prism, and is refracted through, the emergent pencil does not, in general, appear to diverge from a point, but from two focal lines at right angles to each other, so that if the light were received on a lens, instead of getting on the screen a real image of Q, we should get for one position of the screen a real image of the primary line, for another a real image of the secondary. There is, however, it may be shown, one position of the prism for which the primary and secondary focal lines coincide, and when,

therefore, the emergent pencil appears to diverge from one point. If we obtain the formulæ which determine the positions of these focal lines, we shall find that this is the case when the angle which the incident light makes with the face of incidence is equal to that which the emergent light makes with the face of emergence.

Let Q R S T (fig. 111) be the path of the ray through the prism. Let L R M be normal to the face A B at R, L S N, to the face A C at S, and, as before, let Q R M $= \phi$, S R L $= \phi'$, R S L $= \psi'$, T S N $= \psi$. Then, the formulæ show us that the

FIG. 111.



primary and secondary focal lines will coincide if $\phi = \psi$, and therefore $\phi' = \psi'$. Now, if D be the deviation of the light passing through the prism, and i the refracting angle, B A C, we have shown already (p. 52) that

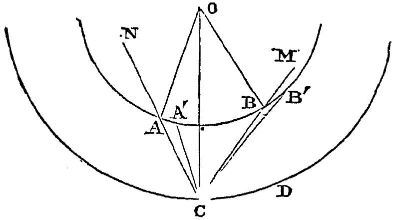
$$D = \phi + \psi - i;$$
$$i = \phi' + \psi'.$$

We shall show now that D is a minimum when $\phi = \psi$. Thus, this being the case, when D is a minimum the primary and secondary focal lines of the emergent pencil will coincide, and it will appear to diverge from a point. To prove that D is a minimum when $\phi = \psi$, we shall have recourse to a simple geometrical construction giving the deviation of a ray refracted in any manner.

With any point, O (fig. 112) as centre, draw two circles, A B, C D, with radii in the ratio of 1 to μ, μ being the refractive index of the medium, supposed denser than air. Draw O A, a radius of the lesser circle, in a direction parallel to that of

the incident light. From A draw N A C parallel to the normal to the surface at the point of incidence to meet the second

FIG. 112.



circle in C. Join O C. Then O C is the direction of the ray in the prism. For N A O is clearly equal to ϕ, the angle of incidence, and $\sin \text{NCO} : \sin \text{NAO} = \text{AO} : \text{CO} = 1 : \mu$,

$$\therefore \sin \text{NCO} = \frac{\sin \phi}{\mu},$$

$\therefore$ N C O $= \phi'$, the angle of refraction,

$\therefore$ C O is parallel to the direction of the refracted ray, and the angle A O C measures the deviation produced by the first refraction.

Draw C B M parallel to the normal at the point in which the ray cuts the second surface of the prism to meet the inner circle in B; join O B. Then, similarly, O B is parallel to the direction of the emergent ray, M B O $= \psi$, M C O $= \psi'$, and the total deviation is A O B. Thus, the deviation will be a minimum when A O B or the arc A B is a minimum. Again, C A and C B being parallel to the normals at the points of incidence and emergence, the angle A C B is equal to the angle of the prism, and is constant. Thus, A B is the arc cut off from the inner circle by two lines inclined at a constant angle to each other.

We shall show that A B is least when C A is equal to C B. For let C B be greater than C A, and draw C B′ inclined at a small angle to C B in such a way that C B′ is greater than C B. Make the angle A′ C B′ = angle A C B. Then C A′ is less than

C A, and angle B C B′ = angle A C A′. If this angle be very small we may treat A A′ and B B′ as if they were straight lines, and the figures A C A′ B C B′ as if they were triangles. But C B′ is greater than C A′, and C B is greater than C A, and angle A C A′ = angle B C B′. Thus, the base B B′ is greater than the base A A′. Hence, the arc A′ B′ is greater than the arc A B. Thus, if C B is greater than C A, the arc A B is increased by increasing the inequality between C A and C B.

Similarly, if C A is greater than C B, the arc will be increased by increasing this inequality. Thus, the arc A B must be least when C A is equal to C B. But, in this case, the angle O C A is equal to the angle O C B, or $\phi' = \psi'$, and therefore $\phi = \psi$. But when the arc A B is least the deviation is least. Thus, the deviation is least when $\phi = \psi$, and in this case the emergent light diverges from a focus, and not from two focal lines.

If, therefore, to obtain our spectrum we use simply a slit and a prism, followed by a lens or telescope, the prism must be placed in the position of minimum deviation.

An instrument fitted to observe a spectrum is called a spectroscope. If in addition it has a graduated circle, by means of which we can measure the deviation of the emergent light, it becomes a spectrometer. But we have not yet overcome all the difficulties attendant on the production of a pure spectrum. The condition that the two focal lines of the emergent pencil may coincide is that ϕ should be equal to ψ. Now, if we are using white light so as to get a spectrum, ϕ is the same for all the colours, ψ, however, is different. And thus the prism cannot be at once in the position of minimum deviation for all colours. Thus, for example, when the red end of the spectrum is in focus, as seen through the telescope, the violet will be out of focus. We must, of course, have an achromatic object glass to our telescope, or we shall be still further troubled by the chromatic aberration produced by the refraction

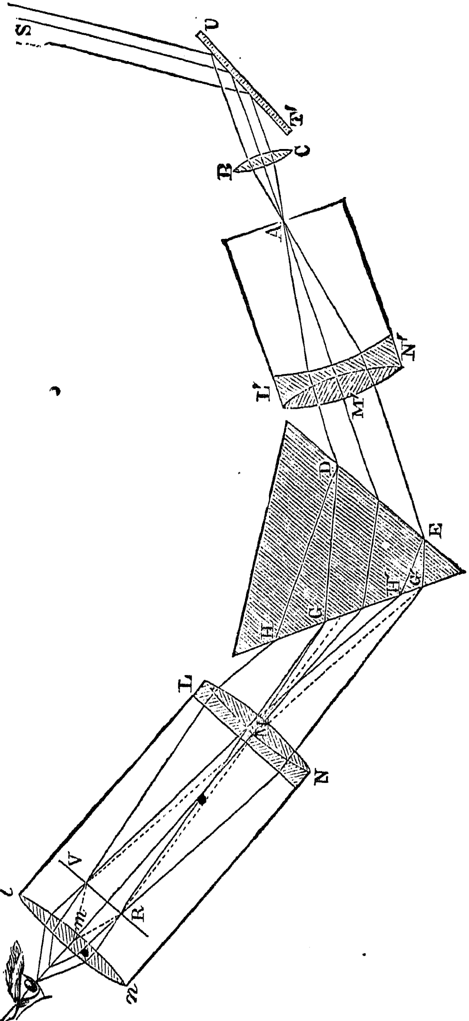


FIG. 113.

through it. This difficulty is best obviated by putting between the slit and the prism another convex lens, which is also achromatic, and adjusting this lens, L′ M′ N′ (fig. 113), so that the distance between it and the slit is equal to its focal length. These being the case, all the rays from it will emerge parallel to A M′. Now, a pencil of parallel rays of given refrangibility remains parallel after any number of refractions at plane surfaces. Thus the violet rays will emerge from the prism all parallel among themselves, while the red rays also will remain parallel among themselves, though, of course, they will be inclined to the violet, owing to the dispersion produced by the prism. If now these differently coloured pencils fall on the achromatic object-glass L M N of a telescope focused to receive parallel rays, the violet rays will be brought to a focus at V, and the red rays will at the same time be brought to a focus at R. Both violet and red will be seen distinctly for the same adjustment of the telescope, or at least any alteration in adjustment which may be necessary will be due to a want of achromatism in the telescope. A lens arranged in the manner described above, to produce parallel rays, is called a collimator. One point requires further notice. If a collimator be used, the rays of any one colour coming from the prism will always be parallel whatever be the angle of incidence. There is, therefore, no need to place the prism in a position of minimum deviation to obtain a pure spectrum. For making measurements and comparisons, however, we must know that the prism always occupies the same position, and this position of minimum deviation is the one which it is most easy to recover. The prism, therefore, even when a collimator is employed, is usually put in the position of minimum deviation.

We have spoken throughout of the sun's rays passing through the slit and falling on the prism, but owing to the earth's rotation, the sun appears to move, and the direction in which they come to the slit is continually changing.

Thus we should have continually to move our prism and lenses to allow the light to reach them. This difficulty is obviated by placing a movable mirror, T U, outside the slit, and adjusting it so as to reflect the sun's rays in any required direction. The mirror is then made to move, following the sun but at only half the rate, round the polar axis of the earth, and the motion is so arranged that the light is always reflected by the mirror in the same direction, and so when the lenses and prisms are once adjusted no alteration is necessary. This apparatus is called a heliostat.

Again, it is clear that of the broad beam of parallel rays which falls on the shutter only a very small portion can pass through the narrow slit. This may be remedied by placing between the slit and the heliostat a convex lens, B C, of somewhat short focal length, and adjusting it to form an image of the sun on the slit. All the light, then, which falls on the lens passes through the slit, for the sun's image will be practically a point, or, if a cylindrical lens is used, a line of light parallel to the slit.

That part of this light which falls on the collimating lens emerges parallel, and falling on the prism forms a spectrum.

Fig. 113 shows the arrangement, T U being the heliostat, and B C the convex lens forming an image of the sun on the slit at A.

The arrangement of the telescope and collimator in a spectrometer has been described already in the Chapter on a diffraction grating and measurement of wave-lengths. We shall now show how the instrument may be used to measure the refractive index of a given medium for a given ray. We have already proved the formulæ

$$D = \phi + \psi - i$$
$$i = \phi' + \psi'$$

And we have shown that when D is a minimum for any given ray $\phi = \psi$, $\phi' = \psi'$.

Let us suppose D_1 is the minimum value of D, then

$$D_1 + i = 2\phi,\ i = 2\phi'.$$

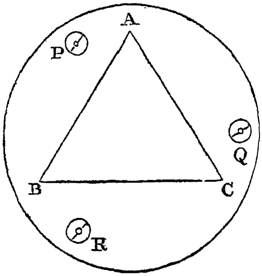
But if μ is the refractive index,

$$\mu = \sin \phi \Big/ \sin \phi' = \sin \frac{D_1 + i}{2} \Big/ \sin \frac{i}{2}$$

If, then, we can measure the minimum deviation of the light passing through a prism of the medium, and also the angle of the prism, we can obtain a value for μ.

Let us suppose that the telescope and collimator have been adjusted for parallel rays, as in Chapter VII. (fig. 91), and that we have also made sure that their axes are in the same plane as the graduated circle. The prism is then fixed with wax or cement on to the levelling stand, which is placed in position on the central table. The first step is to level the prism, that is, to place it so that its faces and therefore the line of their intersection are perpendicular to the plane of the graduated circle. This is done for each face successively, exactly in the same manner as the grating was levelled. One point must be attended to if we do not wish to spend a long time over the levelling. Let A B C (fig. 114) be the prism, P Q R the tops of the levelling screws. Level the face A B, then in general A C will not be adjusted, and in levelling it the position of A B will be altered; while in bringing A B into position again, A C will be moved, and so on.

FIG. 114.

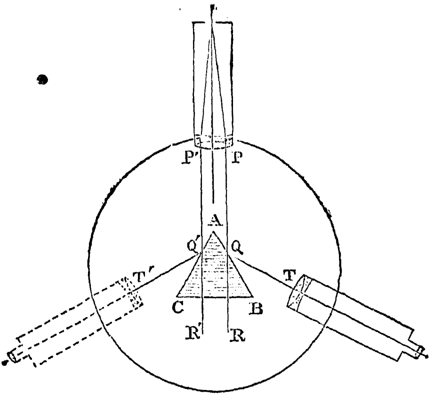


But now let us suppose that the face A B is perpendicular to the line P Q, which joins two of the screws. Then, clearly, altering the third screw R only causes A B to turn in its own plane, and does not affect its inclination to the plane of the graduated circle. If then we level A B first, we can bring A C into position by altering the screw R without affecting A B. Thus we should put the prism on the stand

so that one face is at right angles to the line joining two screws. This can be easily done by eye with sufficient exactness.

We must at the same time be careful that the prism is so fixed that it is possible to place it so as to see the slit by reflexion from both faces on bringing the telescope into the required position; and secondly, so that when the prism is placed in the position of minimum deviation the light from the central portion of the collimator lens falls on it and

FIG. 115.



not beside it. These conditions are generally attained by placing the prism with its edge a little, but not much, in front of the centre of the graduated circle.

The prism being adjusted, we proceed to measure its angle. Turn the table of the spectroscope so that the edge of the prism is towards the collimator, and the light from the collimator lens falls on both faces and is there reflected; any moderately bright source of light may be used. Turn the telescope until the light reflected from the face A B (fig. 115) falls on its object glass and an image of the slit is seen in the field of view. Clamp the telescope, and by means of the

tangent screw bring the vertical cross-wire or needle-point to coincide with the image of the slit, then read the vernier attached to the telescope. Now unclamp and turn the telescope to the position T′ in which the image reflected from the face A C is seen; and again read the vernier. We shall show that the difference between these two readings is twice the angle of the prism. The difference between the two readings is clearly the angle through which the telescope has been turned. Now let P Q T be the path of a ray reflected at A B, P′ Q′ T′ that of a ray reflected at A C; the incident rays being parallel, P Q is parallel to P′ Q′. In the first case, the axis of the telescope coincided with P T, in the second with Q′ T′, so that the angle which has been observed is the angle between Q T and Q′T′.

Let us call this angle δ, and let i be the angle of the prism; produce P Q to R, P′ Q′ to R′. Then since δ is the angle between T Q and T′ Q′, and Q R is parallel to Q′ R′, it is clear that $\delta = \text{TQR} + \text{T}'\text{Q}'\text{R}'$.

Now $\quad \text{TQB} = \text{PQA} = \text{BQR}.$

Thus $\quad \text{TQR} = 2\,\text{BQR}$

similarly $\quad \text{T}'\text{Q}'\text{R}' = 2\,\text{CQ}'\text{R}'$

$\therefore\ \text{TQR} + \text{T}'\text{Q}'\text{R}' = 2\,(\text{BQR} + \text{CQ}'\text{R}') = 2\,\text{BAC},$

for Q R, Q′ R′ are parallel.

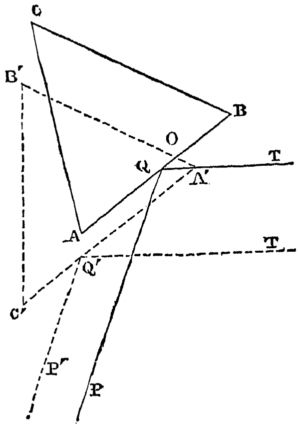
Thus $\delta = 2\,i$; or the angle of the prism is half the angle through which the telescope has been turned, *i.e.* half the difference between the two observed readings.

To ensure accuracy, of course the observation should be made several times, moving the table on which the prism stands between each, so as not to use exactly the same part of the graduated circle.

If, as in many instruments, a vernier is attached to the movable table on which the prism rests, so that its position can be determined, another method may be used. Clamp the telescope to the circle so that its axis is inclined to that of the collimator, and turn the prism until an image of the slit is seen in the telescope by reflexion at the one face. Read the

vernier of the table. Then turn the prism round until the slit is again seen by reflexion from the other face, and bring its image into coincidence with the cross-wire, and again read the vernier. Then the difference between the two readings will be equal to $180° - i$. Let this difference be δ. Let A B C (fig. 116) be the prism in the first position, A′ B′ C′ in the second; then it is clear that the face A′ C′ must be parallel to the face A B in the first position. Let A′ B′ meet A B in O. A′ B′ has been turned from the position A B. Thus B′ O B is δ, the angle through which the prism has been turned; and since A′ C′ is parallel to A B,

FIG. 116.



$$\text{B O A}' = \text{C}' \text{ A}' \text{ B}' = i, \text{ and } \text{B O A}' + \text{B O B}' = 180°.$$

Thus $$i + \delta = 180°$$

or $$i = 180° - \delta.$$

Having thus found the angle of the prism, we require only to measure the minimum deviation of the refracted light in order to calculate the refractive index. But this deviation depends on the nature of the light used and is different for different parts of the spectrum. Let us suppose, then, we are using a homogeneous source of light—a Bunsen burner, with a bead of salt giving the yellow sodium light only. Turn the telescope to view the slit directly, and bring its image into coincidence with the cross-wire. Then read the vernier. (Of course when the telescope is turned to view the collimator directly the prism is between the two, but it can generally be placed so as not to intercept all the light.) Move the prism so as to obtain a refracted image

of the slit, and turn the telescope till this image comes into the field. Let us suppose the telescope has been turned to the right. On moving the prism round the axis of the graduated circle the image of the slit moves across the field. Turn the prism so as to cause the image to move to the left, that is to say, so as to bring it as close as possible to the direction of the direct light, and follow it with the telescope, keeping it always in the field of view. After a time a position will be found for the prism, such that motion in either direction causes the image to move to the right, that is to say, causes an increase in the deviation. The prism is then placed in the position of minimum deviation. Bring the cross-wire of the telescope into coincidence with the image and take the reading of the vernier; the difference between this and the first reading will be the minimum deviation required; and if this is D_1, the value of μ is given by the formula

$$\mu = \sin \frac{(D_1 + i)}{2} \Big/ \sin \frac{i}{2}$$

The value of μ depends on the medium used and the nature of the light.

Thus for a prism of crown glass of refractive angle 60° we might find :—

Direct reading . . .	183°	15′	30″
Deviation reading . .	143	29	10
D_1	39	46	20

$$\frac{D_1 + i}{2} = 49° \quad 53′ \quad 10″$$

$$\frac{i}{2} \quad = 30°$$

and $$\mu = 1 \cdot 5296.$$

We saw, when treating of diffraction, that the pure spectrum of the sun, formed by a grating, was crossed by a number of dark lines. Light of various definite wave-lengths is absent from the sun's rays, and the spaces in the

spectrum at which those rays would have formed images of the slit are therefore black. The same, too, will be the case in the pure spectrum formed by a prism, and these dark lines, which we then used as marks we could recognise, and for which we could accurately determine the wave-length, we can now use as marks to assist us in determining the refractive index. Just as we found the wave-length of one of the dark lines A, B, or H, so we may find the refractive index of a given medium for the same lines. This has been done by many experimenters for various substances. The following table gives some of the results. To determine the refractive index of a fluid, a hollow prism, whose faces are made of carefully worked plane parallel glass, is used. This is filled with the fluid to be observed, and the measurements are made as for solid media.

Table of Refractive Indices.

Line of the Spectrum	Material				
	Hard Crown Glass	Extra Dense Flint Glass	Water at 15° C.	Carbon Disulphide at 11° C.	Alcohol at 15° C.
A	1·5118	1·6391	1·3284	1·6142	1·3600
B	1·5136	1·6429	1·3300	1·6207	1·3612
C	1·5146	1·6449	1·3307	1·6240	1·3621
D	1·5171	1·6504	1·3324	1·6333	1·3638
E	1·5203	1·6576	1·3347	1·6465	1·3661
b	1·5210	1·6591	—	—	—
F	1·5231	1·6642	1·3366	1·6584	1·3683
G	1·5283	1·6770	1·3402	1·6836	1·3720
h	1·5310	1·6836	—	—	—
H	1·5328	1·6886	1·3431	1·7090	1·3751

The numbers in the first two columns were obtained by Dr. Hopkinson ('Proc. Royal Soc.,' June 14, 1877), those in the last three by Messrs. Gladstone and Dale. The index of refraction of air for light in the neighbourhood of the line E is 1·000294.

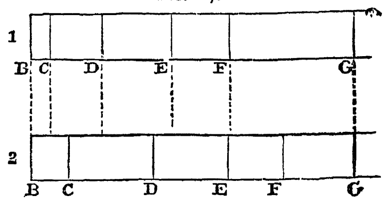
There is one important distinction to be noticed between

the refraction and diffraction spectra. In the latter of the two the sine of the deviation, and, therefore, approximately, the deviation, since it is small, is proportional to the wave-length. Thus, the distance between any two dark lines varies as the difference of their wave-lengths.

No such simple law connects the wave-length and the deviation in the refraction spectrum, and the distance between two dark lines will, as we have seen, be different for two media, even when the dispersion of the extreme rays is the same.

Fig. 117 gives approximately the position of the dark lines in the diffraction spectrum and that produced by a

FIG. 117.



prism of flint glass, the total dispersion between the lines B and G being the same. It will be seen that in the refraction spectrum (fig. 117) (1), towards the red end the lines are closer together, and towards the violet they are wider apart, than in the diffraction one (fig. 117) (2).

There is, of course, a relation between the deviation produced by a prism and the wave-length, but it is not so simple as in the case of diffraction. Cauchy, however, has indicated, from some theoretical considerations, the form of an equation which may express this connection.

From a mathematical standpoint we consider the ether as a body which has been thrown into a state of strain, and the particles of which are vibrating. The laws of these vibrations can be deduced from the mathematical equations of the motion, and we arrive at the ordinary laws of the

propagation, reflexion, and refraction of light. But, in order to simplify our equations, certain small quantities have been neglected, so that our first result, according to which all waves would travel at the same rate, is only approximate. If we proceed to a higher order of approximation we find that there is a relation between the velocity and the wave-length; and, on Cauchy's theory, if μ be the refractive index and λ the wave-length, the relation is expressed by the formula—

$$\mu = a + \frac{b}{\lambda^2} + \frac{c}{\lambda^4} + \dots$$

a, b, c, &c., being constants depending on the medium.

The terms after the third are in general very small; if, then, we know the values of μ for three wave-lengths, we can find values for a, b, c, and this formula would enable us to determine the value of μ for any other wave-length. We can test our formula by measuring the value of μ for that wave. The results of a comparison show us that the formula can only be regarded as approximately true, and various others have been suggested which will express the observed facts with the same amount of accuracy. In fact, the theory of dispersion has not been worked out, and is one of extreme difficulty, depending as it does on the nature of the action between ether and matter. In a vacuum light of all wave-lengths travels at the same rate; this rate is modified, in a way in which we do not yet understand, when the light enters any transparent medium; all we can say is that there is some action between the particles of the medium and the particles of the ether, and that this action depends on the wave-length in some unknown manner.

Cauchy's formula is sufficient to help us to determine in many cases the wave-length of a particular line from observations on its refractive index. Let us suppose we have observed the refractive index μ of the medium for this line, and also the refractive indices μ_1, μ_2, μ_3, for three other

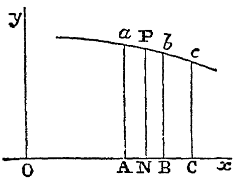
lines not far from it of known wave-length λ_1, λ_2, λ_3. Let λ be the wave-length required. According to Cauchy, μ, λ, &c., are connected by the equations—

$$\mu_1 = a + \frac{b}{\lambda_1^2} + \frac{c}{\lambda_1^4},$$
$$\mu_2 = a + \frac{b}{\lambda_2^2} + \frac{c}{\lambda_2^4},$$
$$\mu_3 = a + \frac{b}{\lambda_3^2} + \frac{c}{\lambda_3^4}.$$

From these three equations we can find a, b, and c; then, in the equation $\mu = a + \frac{b}{\lambda^2} + \frac{c}{\lambda^4}$, between μ and λ, we know μ, a, b, and c, and can, therefore, find λ.

A graphical method of solution may be adopted which is simple in practice. Let us call D_1, D_2, D_3 (fig. 118) the observed deviations of the known rays λ_1, λ_2, λ_3, and lay down abscissæ O A, O B, O C, representing these on a horizontal line O x. From A B C draw vertical ordinates A a, B b, C c, to represent λ_1, λ_2, λ_3, and draw a curve through a, b, c. Let O N represent the observed deviation of the unknown line λ, P N the ordinate of the curve at N. P N will represent the wave-length λ. In applying the method we should choose our lines λ_1, λ_2, λ_3 fairly close together, and in such a way that λ falls between two of them.

FIG. 118.



We will close this chapter with an account of a method due to Professor Schuster for adjusting a collimator for parallel rays without having to turn the telescope to a distant object or remove the prism, both of which it is often inconvenient to do. The method, too, has the advantage that the adjustments are made for light of definite wave-lengths, so that any want of perfect achromatism in the lenses can be corrected for that light.

Let us suppose the slit is illuminated with homogeneous light. Place the telescope so that it is inclined to the direct light at an angle somewhat greater than that of minimum deviation. Now there are always two positions of the prism, one on each side of that giving minimum deviation, for which the deviation will be the same. Thus there are two positions of the prism in which the image can be brought into the field of the telescope. In one of these the angle of incidence is greater than that for minimum deviation ; in the other, less.

Turn the prism into the first of these positions. In general the image will appear blurred and indistinct. Focus the telescope until it is clear. Turn the prism into the second position. The image seen, unless the collimator is in adjustment, will no longer be clear and in focus. Focus the collimator. Turn the prism to the first position and again focus the telescope, then to the second, and focus the collimator. After this has been done two or three times the slit will be in focus without alteration in both positions of the prism, and when this is the case the rays which fall on the telescope must be parallel; for, since the slit remains in focus, its virtual image formed by the prism is at the same distance from the telescope in the two positions of the prism; that is to say, the distance between the prism and the virtual image of the slit is not altered by altering the angle of incidence; but this can only happen when the distance is infinite, that is, when the rays falling on the telescope are parallel; and, since the faces of the prism are plane, the rays emerging from the collimator are parallel also. Thus both telescope and collimator are in adjustment.

CHAPTER IX.

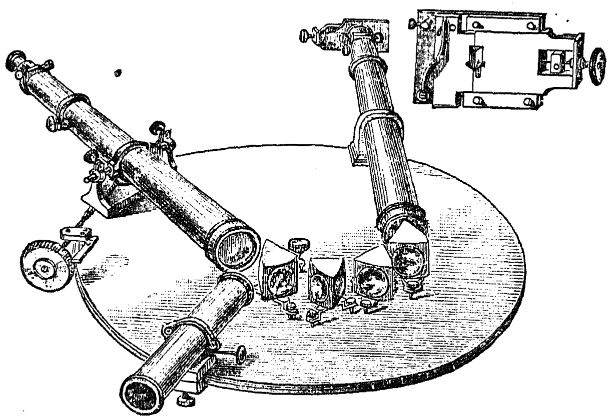
SPECTRUM ANALYSIS.

THE solar spectrum formed by a prism consists, we have seen, of a coloured band of light crossed by a number of dark lines. If now we change our source of light we shall find that the spectrum changes too. If, for example, we look at the light of a candle-flame, or at the white hot lime of the oxyhydrogen lamp, we shall find that the dark lines are gone, and we have left only the continuous spectrum. If, on the other hand, we look at the light coming from an incandescent gas or vapour, we find in general that the spectrum consists of a definite number of bright lines seen on a dark background. Under some circumstances the spectrum of a gas or vapour consists of a number of narrow bands distinctly marked on one side, but which gradually fade away on the other, while in other cases a continuous spectrum may be produced. The spectra then formed by the light from different glowing vapours and gases are different, and a careful examination of their spectra may give us some information as to the bodies emitting the light. The spectrometer described in the preceding chapter might be used for such examination, but the dispersion we could obtain from one prism would be too small for many important observations, and, therefore, a second prism is placed to receive the light emerging from the first, and a third prism after the second, and so on. The dispersion is increased by each prism, and the spectrum seen in the telescope is thereby greatly lengthened.

The prisms are placed successively, each in the position of minimum deviation for some definite ray, for this is an arrangement which can easily be recovered at any future time, though, when a collimator is used, it is, as we have seen, not necessary in order to secure distinct vision.

In many spectroscopes there is a third moveable tube; this tube carries at one end a transparent scale, which can be illuminated by a lamp, and at the other a lens whose focal length is the length of the tube. This tube can be placed so that the light from the scale falls on the second surface of the last prism, and part of it is there reflected. By moving the tube this reflected light can be brought into the field of view of the telescope, and an image of the scale becomes visible. The lens in the scale-tube can be focused so as to give a clear image of the scale in the telescope. By

FIG. 119.



this means we have always visible a scale to which we can refer the position of any given line or band.

Fig. 119 shows a spectroscope.

With this arrangement of scale there is no necessity for a graduated arc to measure the angle through which the telescope is turned. The distances between the lines are directly given in terms of the divisions of the scale.

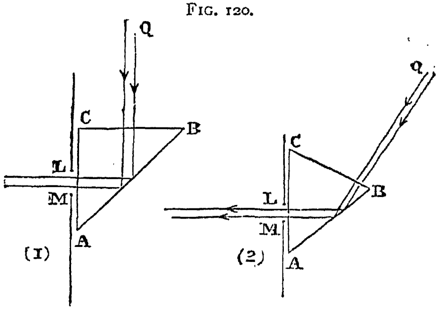
Some spectroscopes are fitted with a micrometer eyepiece, and the distance between two lines is then measured by the number of turns of the screw required to bring the

cross-wire or needle-point from coincidence with one line to coincidence with the other.

The slit is an important part of a spectroscope, and is shown separately in the figure. This is formed by two carefully worked parallel metal jaws, which are moved parallel to themselves by a screw, and so form a narrow rectangular aperture.

Spectrum analysis consists chiefly in comparing two spectra, one of which is known, and for this purpose it is useful to have the two spectra in the field simultaneously. This is attained by placing in front of the lower half of the slit a small rectangular prism of glass. Thus, in fig 120 (1)

FIG. 120.



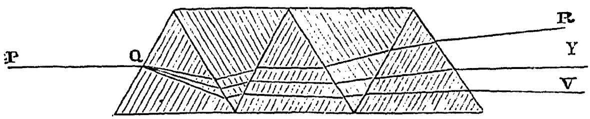
L M is a section of the slit, A B C the glass prism, C A is parallel to the plane of the slit, and C B at right angles to it. Light coming from a source Q falls perpendicularly on the face B C, is totally reflected at the hypothenuse A B, and emerges at right angles to the face A C, that is to say, in the direction of the axis of the collimator. This light falls on the upper half of the prisms, and is formed by them into a spectrum. The upper half of the slit is illuminated by the light coming directly from a second source which is dispersed by the lower portions of the refracting prisms. The two spectra are seen in the telescope, the lower half of the field being occupied by the direct light, the upper half by that from Q.

Fig. 120 shows the slit of a spectroscope with the reflecting prism.

In some instruments the prism used is equilateral, and the path of the light is as in fig. 120 (2).

For many purposes the direct-vision spectroscope is a convenient and useful form. We have seen how it is possible to combine two prisms so as to produce deviation without dispersion. It is equally possible to combine a number of prisms so as to produce dispersion of the extreme rays without deviation for some one given ray. On allowing then the light from a slit to fall on such an arrangement, a spectrum will be formed on a screen placed behind the prisms in the neighbourhood of the point in which the incident light would, if produced, meet the screen. Light

FIG. 121.



of one definite refrangibility passes through the combination without deviation; the other rays are slightly deviated, and form a spectrum. Fig. 121 shows the path of the light through such a spectroscope with five prisms, three of crown and two of flint glass.

To see the spectrum formed by a given source of light we have only to hold the apparatus between our eye and the light at such a distance, that, if the prism were removed, we could see the slit distinctly. The spectrum formed may be magnified by means of a convex lens placed between the observer's eye and the slit, and focussed on the slit. Small pocket instruments of this kind, which show distinctly the dark lines in the solar spectrum, are made by many opticians.

Larger instruments of the kind are fitted with a collimating lens and a telescope to view the spectrum formed,

and are frequently used for observations which do not require a very high dispersive power.

Let us now turn the slit of the spectroscope towards the non-luminous flame of a Bunsen gas-burner, and introduce in turn some of the salts of the various metallic elements into the hottest portion of the flame. The salt becomes volatilised by the heat, and the vapour rises and colours the flame. Thus, if we place on a small platinum spoon a little common salt or chloride of sodium the flame glows with a brilliant yellow light, and on looking through the spectroscope we see the spectrum consists of a narrow line of yellow light. If the instrument be sufficiently powerful we should find that this narrow line is really double—that there are, in fact, two narrow lines very close together. Some other faint lines which occur in the sodium spectrum would not be visible. If for the chloride of sodium we substitute that of some other metal which can be volatilised at the temperature of the Bunsen burner, the colour of our flame alters, and the spectrum changes too. Thus strontium colours the flame red, and its spectrum consists of a number of lines in the red, with an orange line somewhat less refrangible than the sodium line, and a line in the blue part of the spectrum. Lithium, too, colours the flame red, and with the naked eye it would be difficult to say whether the red colour of a given flame was due to the presence of strontium or of lithium; but the spectroscope tells us at once, for the spectrum of lithium consists of one brilliant red line with three fainter lines respectively in the orange, green, and greenish blue. A potassium salt will give us two lines in the red and one in the violet, while the flame would appear to be of a violet tinge. Barium will colour it green, and calcium a yellowish red. The spectrum of calcium, seen in this way, consists of two lines in the red, four in the yellow, one in the green, and one in the violet; while in that of barium we have a series of brilliant lines in the green and greenish blue, with others in the red and orange.

Again, it is immaterial what salt of these metals we use: we always get the same spectrum. We may put into our flame either chloride, carbonate, or nitrate of sodium—we shall still have the two yellow lines close together, and we may use these two yellow lines as a mark of the presence of incandescent sodium vapour. Whenever we see them we know that sodium is present in the flame, and so delicate is the test that Bunsen and Kirchhoff have calculated that the eighteen-millionth part of a grain can be detected by it. In fact, it is difficult to obtain a flame free from some trace of sodium vapour.

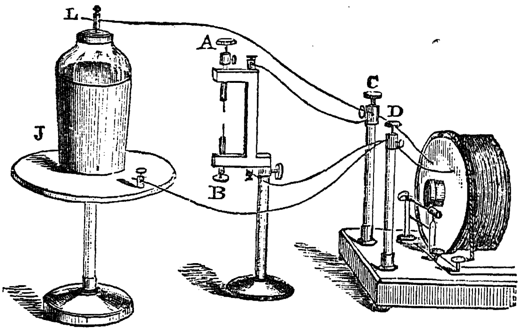
The presence of any other of these metals may with equal certainty be detected by its spectrum. We wish to determine if a certain salt contains lithium. Volatilise it in the Bunsen burner, and examine the spectrum. A little practice enables the observer to detect the distinctive lines of each metal at once, and a glance is almost sufficient to decide, if, for example, the bright red line of lithium is present. If there be a doubt as to whether the line seen is really the lithium line, introduce into the field the spectrum of lithium by means of the reflecting prism at the slit of the spectroscope, and notice if the red line observed, and that of lithium, really coincide.

But there are other metals which we cannot reduce to vapour at the temperature of a Bunsen burner. If we put a piece of iron in the flame it remains solid but becomes red-hot, and from it we get a continuous red spectrum, which extends further and further into the violet as the temperature of the iron rises; so too with copper, lead, gold, or silver. To get the iron or copper spectrum we must use the electric spark. When a spark passes between two conductors of electricity a brilliant flash of light is seen; the spark renders the air between the poles incandescent, but at the same time a very minute portion of the matter of the poles themselves is carried across in a state of vapour, and part of the light seen comes from this. Thus if we put upon

the poles a little sodium the spark looks yellow, and the yellow sodium lines are seen in the spectroscope. In addition to these sodium lines we have the very complicated spectrum due to the air which has been heated and rendered incandescent by the spark.

To obtain the spark we use a Ruhmkorff induction coil. Two pieces of the metal we wish to examine are held in a suitable slip at about a millimètre apart. These two, A, B (fig. 122), are connected by fine wires with C, D, the poles of the secondary wire of the coil, and on working the coil the spark passes between A and B, carrying with it a very minute

FIG. 122.



quantity of the metal in a state of vapour. If the battery power used be only just sufficient to work the coil, some of the most brilliant of the metal lines will be seen separated by dark spaces, which are crossed by bands of light more or less faint, corresponding to the more luminous of the air-lines. If now A and B be connected with the two coatings of a Leyden jar (L, J in figure) the sparks become much more intense, and their spectrum proportionately brighter, but at the same time the air-lines are brought out distinctly, and the spectrum rendered very complex. To decide which of the lines belong to the metal and which to the air, we must introduce the spectrum from a second pair of poles of

a different metal into the field by means of the reflecting prism. Those of the lines which are common to the two spectra are either air-lines or are due to some common impurity in the poles. The non-coincident lines belong to the two metals respectively. It is best to choose for our second spectrum one like that of platinum, which contains comparatively few lines and those fairly distinct.

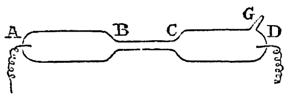
We can now determine in terms of the divisions of our scale the position of the lines in the spectrum of the metal examined: we can map the spectrum. But, of course, a map constructed in this manner refers only to our apparatus, and for the definite position of the prisms at the time of observation we need to reduce it to some permanent scale of measurement. Now a little practice enables an observer to recognise with absolute certainty the more prominent of the air-lines, and the wave-lengths of these lines have been determined with care by various observers. We have here a means of turning the scale of our apparatus into one of wave-lengths. For let a horizontal line, as in fig. 118, represent the scale of our spectroscope; coinciding with certain divisions of the scale in the field of view, we see bright lines whose wave-lengths we know. Draw vertical ordinates at the corresponding divisions of the horizontal line O x (fig. 118) to represent these wave-lengths. Let us suppose A a, B b, &c., are a number of these ordinates; draw a curve $a\,b\,c$ through the points a, b, c, and let P be any other point on this curve, and P N the ordinate at P, then P N represents the wave-length of a bright line seen to coincide with the division of the scale which corresponds to N. If, then, we observe an unknown bright line coinciding with any given division of the scale we must draw the ordinate of our curve at the corresponding point, and this ordinate gives us the wave-length of the unknown line directly.

To observe the spectrum of a permanent gas a vacuum tube is used. The tube consists of two wider portions, A B, C D (fig. 123), connected by a narrow capillary tube, B C.

Two pieces of platinum wire are sealed into the glass at A and D, and can be connected with the poles of the Ruhmkorff coil. The tube is exhausted and, after being filled with the gas to be examined, is hermetically sealed at G. On passing a spark through it the gas becomes incandescent, and a narrow line of light is seen along the capillary part, the spectrum of which can be observed by placing it behind the slit of the spectroscope.

The spectrum depends of course on the gas used, but the same gas under different conditions of pressure and temperature gives different spectra. Thus the spectrum of nitrogen given by a strong spark in nitrogen gas consists of a large number of bright lines; if, however, the discharge be reduced in intensity by removing the Leyden jar from

FIG. 123.



the circuit, we have a number of bands instead of our sharply defined lines. Each of these bands is sharply defined towards the red end, and fades away to blackness towards the violet.

The spectrum of oxygen, again, when the Leyden jar is in connection, and in addition the circuit is broken somewhere, so that the spark has to pass through an interval of air, consists of a number of lines, lying for the most part in the violet end. If the temperature of the incandescent gas is lowered by reducing the intensity of the spark, a spectrum consisting of four lines, one in the red, two in the green, and one in the blue, is seen. As the pressure of the gas is increased the lines, with the exception of the blue line, widen out considerably. If the temperature be still further lowered so that the oxygen is only just luminous, as, for example, in the wide part of a vacuum tube, the spectrum observed is a continuous one. A fourth different spectrum

is obtained from the glow surrounding the negative pole in oxygen.

Dr. Schuster has described (Phil. Trans. clxx.) the manner in which these changes may be observed in a tube filled with pure oxygen and undergoing gradual exhaustion.

If, again, the tube be filled with hydrogen at the atmospheric pressure, the spectrum consists of three lines in the red, green, and violet, with a certain amount of continuous spectrum, there is also a fourth faint line in the extreme violet; as the pressure is decreased the lines become sharper and better defined, while the continuous spectrum grows fainter and fainter, and finally disappears. If the exhaustion be further continued, the red and violet lines disappear, and the green line alone is left. If, on the other hand, instead of exhausting the hydrogen we compress more gas into the tube, the lines grow broader and fainter, while the continuous spectrum becomes more brilliant.

Thus spectrum analysis is able to tell us, by means of the spectrum produced, not only what gas we are investigating, but it can also give us information as to the state of that gas with reference to its pressure and temperature.

But the solar spectrum—the one we started with—consists of a continuous band of colour crossed by a series of dark lines, and, as yet, we have described no other such spectrum. Now Fraunhofer, who first observed these lines, had noticed also that the dark line, which he called D, coincided exactly in position with the yellow line of glowing sodium vapour, and in his recent book on the spectroscope, Mr. Norman Lockyer quotes as follows from an extract from Dr. Brewster's note-book, under the date October 28, 1841:—'I have this evening discovered the remarkable fact that in the combustion of nitre upon charcoal there are definite bright rays corresponding to the double lines of A and B, and the group of lines *a* in the space A B. The coincidence of the two yellow rays with the two deficient ones at D, with the existence of definite bright rays in the

nitre flame, not only at D but also at A *a* and B, is so extraordinary that it indicates some regular connection between the two classes of phenomena.'

The exact nature of this connection was first stated, about the year 1852, by Professor Stokes in conversation with Lord Kelvin.

In a letter written to Kirchhoff in 1860, Lord Kelvin says:—'Professor Stokes said he believed a mechanical explanation of the cause was to be had on some such principles as the following. Vapour of sodium must possess by its molecular structure a tendency to vibrate in the periods corresponding to the degrees of refrangibility of the double line D. Hence the presence of sodium in a source of light must tend to originate light of that quality. On the other hand, vapour of sodium in an atmosphere round a source must have a great tendency to retain in itself, *i.e.*, to absorb and have its temperature raised by, light from the source of the precise quality in question. In the atmosphere round the sun, therefore, there must be present vapour of sodium, which, according to the mechanical explanation thus suggested, being particularly opaque for light of that quality, prevents such of it as is emitted from the sun from penetrating to any considerable distance through the surrounding atmosphere. The test of this theory must be had in ascertaining whether or not vapour of sodium has this special absorbing power anticipated.' Lord Kelvin continues: 'I have the impression some Frenchman did make the experiment, but can find no reference on the point. I am not sure whether Professor Stokes' suggestion of a mechanical theory has ever appeared in print. I have given it in my lectures regularly for many years, always pointing out that solar and stellar chemistry were to be studied by investigating terrestrial substances giving bright lines in the spectra of artificial flames, corresponding to the dark lines of the solar and stellar spectra.' [1]

[1] *Phil. Mag.* Ser. iv. vol. xx. 1860, page 20.

The Frenchman referred to was M. Leon Foucault, and the test had been made by him in 1849.

Professor Stokes' anticipations received, independently of him, their complete fulfilment in some experiments of Kirchhoff in 1859. Lord Kelvin's letter, quoted above, was written in consequence of these experiments. Kirchhoff's paper was translated by Professor Stokes, and we will give his own words in describing the observations.[1]

He says: 'Fraunhofer had remarked that in the spectrum of the flame of a candle there appear two bright lines which coincide with the two dark lines D of the solar spectrum. The same bright lines are obtained of greater intensity from a flame into which some common salt is put. I formed a solar spectrum by projection, and allowed the solar rays concerned, before they fell on the slit, to pass through a powerful salt flame. If the sunlight were sufficiently reduced there appeared in place of the two dark lines D two bright lines; if, on the other hand, its intensity surpassed a certain limit, the two dark lines D showed themselves in much greater distinctness than without the salt flame. The spectrum of the Drummond light (oxyhydrogen lime light) contains, as a general rule, the two bright lines of sodium if the luminous spot of the cylinder of lime has not long been exposed to the white heat. If the cylinder remains unmoved these lines become weaker, and finally vanish altogether. If they have vanished or only faintly appear, an alcohol flame into which salt has been put, and which is placed between the cylinder of lime and the slit, causes two dark lines of remarkable sharpness and fineness, which in that respect agree with the lines D of the solar spectrum, to show themselves. Thus the lines D of the solar spectrum are artificially evoked in a spectrum in which naturally they are not present. If chloride of lithium be brought into the flame of Bunsen's gas lamp, the spectrum of the flame shows a very bright sharply defined line which

[1] *Phil. Mag.* Ser. iv. vol. xx. 1860.

lies midway between Fraunhofer's lines B and C. If now solar rays of moderate intensity are allowed to fall through the flame on the slit, the line at the place pointed out is seen bright on a darker ground, but with greater strength of sunlight there appears in its place a dark line, which has quite the same character as Fraunhofer's lines. If the flame be taken away the line disappears, as far as I have been able to see, completely. I conclude from these observations that coloured flames, in the spectra of which bright lines present themselves, so weaken rays of the colour of these lines when such rays pass through them, that in place of the bright lines dark ones appear as soon as there is brought behind the flame a source of light of sufficient intensity, in which these lines are otherwise wanting. I conclude further that the dark lines of the solar spectrum, which are not evoked by the atmosphere of the earth, exist in consequence of the presence in the atmosphere of the sun of those substances which, in the spectrum of a flame, produce bright lines at the same place. We may assume that the bright lines agreeing with D in the spectrum of a flame always arise from the presence of sodium contained in it; the dark line D in the solar spectrum allows us therefore to conclude that there exists sodium in the sun's atmosphere. Brewster has found bright lines in the spectrum of the flame of saltpetre at the place of Fraunhofer's lines, A, *a*, B; these lines point to the existence of potassium in the sun's atmosphere. From my observation, according to which no dark line in the solar spectrum answers to the red line of lithium, it would follow with probability that in the atmosphere of the sun lithium is either absent or is present in comparatively small quantity.' In a note appended to the translation Professor Stokes gives the following dynamical illustration of the phenomena: 'We know that a stretched string, which on being struck gives out a certain note (its fundamental note suppose), is capable of being thrown into the same state of vibration by aerial

vibrations corresponding to the same note. Suppose now a portion of space to contain a great number of such strings, forming thus the analogue of a medium. It is evident that such a medium, on being agitated, would give out the note above mentioned, while, on the other hand, if the note were sounded in air at a distance the incident vibrations would throw the strings into vibration, and consequently would themselves be gradually extinguished, since otherwise there would be a creation of *vis viva*, that is, of kinetic energy.'

We may perhaps carry the illustration a little further. If instead of sounding the note corresponding to the strings we sound some other, it will not set the strings in vibration, and will pass on unchanged. If then we produce a complex sound near such a medium, the strings will select from the sound and absorb the note which they would themselves emit if put into a state of vibration.

If then white light falls on a gas, the gas absorbs from the light just those rays which it would itself emit if incandescent, and dark lines are seen in the spectrum of the light. If the gas be itself incandescent the absorption still takes place, but the light of the gas itself is substituted for the light absorbed. Thus if the light from the incandescent gas be more brilliant than the corresponding rays which have been absorbed, we see in the spectrum the bright lines of the gas itself. If now the brilliancy of the gas be diminished, or that of the source of white light be increased, the rays emitted by the gas become less intense than those which have been absorbed, less intense also than the light in their immediate neighbourhood, and so the part of the spectrum which is illuminated by them appears dark by contrast; we have a series of black lines crossing our band of light. This explains how it is that when a faint beam of solar light passes through a flame containing lithium or sodium vapour we see the bright lines of sodium or lithium in the spectrum, while as the intensity of the solar light is

increased the sodium or lithium lines grow fainter, and finally appear as dark lines instead of bright.

It is easy to show directly that sodium vapour absorbs rays of sodium light which fall upon it. If we place a lighted spirit lamp with a salted wick before a Bunsen flame, in which a sodium bead is incandescent, the flame of the spirit lamp is quite black. The temperature of the spirit lamp is much lower than that of the Bunsen. It absorbs the rays from the Bunsen burner, and replaces them by its own; these latter are so much less intense that the flame actually looks black when viewed against the more brilliant Bunsen light.

These experiments then lead us to the conclusion that a body which emits certain definite radiations absorbs readily those radiations when they fall on it, that good radiators are good absorbers, the radiation and absorption of course having reference to light of the same wave-length.

To go more fully into the question of spectrum analysis, or even to attempt to describe the work that is being done by the spectroscope on the sun and stars, would occupy more space than we can afford, and for this the reader must be referred to works dealing specially with the question.

CHAPTER X.

ABSORPTION AND ANOMALOUS DISPERSION.

THE dark lines of the solar spectrum are, we have seen, due to the absorption of rays of certain definite wave-lengths by the solar atmosphere. We turn now to other phenomena of absorption. Why does a green leaf look green, a white lily white, or a red rose red? What is it which gives their distinctive colours to natural bodies? We may say, and that with truth, that the green leaf reflects only

green rays, while the lily reflects all those which fall upon it; but when and where does this selection take place? It is not true in these cases, at least, that it happens in the act of reflexion. We shall best arrive at an answer to our question by considering the phenomena of absorption produced by coloured liquid solutions or gases. The vapour of iodine, when viewed in the sunlight, has a beautiful violet colour. Let us suppose that we allow the sun's rays or the light of a powerful lamp to pass through a vessel containing iodine vapour, and then to fall on the slit of a spectroscope, and examine the spectrum produced. It will be found that this is crossed by a number of dark lines, chiefly in the orange, yellow, and green parts of the spectrum; these are grouped together so closely in some parts as to appear like a dark band across the field. Just as the sodium vapour at the low temperature is opaque to the yellow D lines, and causes them to appear as two dark lines across the spectrum, so the vapour of iodine is opaque to the many rays whose places are indicated by these dark lines. Sodium vapour, if not sufficiently hot to appear self-luminous, would seem to the naked eye colourless when placed between it and the lamp; the absorption of the one or two rays would not be noticed. In the case of the iodine vapour, when viewed by transmitted light, so much light is absorbed from the yellow and green part of the spectrum, that only the blue, violet, and some of the red rays get through, and the vapour looks violet to the eye. Other vapours placed between the lamp and the slit give us different spectra, and in all cases the colour of the vapour seen thus by transmitted light depends on the rays which have passed through it without suffering absorption.

Just as we get absorption spectra by examining light passed through vapours, so too we can observe them when the light has been transmitted through coloured liquid solutions or transparent solids, such as coloured glasses. Liquid solutions are perhaps the best to work with, be-

cause we can more easily trace the effects of greater concentration of the colouring matter, or of greater thickness of the absorbing medium. In a lecture on the absorption of light, delivered at South Kensington in 1876, Professor Stokes describes several simple experiments illustrating the importance of the subject.

We will quote a few lines describing the phenomena observed when a solution of permanganate of potash is placed in the path of the light. He says: 'If the solution is considerably diluted, and you analyse the light transmitted through, you will see a broad dark band in the spectrum. If you have it more diluted you obtain a spectrum highly characteristic, in which are seen five dark bands in the green part. Those are highly characteristic of the permanganates. There is just a trace of a sixth band, which comes in when the solution is stronger. There are alternations of transparency and opacity, not that the fluid is perfectly transparent, but these intervening spaces are really alternations of greater and less absorption. When the quantity present is sufficient, the whole of this region is absorbed, and then the characteristics are lost, because there are a great variety of purple substances which would give a spectrum not very different. In examining a substance you must dilute the solution to make sure of breaking up any such broad dark region, and then you see the dark bands, if any, which are characteristic of the substance.'

A mixture of sulphate of copper and carbonate of ammonia gives us a solution which transmits only the more refrangible end of the spectrum, and in which the absorption increases nearly regularly from the violet to the red, while a solution of bichromate of potash allows only the red and yellow rays to pass; so that if we place two cells containing these two solutions one behind the other, we obtain a combination opaque to all the visible rays.

A dilute solution of blood gives a highly characteristic spectrum, marked by two dark bands, in the neighbourhood of

the lines D and E, while the violet portion is wanting altogether. If the blood be deoxidised by the introduction of a suitable reducing agent, the colour of the solution becomes more purple, and the spectrum is altered greatly. The two dark bands disappear, and a new band comes into view, in position somewhere between the two. By shaking the solution with air, so as to reoxidise it, the original spectrum is recovered. The absorption spectrum produced by chlorophyll, the green colouring matter of the leaves of plants, has been carefully observed by Professor Stokes, and consists, if different thicknesses of the solution be used, of five distinct bands. The first two of these are in the red, the next between the yellow and yellow green, the fourth in the green, and the last in the blue. The first is, if the thickness of the liquid be small, by far the most intense. The extreme violet end of the spectrum is wanting. A red glass coloured with oxide of copper stops the passage of all rays after the sodium line D, while the blue cobalt glasses, which are common, allow only the blue and the extreme red beyond B to pass through. If, then, we combine together two of these, we permit only the extreme red light to reach the eye and thus obtain a deep red of great purity. The blue cobalt glass allows the invisible rays of high refrangibility to pass freely, and is useful in experiments in which they only are required, while many pale brown and yellow glasses may be found that stop these rays entirely, and permit the passage only of the yellow ; these are employed in photographic operations.

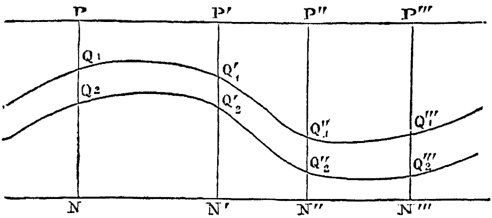
The following list of solutions will be found useful if we wish to quench the light from the parts of the spectrum mentioned and allow the rest to pass through :—

To absorb the red rays only ;—neutral solution of sulphate of copper. To absorb the yellow ;—alkaline litmus, chromium chloride or aniline blue. For the green ;—permanganate of potash. For the blue green, the aniline dye called aurine. For the blue chromate of potash.

To allow only definite colours to pass, we use combinations of these absorbing media. Thus, for example, if we take two cells, one containing aniline blue, and the other chromate of potash, and allow light to pass through the two, we effectually absorb all the yellow and blue rays, the emergent light is a mixture of red and green. If in addition to these we pass the light through sulphate of copper, the red rays are absorbed and the green light alone is transmitted.

A simple graphical method of determining the intensity of the various components in a ray of light transmitted through a known thickness of a given medium when we know the absorption of that medium for some standard

FIG. 124.



thickness, is given by Sir John Herschel in his treatise on light. It depends on the following principle. If we know that a given thickness, one centimètre say, of the medium absorbs a given proportion of the light which falls on it, then the next centimètre will absorb the same proportion of the light that is left, and so on continually; that is to say, if after passing through the first centimètre we find that one-half the light is left, then after traversing the second one-half of this, or one-fourth of the whole incident light is left, after passing through a third centimètre one-eighth, and so on.

Now let us suppose we represent by points on a horizontal straight line (fig. 124) the various points on a spectrum, and draw ordinates to represent the intensity of the light of

each colour falling on a certain absorbing medium. The extremities of these ordinates will lie on a curve which will give us the intensity of the incident light at every point of the spectrum. Let us suppose that in the case considered rays of all colours are equally intense, so that our curve is a horizontal straight line, and that we know the proportion of light transmitted by a certain thickness, say one centimètre of our medium for rays of all colours, this proportion is of course different for different colours. Let P N now be any ordinate, and in it take a point Q_1 such that the ratio $Q_1 N : PN$ = ratio of transmitted light to incident light for the colour represented by the point N. Do this for all colours, we thus arrive at another curve $Q_1 Q_1' Q_1''$, &c., which represents the intensity of the light after passing through one centimètre of the medium.

Suppose now we wish to find the intensity after passing through two centimètres. In every ordinate such as P N, take a point Q_2 such that $Q_2 N : Q_1 N = Q_1 N : P N$. Then clearly $Q_2 N$ represents the amount of light which emerges after passing through the second centimètre, and a third curve drawn in this manner gives us the intensities at all points of the spectrum. To find the effect of a third centimètre we proceed in the same manner.

A curve of this kind enables us to explain simply the phenomenon known as dichromatism, or the variations in the apparent colour of an absorbing medium when different thicknesses are used. Thus glass coloured with cobalt appears blue by transmitted light if the thickness is small; as it increases the blue becomes purple, and finally, if a sufficiently great thickness be employed, the colour is a deep red. Or again, if we fill a thin wedge-shaped vessel with a solution of muriate of chromium and look at the light from a white cloud through it, near the edge the liquid appears green, but if we slide the wedge forward so as to bring greater and greater thicknesses between the eye and the cloud, the green passes through a

brownish tint to a deep red. Alkaline solution of archil is another medium which is purple when thin, but becomes red as the thickness through which the light is transmitted is increased. To explain this, let us suppose that on falling on the medium the intensity of the red rays which can pass through may be represented by 10, while that of the violet and blue is 100. For clearness we may, if we like, take these numbers as the number of rays of a certain description, all equally intense, which fall on the medium. Let us further suppose that some small thickness of the medium, say ·1 centimètre, absorbs one-tenth of the red rays and one-half of the blue and violet, thus allowing ·9 of the red and ·5 of the blue to pass through. After passing through various thicknesses of the medium, the intensities will be as given in the Table below.

Thickness	Intensity	
	Red	Blue
0	10	100
·1	9	50
·2	8·1	25
·3	7·29	12·5
·4	6·56	6·25
·5	5·9	3·12

Thus taking these numbers it is clear that the relative proportion of the blue to red decreases very rapidly and has changed after passing through half a centimètre of the medium from 10 to 1, to 3·1 to 5·9. Thus the red light has the predominance, and the apparent colour has been altered from a blue purple to a reddish purple, and finally, if we increase the thickness still more, to red.

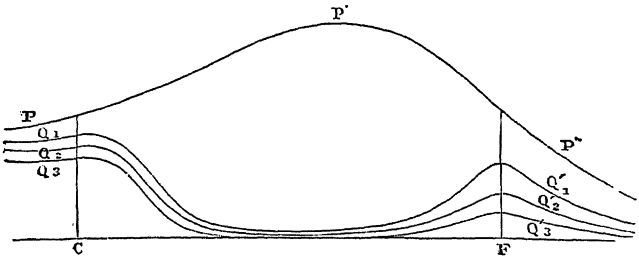
We may represent this by a curve in the following manner. Let us suppose the medium absorbs nearly all the rays between the fixed lines C and F, and that if we represent the amount of light between the extreme red and C by 10, we

may represent that between F and the extreme violet by 100. Let the ordinates of the curve P P′ (fig. 125) represent the intensities of the incident light.

Then of this incident light between A and C about ·9 gets through; from C to F there is almost nothing transmitted, and from F on to the extreme violet about ·5.

Then the curves $Q_1 Q_1'$, $Q_2 Q_2'$, &c., represent the intensities after traversing ·1, ·2, &c., centimètres. The ordinates of $Q_1 Q_1'$ between A and C are about ·9 of those of P P′, from C to F there is hardly any light transmitted, the ordinates of Q Q′ are therefore practically zero, while from F outwards the ordinates of $Q_1 Q_1'$ are about half those of P P′; $Q_2 Q_2'$ is got from $Q_1 Q_1'$ in the same manner.

FIG. 125.



The amount of light between certain definite wave-lengths which is transmitted, is represented by the area bounded by the curve, the horizontal line and ordinates drawn from points on the horizontal line corresponding to the wave-lengths considered. The curves show the changes which take place in the relative intensities of the red and blue light, and consequently in the apparent colour of the mixture.

Let us now proceed to see how the phenomena of absorption enable us to explain the natural colours of bodies, to say why a leaf looks green and a lily white. Let us suppose that we enclose a perfectly clear coloured solution, sulphate of copper, for example, in a black vessel open at

the top, so that no light can be transmitted through it or reflected from the sides of the vessel, and look at it by light from above. The solution looks black, and if we examine the image of a white object seen by reflection at the surface the image is colourless. All the rays are reflected according to the same law, and so nearly in the same proportion that we cannot distinguish any alteration in colour.

The clear transparent blue liquid reflects all rays practically alike, and itself looks colourless; but now suppose we make our solution slightly turbid by introducing some fine powdered chalk. The solution appears to be a brilliant blue; let us follow the course of an incident white ray; some of it is reflected from the surface, but the rest is refracted into the liquid. Now our blue solution absorbs all red light which falls on it, and therefore after passing through a very small thickness of the liquid this refracted ray is practically a blue ray. This blue ray falls on a small particle of the powdered chalk and is reflected by it upwards, and on emerging again from the surface reaches our eye. The blue light which we see and which gives to the solution its colour, has been reflected to us, not by the surface of the liquid, but by the small particles in its interior. It has passed through some thickness greater or less, as the case may be, of the solution, and thus the red and yellow constituents in the incident white light have been removed; the blue alone remains to emerge and reach our eye. And just this has happened in our leaf or flower. The green colouring matter, chlorophyll, absorbs readily the violet and some of the red rays; a very slight thickness is sufficient to prevent all trace of the violet and of the most brilliant part of the red between D and C from passing through. The white sunlight falls upon a leaf, and some of the rays penetrate a little way within; the interior structure is highly irregular, like the fluid with the chalk; there are constant reflexions back and forwards of this light that has entered, but some of it escapes again and reaches our eye robbed of its blue and red components; the

leaf, therefore, looks green. If the interior of the leaf were quite regular and homogeneous like the clear copper solution, so that no light could be reflected from it in such a way as to reach the eye, the leaf viewed by reflected light would, like the copper sulphate, look black; but if we held it up to the sky so as to view it by transmitted light, it would appear green, just as the copper appears blue.

The structure of the white lily, too, is irregular, and we see it by means of light reflected from the interior, but there is nothing in the lily which absorbs one colour more readily than another; all rays are reflected equally or nearly so, and the emergent light is white. A poppy, again, looks red because its cells are filled with a liquid which absorbs the green and blue light, and transmits the red.

We can easily show the difference between the absorption of the lily, diminishing all rays in about the same proportion, and that of the poppy, which selects the green and blue, refusing them passage almost entirely, and transmits the red, by viewing them in coloured light, coloured either by transmission through suitable glasses or solutions, or by means of the dispersion produced by a prism. If we allow the spectrum formed by a prism to fall on a white screen, all the colours are seen, and a white object placed in the path of the light assumes the colour of that part of the spectrum which falls on it. In the red end our lily is red, in the green it is green, and in the blue, blue.

But this is not the case with the poppy; placed in the red light it shines with a brilliant red, placed in the green or blue it is black and colourless. In the first case only red rays fall on the flower and are reflected from its interior surfaces, and of these it absorbs but few; it therefore looks red. When in the green there are only green rays to illuminate it, and these are all absorbed, there is no red in the incident light; no red, therefore, can be reflected, and the poppy looks black.

The lily, on the other hand, absorbs but little of any

colour. When red light falls on it, red light can be, and is, reflected; when it is placed in the green, green can be, and is reflected. The lily returns the light unchanged in colour, and appears of the same tint as the light which illuminates it.

Another method of illustrating this is to receive our spectrum on a coloured screen. If the screen be red the green and greenish blue portions will seem to be absent from the spectrum, if again the screen be green there will be no red visible.

The natural colours, then, of most bodies are part of the phenomena of absorption. Of the light falling on a coloured body some part penetrates a little way into the interior, is there reflected and emerges, and by this light we see the object; but in its passage through the small thickness of the body it has traversed certain of its components have been absorbed much more than others, and the emergent light is therefore coloured and gives its colour to the object.

But there are some bodies in which a preferential selection is made in the act of reflection. A polished plate of gold is opaque but reflects light regularly from its surface; the reflected light, however, is yellow, while that reflected from copper is red.

If, again, we make solutions of many of the aniline dyes in alcohol, and spreading them on a plate of glass allow the alcohol to evaporate so as to leave a thin film of colour, we shall find that the plate appears of one colour when we look through it, and of another when we receive in our eyes light reflected from its front surface. The colour of the reflected light depends somewhat on the angle of incidence; as will be explained when we come to the polarisation of light, there is for most isotropic transparent substances a particular angle of incidence such that light reflected at that angle undergoes a change in its character called polarisation, and the reflected beam is said to be plane polarised. If we examine plane

polarised light by transmitting it through a crystal of Iceland spar cut in a particular manner so as to form a Nicol's prism, we can always find one position of the Nicol for which no light will pass through, so that if we look at the surface of a transparent medium on which light falls at this particular angle, depending on the medium, through a Nicol's prism and turn the Nicol round, the light coming from the surface gets fainter and fainter, and finally disappears entirely. But if we examine light reflected from a metal surface or from one of these films of anilin and some other substances, we find that it cannot be made to disappear for any angle of incidence or for any position of the Nicol. The reflected ray is never plane polarised. The colour of the surface viewed in this way through a Nicol adjusted to extinguish light reflected from the glass is different from its colour when looked at without the Nicol, and changes as we rotate the Nicol.

In the following Table the colours of the transmitted light and of the reflected light seen with or without the Nicol are given.

Name	Colour		
	Transmitted	Reflected	Reflected through a Nicol's prism
Rose anilin or fuchsin	Rose . . .	Green . .	Peacock blue
Mauve anilin . .	Mauve . .	Apple green	Emerald green
Malachite green .	Deep green	Plum colour	Orange gold
Blue anilin . . .	Blue . . .	Bronze . .	Olive green

A large number of substances have been shown by Haidinger, of Vienna, to possess this difference between surface colour and body colour in a greater or less degree, and thus to resemble metals at least for certain rays of the spectrum.

It is this last fact that they resemble metals for certain

rays only which produces the difference in the colour of the reflected light as viewed by the naked eye and through a Nicol's prism.

In the first case the reflected beam is composed of two parts. One of these is reflected in the ordinary manner from the particles of the medium just below the surface, and would be of the same colour as the transmitted light just as in the case of a flower or leaf; this part of the reflected beam is plane polarised at a proper angle of incidence, and can be quenched by a Nicol's prism. The other part of the beam consists of light reflected as from a metal surface, and is in colour complementary to the light which has been transmitted. It is never plane polarised, and therefore never quenched when viewed through the Nicol.

In the second case, therefore, it is this second part of the reflected pencil alone which we see. The plane polarised part due to the ordinary reflexion, and of the same colour as the transmitted beam, has been extinguished, the anomalous portion alone remains. And this anomalous portion is just that which is wanting from the transmitted beam. If we could place side by side the spectra of the transmitted and reflected light, we should find that where the one was crossed by dark bands showing strong absorption the other was most intense.

This is particularly marked in the case of crystals of permanganate of potash which have been examined by Professor Stokes. The absorption spectrum of a dilute solution of these crystals—they are too opaque in the solid form to transmit light—is marked by five distant black bands lying in the yellow green of the spectrum between D and F. The light reflected from the surface of the crystal is green in colour, and when Professor Stokes examined it with a spectroscope and a Nicol's prism placed so as to allow only the anomalous portion to pass, he found that the spectrum consisted of four bright bands which coincided exactly with the first four of the dark bands seen in the absorption

spectrum. The light was too faint in the neighbourhood of the fifth absorption band to permit a fifth bright band to be seen.

The dark bands in the transmitted spectrum then were due, not so much to an absorption of the light by the medium as to a selection exercised in the act of reflexion. Certain definite rays are by these quasi metallic substances, as it were, entirely refused admission and turned back, and these rays are never plane polarised by the reflexion. These rays give their peculiar colour to metallic surfaces. Gold, for example, is yellow, because the yellow rays are entirely reflected. The transmitted light is blue or bluish green, and may be seen by putting a little protosulphate of iron into a solution of chloride of gold, or by looking at a bright light through a very thin piece of gold leaf.

But there is another effect produced by these substances on light which is intimately connected with this metallic reflexion, and has received the name of anomalous dispersion. Professors Christiansen, Kundt, and others, have shown that if we pass the sun's rays through a hollow glass prism containing a strong solution in alcohol of many of these same substances, the order of the colours in the spectrum formed is different to that in the spectrum given by glass or other transparent media. Thus, according to Herr Kundt, with fuchsin, cyanin, mauve anilin, and anilin blue, the order of the colours is green, blue, red, green being the least deviated. The spectrum of cyanin is specially remarkable, for between the blue and the red there is a dark space without light, and beyond the red orange is visible. The dark lines of the solar spectrum can be seen and recognised in their changed positions, and the refractive indices of fuchsin, cyanin, and permanganate of potash, have been carefully measured for several of them.

In fuchsin and cyanin the order of the lines is F G H A B C D, F being the least refracted.

At first anomalous dispersion was not detected in per-

manganate of potash, but more careful observation showed that it occurred to a slight extent between D and F, that is, in the neighbourhood of the bands seen when light is transmitted through a dilute solution.

We have learnt already that as we go from red to violet, the time of vibration of the ether particles whose motion produces the light decreases; let us suppose that by the term a 'high co-efficient of absorption for certain rays,' we mean that the medium considered absorbs largely those rays so that they are absent from the spectrum of the transmitted light, and let us suppose further that we speak of going from a part of the spectrum in which the period of vibration is long to one in which it is short as going *up* the spectrum. Then Professor Kundt has summed up his results as follows:

If, as we pass up the spectrum, the co-efficient of absorption increases rapidly, then the refractive index is increased by the absorption.

If, however, as we go down the spectrum, the co-efficient of absorption increases, then the refractive index is decreased by the absorption.

In the case of fuchsin, for example, the absorption is very strong between D and F. As we approach D from the red end of the spectrum the period of vibration decreases, and the co-efficient of absorption increases very rapidly, so the refractive index is increased greatly by the absorption, and the deviation of the red ray becomes abnormally large. Again, as we approach F from the violet end the absorption increases rapidly, but we are going from a part of the spectrum in which the period is small to one in which it is greater, and so, in accordance with the second half of our rule, the refractive index is greatly decreased—decreased so much, in fact, that the violet end of the spectrum is less deviated than the red, and the order of the colours is green, blue, violet, then a dark space followed by red and orange, so that, as Kundt continues, when the absorption is sufficiently great, of two portions of the spectrum separated by strongly

absorbed rays, the one in which the period is large may be more deviated than the second in which the period is small.'

If, as above, we speak of going from red to violet in a normal spectrum as going up the spectrum, then we may say that owing to abnormal dispersion the ray immediately below the absorption band has its deviation abnormally increased, while that immediately above the absorption band has its deviation abnormally diminished.

In ordinary dispersing media the refractive index increases as the wave-length and the period of vibration decrease; and we have seen that we can find a relation of the form $\mu = a + \frac{b}{\lambda^2} + \frac{c}{\lambda^4} + \ldots$, which expresses, with considerable accuracy, the connection between the two. In media in which anomalous dispersion takes place this is not the case, for it is no longer true that waves of short period have a greater refractive index than waves of long period.

We must, of course, distinguish between the spectrum formed by a hollow prism containing one of these media and the absorption spectrum produced by allowing the rays dispersed by a glass prism to pass through a dilute solution of cyanin or fuchsin. In this latter the order of the colours will be that given by the glass prism with dark bands across corresponding to the rays, which are powerfully absorbed. In the former the dispersion is due to the variation in velocity of different colours in the anilin medium, and the order of the colours depends on the law connecting the velocity with the period of vibration.

Another curious phenomenon which is presented by certain coloured media, and to which Professor Stokes has given the name of fluorescence, remains to be described. This was first observed by Sir David Brewster in a solution of chlorophyll, and named by him internal dispersion. He found that on passing the sun's light through a solution in alcohol of the green colouring matter of leaves, the path of the beam was marked by a brilliant red light some time

afterwards. Sir John Herschel noticed that when the sun's light is allowed to fall on a vessel containing a dilute solution of sulphate of quinine, the surface of the liquid on which the light first falls is of a bright blue colour, which extends some little way, though not far, into the liquid. The best way of observing this is to reflect the sun's rays vertically downwards on to the surface of the solution, and, placing the eye about level with the surface, look at a black cloth behind it. If we allow the light, after passing through this solution, to fall on a second, the bright blue colour hardly appears at all. Sir John Herschel called this phenomenon the epipolic dispersion of light, and a beam which had experienced this change was said to be epipolised, so that we conclude that an epipolised beam is incapable of again producing epipolic dispersion. Sir David Brewster showed, by concentrating the sun's rays by means of a convex lens to a focus within the solution, that the whole of the path of the beam was marked by this blue light, though it was most intense near the first surface, and concluded hence that epipolic dispersion was part of the general phenomena of internal dispeısion which he had observed in chlorophyll, fluorspar, and some other substances. It was left for Professor Stokes to show that these phenomena, to which he gave the name of fluorescence, may be observed in many other media, and to explain the action which gives rise to them. He proved, in a paper in the 'Philosophical Transactions' for 1852, that they were due to a change produced by the medium in the refrangibility of the light passing through it. If we examine with a prism light that has passed through a dilute solution of sulphate of quinine so as to observe its absorption spectrum, we find that the violet part of the spectrum is wanting—all beyond the line H is missing. Now, we shall show that it is just these rays which are wanting from the absorption spectrum, which are, that is, strongly absorbed by the solution, that produce the fluorescence. They are absorbed by the medium, but given out

again as rays of lower refrangibility—that is, as blue light, or, it may be, as in the case of chlorophyll, as red light. To prove this we must observe the effects of allowing light of different colours to fall on the solution. Let us form on a screen a spectrum of the sun's rays by means of prisms and a slit, and hold a test-tube containing a solution of sulphate of quinine in the different colours. So long as it is in the red and green part of the spectrum we observe nothing strange. In the red our solution looks red; in the green it is green, but, as we pass on to the blue and violet, the appearances change in a very marked manner. The pale blue light seen when the sun's rays fell on the solution begins to appear, and gets strongest and best marked in the violet, but if we carry on the test-tube far beyond the limits of the visible spectrum, this blue shimmer is still visible; the mixture still shows fluorescence.

The red and green rays which pass most easily through the sulphate of quinine have no tendency to make it fluoresce; the violet and ultra-violet rays alone, which it most readily absorbs, are those to which the effect is due. We can test this in another manner, even more simply and with less apparatus. We have seen that cobalt glass allows the extreme violet rays to pass freely through, and also some of the extreme red. If, in addition to this, we take a glass coloured a deep violet by manganese, and allow the sun's rays to pass through the combination and fall on the solution of quinine, only the violet light can get through the glass, but the blue colour is seen just as when the full light of the sun was used. If, on the other hand, to sift our beam we use a yellow glass of the kind employed by photographers, we shall find that no fluorescence is visible. The yellow glass absorbs the very rays which are capable of producing it.

If we analyse, by means of a prism and a slit, this blue stratum of light, we find that it is not homogeneous, but is composed of rays from different parts of the spectrum, and

of different refrangibilities. All, however, are less refrangible than the violet light from which they come.

Another method of experimenting given by Professor Stokes allows us to show in a very marked way the change in refrangibility. If we place behind the purple manganese glass one coloured yellow by nitrate of silver and slightly overburnt, we have a combination through which no light can pass. The manganese and cobalt glass absorbs all rays but the extreme violet; the yellow glass absorbs these.

If, now, the light from the sky enters a darkened room through a hole in the shutter covered by the purple glass, and we look at this through the silver glass, it appears quite black, and if we place behind the combination a fluorescent substance such as sulphate of quinine no trace of fluorescence is seen. But suppose now we remove the second glass, the blue glimmer shows itself at once on the sulphate of quinine, and remains visible even if we put the yellow glass between it and our eyes. The yellow glass in its first position absorbed the rays which produce fluorescence. These same rays have been so altered by the fluorescence that in its second position before the observer's eye they can traverse it quite freely. Instead of darkening a room and cutting a hole in the shutter, we may, as Professor Stokes has suggested, use a box or packing-case blackened on the inside, with two holes cut in opposite ends. One of these, which is turned to the sky, is covered by the purple glass or other absorbing medium. Behind this there is a suitable table or shelf to hold the substance to be examined. The observer looks through the hole at the other end of the box, which should be large enough to allow him to put his hands conveniently inside, while at the same time his head and the end of the box are covered with a dark cloth. In this manner we can easily assure ourselves that fluorescence is shown by a large number of substances. If, in addition, we use a prism and a slit, and have on the table in the box on which the fluorescent substance stands

a plate of white porcelain, we can determine the part of the spectrum to which the fluorescent light belongs. For this purpose we turn our prism and slit to view the plate of porcelain, having first made sure that it gives no fluorescent effects. The spectrum which we see will be identical with that obtained by looking directly at the absorbing medium which covers the first hole, only less bright. A blue cobalt glass is rather better for this purpose than one coloured with manganese, for the red rays which it transmits do not interfere with the appearances presented, and serve as a point of reference. Place now on the porcelain plate the substance to be examined, and hold the slit close down to it in such a way that the plate is seen through one part, the fluorescing substance through the other. The spectrum of the light from the plate remains as before; the other half of the slit shows the peculiar spectrum of the fluorescing medium. By this method Professor Stokes has detected the fluorescence of white paper, cotton wool, bone, ivory, white leather, the white part of a quill, and many other substances.

The fluorescence of sulphate of quinine has been shown to be due to the extreme violet rays. In chlorophyll, however, the greater part of the effect is produced by the visible light of the spectrum, and begins in the red end of the spectrum near the line B. It will be remembered that the absorption spectrum of chlorophyll has a strongly marked dark band between B and C. If a test-tube containing the solution be held between these lines in the spectrum formed by a set of prisms it glows with a deep red light; and this light, when analysed by a prism, is found to be rather less refrangible than the dark absorption band. As we move the test-tube up the spectrum the red glow diminishes somewhat, getting more intense again in the neighbourhood of the other absorption bands; and as we pass on to the blue and violet, the red glow becomes more continuous, turning to a brownish colour in the neighbourhood of H.

If we analyse the fluorescent light with a prism, we find that there is some green visible, which produces this brownish tinge in the spectrum.

By the aid of these fluorescent solutions we are able to map the solar spectrum far beyond the visible violet rays. To do this we form with suitable prisms and lenses a pure spectrum, and receive this pure spectrum on a fluorescent substance. The dark lines in the visible spectrum are due to the absence of light of certain definite refrangibilities. From the invisible rays waves of certain periods are absent, so that at the places on the screen which correspond to these waves there is nothing to produce fluorescence, and the spectrum is there crossed by dark lines just as in the visible portion.

Glass absorbs somewhat readily these highly refrangible rays, and therefore we are compelled to have recourse to some other material for our lenses and prisms. Quartz or rock crystal is the most suitable, and by the aid of this substance the solar spectrum has been mapped far beyond the visible violet.

Phosphorescence is another property of some media which is closely allied to fluorescence. The effects of fluorescence last only so long as the light continues to illuminate the substance. On closing with a shutter the hole through which the light comes on to the quinine solution in the experiments described above, the blue shimmer ceases at once.

But some substances, notably sulphur-compounds of barium, strontium, and calcium, continue to shine after the light has ceased to illuminate them. This phenomenon is called 'phosphorescence.'

The rays which produce phosphorescence are, for the most part, those of high refrangibility, *i.e.* those which are efficient in fluorescence; and, in general, the refrangibility is lowered by the phosphorescence. Both fluorescence and phosphorescence are effects of the absorbed light.

The former lasts only so long as the light continues to fall on the active substance ; the latter continues for some time after.

Let us consider now in conclusion, very briefly, how we are to explain these phenomena of absorption, fluorescence, and phosphorescence on the undulatory theory. Waves of different periods in the ether fall on a transparent body: some are absorbed, some pass through almost unaltered.

Now, we may say almost certainly that the body consists of a number of very small particles, called molecules, which are capable of vibrating slightly about a mean position, and that each of these molecules, again, is a collection of smaller particles, called atoms. Any one molecule—so long as the substance remains the same—probably contains continually the same atoms, but these atoms may vibrate among themselves and change their relative positions.

Again, the ether is very closely bound up with these molecules and atoms, and it is not unreasonable to suppose that when light falls on our body they are set into some state of vibration. According to the simplest hypothesis we can make as to the relation between the forces produced by this vibration in our medium and the amount of displacement of the individual particles, the period of vibration would be necessarily the same as that of the ether particles which set it up; and if it happened that this period was one in which the molecules of the body could vibrate freely, all, or nearly all, of the energy in the incident vibration would be employed in setting up in the body vibrations of this period. This possibly represents what happens when light from a sodium flame falls on sodium vapour at a lower temperature. The period of vibration in the vapour is the same as that in the incident light, and the energy of this light is used in setting the sodium vapour into vibration and in raising the temperature of the whole. The original source of light is at a high temperature ; the sodium molecules in it are vibrating very energetically, and set the

ether particles into a state of vibration of the same period as their own. Those of these ether vibrations which fall on the second layer of sodium vapour give up their energy to the sodium particles in it, for they are vibrating in the period of those particles ; the light is absorbed.

But it by no means follows that in all cases the relation between the elastic forces in our medium and the displacements is of the simple nature this supposes. It is quite possible that they may follow some other law, and in that case it is not true that the period of vibration of the particles of the body will be the same as that of the ether particles which give rise to their motion.

The period will depend on the law of the force and also on the amplitude of the incident vibration, and may be anything *greater* than the period of that incident vibration.

Thus light vibrations falling on a body may set that body into a state of motion in which the periodic time will be greater than that of the original vibration. But the particles of the body thus set in motion themselves react upon the ether and cause it to vibrate. The period of this ether vibration may be the same or greater than that of the molecules of the body, that is, it may be the same or greater than that of the incident light, and these ether vibrations become known to us as light, light which may be of the same or less refrangibility than that which falls on the body, but cannot be of greater.

We suppose that the molecules of the body are capable of vibrating in certain definite periods, and in so doing set up in the ether vibrations of the same or greater periods, that is, they give rise to light of a definite refrangibility. Owing to the action between the ether and the molecules of the body, these vibrations in the body can be set up by vibrations in the ether of certain other definite periods. If in the light falling on the body there be rays of these periods, the energy in them is used in setting the molecules of the body into

vibration; the light is absorbed, and its place in the spectrum of the transmitted light is marked by a dark band, more or less dark according to the avidity with which the absorption takes place; these moving particles of our medium react on the ether round them and cause it to vibrate in periods the same or greater than their own, their motion thus becomes known to us as light of the same or of less refrangibility than that which set up the motion in the body.

In the first case we have ordinary transparent media, in the second we get fluorescence or phosphorescence. The medium is fluorescent if the disturbance set up by the incident light acts only so long as the light falls on it; it is phosphorescent if the effect continues after the incident beam is cut off.

We may give an illustration taken from the lecture of Professor Stokes, already referred to:—

'Suppose you had a number of ships at rest on an ocean perfectly calm; supposing now a series of waves without any wind were propagated from a storm at a distance along the ocean, they would agitate the ships, which would move backwards and forwards; but the time of swing of each ship would depend on the time of its natural oscillation, and would not necessarily synchronise with the periodic time of the waves which agitated the ship in the first instance. The ship being thus thrown into a state of agitation would itself become a centre of agitation, and would produce waves which would be propagated from it in all directions.'

These waves correspond to the fluorescent or phosphorescent light, phosphorescent in this case, for the ship would not come at once to rest the moment the original agitation ceased.

One other point this illustration brings out, the period of the ship would probably be greater—it could not be less—than that of the incident waves; the secondary series of waves which it would produce would then be of a greater

period than the original series, and would correspond to light of a lower refrangibility.

The connection between phosphorescence and fluorescence has been examined by Mons. E. Becquerel, who has shown by means of an instrument called the phosphoroscope that fluorescent effects are visible for a short time after the light which gives rise to them has ceased to illuminate the body.

Two circular discs are arranged to rotate on the same axis, and holes are cut in these discs. Each hole in one disc comes exactly opposite to the space between two holes in the other. These discs are fixed in a closed box, and two holes are made in the sides of the box, exactly opposite to each other, and in such a manner that when the discs are turned, the holes in the discs are brought in succession opposite to those in the box. Light is admitted into the box through one aperture, and the observer looks through the other. If there be nothing else, of course nothing is visible, for when, as the discs move, light passes through the aperture in the first disc it falls on the second disc, and when an aperture in that is opposite the observer's eye, the incident light is stopped by the first disc.

But between the discs there is a fluorescent substance, generally a crystal of some salt of uranium. At one instant, light falls on it through the hole in the first disc, the next it is viewed through the hole in the second, and if the discs are turned with sufficient rapidity so that the interval between these two instants is very small, a continuous illumination is seen. The fluorescent effect is prolonged one moment lest until the hole in the second disc comes opposite the observer's eye, and thus allows the crystal to be seen shining with light that had fallen on it some short time previously.

Various theories have been proposed to account for the phenomena of dispersion, anomalous dispersion, &c. According to some of the earliest of these it is due to the coarse grainedness of the ether. The simple theory of the

propagation of waves assumes the molecules of the ether to be very small compared with the wave-length of light, so that a large number of molecules are included in the length of a single wave. When this is the case the velocity of wave propagation will be independent of the wave-length, light of all degrees of refrangibility will travel at the same rate.

If, however, the number of molecules contained within the limits of a wave be not very large, this result will no longer be true. The wave velocity will to some extent depend on the wave-length, and it is on this hypothesis that Cauchy's formula, $\mu = a + b/\lambda + c/\lambda^2 + \dots$, was obtained. But the hypothesis is not satisfactory, for Lord Kelvin has shown that in order to produce the observed difference of velocity between red and blue light which is actually observed in such substances as dense glass or carbon bisulphide there ought not to be more than, say, ten molecules in the wave-length, while the best results as to the size of molecules lead to the conclusion that there are something like 600 molecules in the wave-length of mean light, and certainly not fewer than 200 or 300. The coarse-grainedness which would be requisite to explain dispersion is much greater than is possible. Again, there is no dispersion in air, and very little in vacuo. But there is no reason why the ether in vacuo should be much less coarse-grained than in glass or water, so that if dispersion is due to coarse-grainedness we should expect to find it in all media; while, thirdly, a coarse-grained structure will not account for anomalous dispersion. We are forced therefore to seek for some other cause in explanation of dispersion.

Now dispersion has been found to be intimately connected with absorption. To understand this let us consider the motion of a pendulum the point of support of which is itself capable of motion. This may be realised by attaching a string to the bob of a pendulum and fixing a second bob on to the string. On displacing the system from the vertical the lower pendulum vibrates, but its point of

support, the bob of the upper pendulum, is itself in motion.

Let us call the upper pendulum A, the second one B, and for the present let A be more massive than B. Start A vibrating, and observe the effect of altering the length of B. It will be found that for most lengths the motion of B is not very great. Suppose at first the length of B less than that of A, then as B is increased in length its motion gets more vigorous until a state is reached for which B's length is equal to that of A, so that the periods of the two synchronise; when this happens B is put into a state of violent agitation and absorbs a large portion of the energy originally given to A.

There are, of course, many other examples of this phenomenon in which a violent agitation of a vibrating body is produced by a series of small impulses synchronising with the free period of that body.

Now let us also observe the time of vibration of A. So long as there is no close approach to synchronism between the periods, and the mass of A is large compared with that of B, there is not much effect on the period, but as, by altering the length of B, we approach the condition of synchronism an effect becomes noticeable; moreover, this effect is in all cases such as makes the period of A differ more from that of B. Thus, when B is shorter than A, and therefore vibrates more quickly, the vibrations of A are made less rapid by the action, its period is lengthened; while if B is longer than A, the vibrations of A become more rapid and its period is shortened.

It can, in fact, be shown that if a and b are the lengths of the two pendulums, A and B their masses, A being great compared with B, and T_a the free period of A, then T, the actual observed period, is given by the equation

$$T = T_a\left\{1 + \frac{1}{2}\frac{B}{A}\frac{b}{a-b}\right\}.$$

So long as b is less than a, T is greater than T_a, but since B / A is small the increase is not much until b is nearly equal to a; while if b is greater than a, $a - b$ is negative, and hence T is less than T_a. If $a = b$, so that the two pendulums synchronise, the solution fails.

Or, again, suppose we have a light particle of mass m connected by a spring to a heavy particle of mass M. Let the two be capable of motion in the line joining them, and let the heavy particle be controlled by a second spring, the strength of this spring being such that the free periods of each particle when under the action of its own spring alone are comparable, then if t_0, T_0 be these periods, t the period of the particle m when the whole can move, we can show that

$$t = t_0 \left\{ 1 - \frac{1}{2} \frac{m}{M} \frac{T_0^2}{T_0^2 - t_0^2} \right\}.$$

Thus if t_0 is less than T_0 the period of m is reduced by the action of M, while if t_0 is greater than T_0 the reverse is the case.

Now let us suppose we have a number of particles of mass m arranged in a line connected by springs whose unstretched length is a, and that the first particle is made to move in this line in a simple harmonic vibration of T; suppose, also, that t is the free period of one particle vibrating when controlled by its own spring only. Then waves travel down the string, and if a be the distance between consecutive particles, V the velocity of wave propogation, we can show that

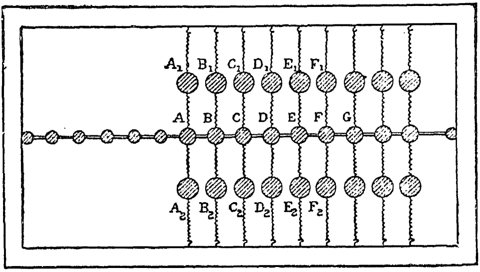
$$V = \frac{\pi a}{T} \frac{1}{\sin^{-1}\left(\frac{1}{2} \frac{t}{T}\right)}.$$

Moreover, if the motion be such that there are a large number, N, of particles in a wave length, we have $t / 2T = \pi / N$.

Hence T is large compared with t, and we may put

$t/2T$ for $\sin^{-1} t/2T$. We thus find $V = 2\pi a/t$. Thus, in such an arrangement the velocity of wave propagation is inversely proportional to the period. If, now, we suppose each of the small masses m to be connected to a large mass M, the first effect will be to modify the period in the manner already described, and consequently to modify in the inverse way the velocity with which a wave will traverse the medium. Thus, if t_0 is greater than T_0, *i.e.* if we are below an absorption band, the period t is increased, and therefore the velocity V is decreased, by the action of M, and the refractive index increased, while if t_0 is less than T_0, and we increase it, thus approaching the absorption band from above, the reverse is the case, V is increased and the refraction decreased.

FIG. 125*a*.



The arrangement considered may be illustrated as in fig. 125*a*. A, B, C, &c., represent a series of particles, each of mass m, connected together by strings. Such a series can vibrate, in the plane of the paper suppose, and if the first particle, A, be made to execute simple harmonic movements a wave will traverse the row. If the number of particles included in the wave be sufficient the velocity will be $2\pi a/t$, where t is the period of any one particle, all the others being held fixed. Such an arrangement may stand for a row of ether particles in free space. Now suppose each particle connected by two springs to fixed masses,

A_1, A_2, B_1, B_2, &c., respectively symmetrically placed on either side of the row.

This will reduce t, the free period, and therefore it will increase the velocity; but if at the same time we suppose the mass of each particle to be increased, and that in a greater ratio than the force to which it is subject, t will be increased and the velocity of wave propagation reduced. In such a case the velocity, V, will be independent of the wave-length; waves of all periods, within certain limits, which will be referred to later, will travel at the same rate. Such a row of particles may represent the ether in an ordinary transparent medium, such as glass or water, if we neglect the phenomena of dispersion.

Now let us make the assumption that the masses A_1, A_2, B_1, B_2, &c., are free to move, and are controlled by a series of springs, but that the mass, M, of each is very great compared with that of m. Let t_0 denote the free period of A when all the particles except itself are held fixed, and the strings connecting it to the neighbouring particles, B, C, are cut; T_0 the free period of one of the heavy particles under its own spring only; and, further, let these springs be so stiff that, in spite of the great mass of A_1, A_2, &c., T_0 may be comparable with the period of the impressed vibrations.

Suppose, now, that the first particle is made to vibrate in period T.

Then an expression can be found for V, the velocity of wave propagation, and it can be shown that

$$\frac{1}{V^2} = \frac{m}{Fa}\left(1 - \frac{T^2}{t_0^2} + \frac{m}{M}\,\frac{T^4}{t_0^4}\,\frac{T_0^2}{T^2 - T_0^2}\right),$$

where m is the mass of one of the particles, a the distance between two consecutive particles, F the tension of the string joining them; T_0 is the period in which the massive particles vibrate under their own springs, while t_0 is the period in which the light particles vibrate under the action of the springs which join them to the massive ones.

We can deduce from this model most of the observed facts as to the behaviour of light. We may make it rather more complete by supposing the string A, B, &c., produced as in the figure, the tension remaining the same, and loaded at equal intervals a with particles of mass m_0. Let V_0 be the velocity of wave propagation in this part of the string, and let V_1 be the velocity when the term in m / M is omitted, *i.e.* when the large masses are fixed.

$$\frac{1}{V_0^2} = \frac{m_0}{Fa};$$

$$\therefore \frac{1}{V_1^2} = \frac{m}{m_0} \frac{1}{V_0^2} \left(1 - \frac{T^2}{t_0^2}\right).$$

In applying this model to the ether, m_0 represents the mass of the ether particles in the free space, m in the transparent medium. Forces are called into play between the ether and the matter, and t_0 is the period of vibration of an ether particle under the action of the matter which surrounds it.

We shall see reason for supposing that direct action between matter and ether is small, so that, in spite of the small mass of an ether particle, t_0 is large. T_0 is the period of the matter particles when vibrating under the forces arising from other matter particles.

If we suppose, in the first instance, that the term in m / M may be neglected, we have for the refractive index μ, which is equal to V_0 / V, the value

$$\mu^2 = \frac{m}{m_0}\left(1 - \frac{T^2}{t_0^2}\right)$$

$$= \frac{m}{m_0}\left(1 - \frac{\lambda^2}{V_0^2 t_0^2}\right),$$

if λ is the wave-length in free space.

Now the most accurate experiments on dispersion indicate that a term of this kind in the expression for the refractive index is needed to account for the results, but that

it is very small, at any rate in transparent media. Thus in a transparent medium we must have $V_0 t_0$ large compared with the wave-length of light, and therefore t_0 large compared with T, the period of the light; thus the action between ether and matter must be small.

In this case we have $\mu = \sqrt{m/m_0} = \sqrt{\rho/\rho_0}$, if ρ and ρ_0 are the densities or masses per unit volume, and this is in accordance with optical theory.

Again, the velocity vanishes if $T = t_0$, and its square becomes negative if T is greater than t_0. In this case the waves cease to be propagated as such, the whole of the incident vibration is reflected, and the medium has the properties of a metal.

Returning now, to the more complete expression, we have

$$\mu^2 = \frac{m}{m_0}\left\{1 - \frac{T^2}{t_0^2} + \frac{m}{M}\frac{T^4}{t_0^4}\frac{T_0^2}{T^2 - T_0^2}\right\}.$$

Let us now start with T larger than T_0, and consider the changes in μ as T is decreased down to and through T_0, *i.e.* as we go up the spectrum. As T decreases, the value of $1 - \frac{T^2}{t_0^2}$ is increased, and that of the term in m/M is also increased; thus the refractive index increases until the value $T = T_0$ is reached, when it becomes infinite, and the wave is quenched, all the energy being absorbed. Just before reaching this point there is an abnormally large increase in μ^2, owing to the very sudden increase in $1/(T^2 - T_0^2)$. As T decreases still further, the value of $1/(T^2 - T_0^2)$ is negative, and at first it is very great; thus the whole expression is negative. Just above an absorption band the medium has the properties of a metal.

As T is further increased the negative term is reduced, the value of μ^2 becomes positive, but less than unity, and increases as we get further from the band. Above the absorption band there is an abnormally small amount of refraction, the velocity is abnormally increased. Thus a medium so

constituted would reproduce some of the effects we meet with in a substance showing dispersion.

To explain the phenomena exhibited by glass, water, or any ordinary transparent substance, we must suppose that within the limits of the visible spectrum T is less than t_0 and greater than T_0.

Thus, as we go up the spectrum μ^2 increases regularly from the red to the violet. The formula, which may, by some transformations, be written in the form

$$\mu^2 = a^2 - k^2 \lambda^2 + \frac{D \lambda_0^2}{\lambda^2 - \lambda_0^2},$$

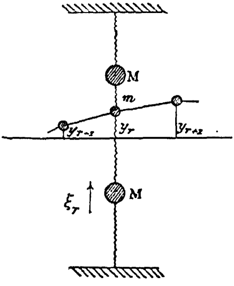
has been shown by Ketteler, to whom it is originally due, to agree well with the experimental results of S. P. Langley over a long range.

This mode of accounting for dispersion, originally due to Sellmeier, has been developed in various papers by Von Helmholtz, Ketteler, and Lord Kelvin.

It must not, of course, be supposed that the connections between the ether and matter are of the very crude kind suggested by the model, or that the analogy is complete. It is, however, interesting and important to work out the problem of a mechanical system which has properties enabling it to produce effects very closely analogous to those exhibited by the ether, both in transparent and in metallic bodies.

The following is the mathematical investigation of the problem.

FIG. 125*b*.



Let y_{r-1}, y_r, y_{r+1} be the displacements of three consecutive masses. Let ξ be the displacement of one of the two masses M attached to m_r.

Suppose the springs attached to these masses are unstretched in the equilibrium position, then when the particle m_r is on either side of the equilibrium line the spring which attaches it to a mass M on the same side is loose and does not affect the motion. Thus we get for the equations of motion

$$m y_r'' = \frac{F}{a}(y_{r+1} + y_{r-1} - 2y_r) - \mu(y_r - \xi_r),$$

$$M \xi_r'' = \mu(y_r - \xi_r) - \nu \xi_r.$$

Now take as the solution,

$$y_r = Y \cos r\beta \cos \kappa t,$$

$$\xi_r = X \cos r\beta \cos \kappa t.$$

Then

$$y_{r+1} + y_{r-1} - 2y_r = 2Y \cos \kappa t \cos r\beta (\cos\beta - 1)$$

$$= -4Y \cos\kappa t \cos r\beta \sin^2 \tfrac{1}{2}\beta,$$

and substituting in the equations, we have

$$Y\left\{m\kappa^2 - \frac{4F}{a}\sin^2\tfrac{1}{2}\beta - \mu\right\} = -\mu X,$$

$$X\{M\kappa^2 - \nu - \mu\} = -\mu Y.$$

Let there be N particles in a wave-length λ.

Then the displacements are the same for r and $r + N$.

Hence $$N\beta = 2\pi, \text{ and } \tfrac{1}{2}\beta = \frac{\pi}{N}.$$

Moreover, $\lambda = Na = VT$, if $T = 2\pi/\kappa$ be the period, and V the wave velocity.

Now let t_0 be the period of m under the force μ, and T_0 the period of M under the force ν; then

$$\frac{\mu}{m} = \frac{4\pi^2}{t_0^2}, \quad \frac{\nu}{M} = \frac{4\pi^2}{T_0^2},$$

$$\frac{\mu}{M} = \frac{m}{M} \times \frac{\mu}{m} = \frac{4\pi^2}{t_0^2} \times \frac{m}{M}.$$

Substituting these values in the equation of motion, and multiplying so as to eliminate X and Y, we find,

$$\left\{\frac{1}{T^2} - \frac{Fa}{mV^2T^2} - \frac{1}{t_0^2}\right\}\left\{\frac{1}{T^2} - \frac{1}{T_0^2} - \frac{m}{M}\frac{1}{t_0^2}\right\} = \frac{m}{M}\frac{1}{t_0^4}.$$

Whence, on reduction, omitting $(m/M)^2$,

$$1 - \frac{Fa}{mV^2} = \frac{T^2}{t_0^2}\left\{1 - \frac{m}{M}\frac{T^2 T_0^2}{t_0^2(T^2 - T_0^2)}\right\},$$

and

$$\frac{1}{V^2} = \frac{m}{Fa}\left\{1 - \frac{T^2}{t_0^2} + \frac{m}{M}\frac{T^4}{t_0^4}\frac{T_0^2}{(T^2 - T_0^2)}\right\}.$$

To obtain the equation in Ketteler's form we put

$$\frac{m}{M}\,\frac{T^4\,T_0^2}{t_0^4\,(T^2-T_0^2)} = \frac{m}{M}\,\frac{T_0^2\,(T^4-T_0^4)+T_0^6}{t_0^4\,(T^2-T_0^2)}$$

$$= \frac{m}{M}\,\frac{T_0^2}{t_0^4}\,(T^2+T_0^2) + \frac{m}{M}\,\frac{T_0^6}{(T^2-T_0^2)}\,t_0^4\,;$$

$$\frac{1}{V^2} = \frac{m}{m_0}\,\frac{m_0}{F\,a}\left\{1 + \frac{m}{M}\,\frac{T_0^4}{t_0^4} - \frac{T^2}{t_0^2}\left(1 - \frac{m}{M}\,\frac{T_0^2}{t_0^2}\right)\right.$$

$$\left. + \frac{m}{M}\,\frac{T_0^4}{t_0^4}\,\frac{T_0^2}{T^2-T_0^2}\right\};$$

and

$$\frac{m_0}{F\,a} = \frac{1}{V_0^2}.$$

Remembering, now, that T is proportional to λ, being equal to λ/V_0, we can write thus,

$$\mu^2 = a^2 - k^2\,\lambda^2 + \frac{D\,\lambda_0^2}{\lambda^2-\lambda_0^2}$$

where

$$a^2 = \frac{m}{m_0}\left(1 + \frac{m}{M}\,\frac{T_0^4}{t_0^4}\right),$$

$$k^2 = \frac{m}{m_0}\,\frac{1}{V_0^2\,t_0^2}\left(1 - \frac{m}{M}\,\frac{T_0^2}{t_0^2}\right),$$

CHAPTER XI.

DOUBLE REFRACTION.

WE have seen that light falling on the surface of glass or water is there refracted according to a certain law, and we have shown how this law follows as a consequence of the undulatory theory. But there are many natural substances in which the refraction is different. In these, instead of having one refracted ray obeying this law, we have two. In some of these substances one ray obeys the ordinary law of refraction, while the other does not. In other substances neither ray follows the ordinary law.

These media are called doubly-refracting. In addition to this we shall find that the light which emerges in each of these waves from a doubly-refracting medium has undergone another important change; it is said to be plane polarised, The phenomena of polarisation and double-refraction are intimately connected, and we shall consider them together. Iceland spar, which is a crystallized form of carbonate of lime, is the medium which shows double refraction in the most marked manner. Crystals of Iceland spar can easily be split along cleavage planes into rhombohedra. (A rhombohedron is a solid figure bounded by six plane figures, each of which is a rhombus.) The acute angles of each rhombus are the same, and the angles between the faces of the crystal are 74° 55′ 35″, and 105° 4′ 25″, according as they are acute or oblique.

FIG. 126.

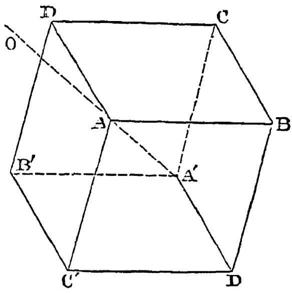


Fig. 126 is a representation of a rhombohedron of Iceland spar. At two opposite corners, A and A′, the three

angles of the rhombic faces, which meet and form the corner, are all obtuse. A line drawn through A, equally inclined to the three edges which meet there, is equally inclined to the six faces of the crystal, and the whole solid is symmetrical with reference to this line, which is called the axis of the crystal. The angle between this and any one of the rhombic faces is about 26° 15′ 30″; any plane passing through the axis of the spar is called a principal plane.

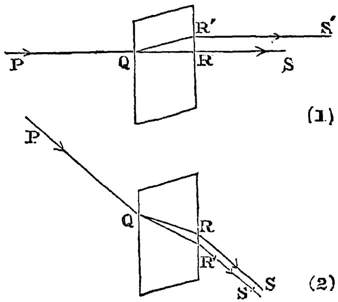
Crystals in general have three axes at right angles, and the three planes, each containing two of these axes, are called principal planes.

A plane passing through the axis and cutting at right angles one of the rhombic faces is the principal plane of that face. Iceland spar has only one axis, and any plane through it is a principal plane.

If now we take a moderately thick rhomb of Iceland spar and, laying it on a sheet of paper on which a black dot has been made, look through it, we shall see two dots. If we look straight down from above the dot and turn the crystal round, keeping it on the paper, we shall see that one dot remains fixed while the other appears to move round it in a circle. The line joining the two dots will always be parallel to the bisectors of the oblique angles of the rhombic face through which we look. The fixed dot, too, will seem to be rather nearer to us than the movable one. We may vary the experiment by looking through the spar at a pin-hole pierced in a sheet of paper, and held up to the light. Two pinholes will be seen, and as we turn the crystal round one will seem to move round the other. The ray of light from the pinhole is divided by the crystal into two, which are refracted differently, and so separated into two on emergence. The paths of the light are given in fig. 127. If the light fall normally on the crystal, as in fig. 127 (1), one ray, Q R, passes straight through, as would be the case in glass; the other, Q R′, is refracted at incidence at Q, along Q R′,

again refracted at R′, and emerges parallel to its original direction. If the incidence be oblique, as in fig. 127 (2), one ray is refracted at Q along Q R, and it is found from careful observation in the case of Iceland spar that the law connecting the angles of incidence and refraction ϕ and ϕ' is the same as it would be in glass, so that the ratio of sin ϕ to sin ϕ' depends only on the nature of the light used. This ray is called the ordinary ray, and after refraction at the second surface at R emerges parallel to its original direction.

FIG. 127.



The second ray is refracted at Q along Q R′, according to a law which we shall explain presently, and emerges along R′ S′, also parallel to its original direction. This ray is known as the extraordinary ray. Thus, in general, when light falls on Iceland spar it is split up into rays, the ordinary and extraordinary, refracted according to certain laws. We shall denote the ordinary ray by the letter O, the extraordinary ray by the letter E.

Now, suppose we take a second rhomb of the spar and placing it on the top of the first, turn it round until the plane surfaces which bound it are parallel to the corresponding surfaces of the first rhomb. On looking down, we still see two images of our dot or pinhole. The two rhombs behave just as one rhomb of the joint thickness would do. The ray O, emerging from the first rhomb, remains an ordinary ray in the second; while the ray E is still refracted according to the extraordinary law.

But now turn the upper rhomb round through a small angle; four dots are seen.

Each of the rays, O and E, which fall on the second

rhomb, are doubly refracted; O gives rise to a second ordinary ray O_O and a second extraordinary ray O_E, while from E also we have two, E_O obeying the ordinary law, and E_E the extraordinary.

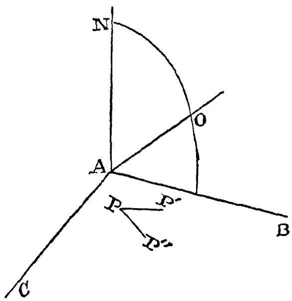
In the first case, the axes of the two crystals were parallel, and the principal planes of the surface of separation of the two were parallel also. Our two incident rays, O and E, only produced two rays in the second crystal. When the upper rhomb is turned so that the principal planes of the common surface of the two are no longer parallel, but inclined at some angle less than 90°, we have four rays O_O, O_E, E_O, and E_E. The two seen in the first position were O_O and E_E. At first, on turning the crystals, the other pair O_E and E_O are very faint and close together, but as the upper rhomb is turned further they get brighter and the distance between them grows larger, and when the angle between the principal planes has increased to 90°, O_O and E_E have disappeared and O_E and E_O alone are left. If the rotation be continued, O_O and E_E come into view again, and the other two grow more faint until when the rhomb has been turned through 180°, so that the principal planes are again parallel, they have disappeared and only O_O and E_E are visible.

They, too, are closer together than in the first position, and, if the rhombs be of the same thickness, they will coincide exactly and give us one image of our dot. As we continue to rotate the rhomb into its first position, the appearances recur, but in the reverse order. Thus we learn that in general a ray of light, which has passed through a crystal of Iceland spar and is incident on a second, is like common light split into two and doubly refracted, but if the principal planes of the faces of emergence and incidence in the two crystals respectively be parallel or at right angles to each other, the ray is not doubly refracted by the second rhomb.

If the principal planes are parallel, the ordinary ray emerging from the first suffers ordinary refraction at the surface of the second, while the extraordinary ray from the

first is an extraordinary ray in the second. If, however, the principal planes are at right angles, the ordinary ray O gives rise to an extraordinary ray O_E, while the extraordinary E produces an ordinary ray E_O. Common light, on the contrary, always gives rise to two refracted rays when passing in the direction considered through such a rhomb. The light, then, which has been refracted through a crystal of spar, has acquired properties which distinguish it from common light, and which are related to the position of the principal planes of the rhombic faces across which it passes; it is said to be plane polarised.

FIG. 128.



Let A B C (fig. 128) be any plane wave in a crystal of Iceland spar, let A N be drawn at right angles to the wave; A N is, as we have learnt, called the wave-normal. Draw A O parallel to the axis of the spar, and through A O and A N draw a plane B O N, cutting the wave in A B. This plane is clearly perpendicular to the wave, and it passes through the axis. It is, therefore, the principal plane of the wave. If A B C be an ordinary wave, it is said to be plane polarised in this principal plane; if it be an extraordinary wave, it is plane polarised in a plane at right angles to this.

An ordinary wave, then, consists of light plane polarised in the principal plane of the wave; an extraordinary wave, of light which is plane polarised in a plane at right angles to the principal plane.

If we wish to determine if light is plane polarised, we can do so by allowing it to fall normally on a rhomb of spar. In general it will be split into two rays. If by turning the spar round an axis parallel to the direction in which the light is coming we can find a position for it such

that only one ray is transmitted through, then we infer that the light is plane polarised, and the plane of polarisation coincides with that of this light in the spar.

But we learn still more from our two rhombs of spar. Consider the incident ray O. In the first case, when the two principal planes are parallel, we got from it a ray O_0. This is polarised in the principal plane of the second crystal, that is, in a plane coincident with that of the incident light. The ray O_E, did it exist, would be polarised in a plane at right angles to that of the incident light, but no such ray is produced. Thus we see in this case that light polarised in one plane will not produce light polarised in a perpendicular plane. If we consider what happens when the upper rhomb is turned through 90°, we arrive at the same result: O_0 is then quenched, and it would be polarised in the principal plane of the second crystal, that is, in a plane at right angles to the plane of polarisation of the incident light. Thus in these two cases, we observe that light polarised in one plane cannot give rise to light polarised in a perpendicular plane, and this we shall find is a general law.

Let us now consider what quality there is in which polarised light can differ from common light. The colour of a wave depends, we have seen, on the periodic time of the vibration which constitutes it; the intensity is known if we know the amplitude of that vibration; but as yet we have said nothing about the direction of the vibration, except that it must be in the wave front. If A B C be a plane wave front, then at any instant all the particles in that front are moving exactly alike: they are vibrating in the same direction and with the same velocity.

But, for all we know, the direction of motion may be different from instant to instant. A particle P (fig. 128) may at one moment be vibrating in the straight line P P′, and of course at that moment all the particles in the plane are moving parallel to P P′, but the next moment this direction may

have changed—instead of P P$'$, P P$''$ may be the path of the particle P, and those of all the other particles will then be parallel to P P$''$.

It is not even necessary for our previous work that the paths of the particles should be straight lines, and, in fact, we can show without much difficulty that in the most general case consistent with the nature of the vibration considered, each particle will describe a small ellipse about its position of rest as centre, and that the eccentricity of this ellipse, and the position of its axes, will continually be changing.

Thus all the particles in the front A B C are any moment describing equal ellipses, but these ellipses are continually varying both in shape and position. The particles in a neighbouring front are all describing ellipses equal to one another, but differing from those in the front A B C.

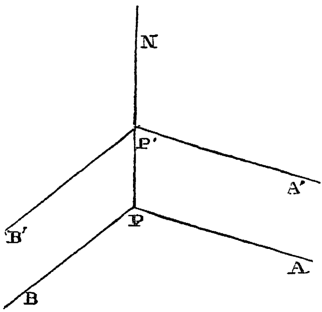
All then that we can say of the vibrations in a ray of common light is that they are in the wave front, and that if the light be homogeneous, each particle of ether is moving at any moment in an ellipse, which is ever varying in shape and position. Let us now suppose that in a plane polarised ray each particle of ether moves in a straight line, and that the direction of this straight line is the same at all points of the ray, and let us further assume that this direction of vibration is at right angles to the plane of polarisation, and endeavour to see how these additional hypotheses will enable us to explain the facts before us.

We may illustrate the difference between a plane polarised wave and a wave of common light by referring to the tube filled with sand, used in the first Chapter to explain wave-motion. A slightly elastic tube, A B, was supposed to be suspended vertically from A, and an observer, grasping it at B, and slightly stretching it, moves his hand backwards and forwards periodically. Waves are thus propagated along the tube. Let us suppose that B is moved backwards and forwards in a straight line perpendicular to the plane

of the paper. Then all other points in the tube will move in lines parallel to this and at right angles, therefore, to the paper. The wave which runs up the tube will be plane polarised, and the plane of the paper will be that of polarisation. If, however, we suppose that the end of the tube is made to describe a small ellipse about B, in a plane at right angles to its length, and that the eccentricity and position of the axes of this ellipse are varied in any manner subject to one condition determining the intensity of the wave-motion, then all the other particles will describe ellipses with varying axes in planes at right angles to A B, and the wave will correspond to one of common light.

In addition to the above two, we shall make the third assumption that the optical difference between a crystal such as Iceland spar and an isotropic medium such as air or water, lies in the fact that if a wave be travelling through the isotropic medium, it is possible, consistently with the propagation of the wave, for the ether particles to vibrate in any direction in its front, while if the medium through which the wave be travelling be a crystal, for each position of the wave-front there are two, and only two, possible directions of vibration, and these are the same for all parallel wave-fronts.

FIG. 129.



Thus, if A P B, A′ P′ B′ (fig. 129) be two parallel wave-fronts in air, particles of ether P and P′ in A P B and A′ P′ B′ respectively may each be vibrating in any direction whatever, but if the medium be crystalline, P can only move in one or other of two definite directions P A, P B, which are at right angles to each other, while P′ must move in one or other of the two directions, P′ A′, P′ B′ parallel to P A and P B respec-

tively; if P moves along P A, P′ must move along P′ A′. Thus a ray of light traversing a crystal must be plane polarised. These directions, P A and P B, depend on the position of the wave-front with reference to the axis of the crystal, and lie respectively in and at right angles to the principal plane of the wave-front.

When, then, a ray of common light falls on a crystal of Iceland spar, our hypothesis leads us to suppose that the vibration in the incident ray is resolved by the spar into two. One of these is at right angles to the principal plane and produces the ordinary ray; the other is in the principal plane and gives rise to the extraordinary ray. Now we know that the refraction of light at the bounding surface of any two transparent media depends on the fact that the velocity of the light is different in the two media, and since the ordinary and extraordinary rays are differently refracted, we infer that they traverse the crystal with different velocities. Again, the ordinary ray is refracted according to the law of sines discovered by Snell, and this law has been deduced from the undulatory theory on the assumption that the second medium was one like air or water in which light travelled at the same rate in all directions. Thus we see that the velocity of the light in the ordinary ray is the same whatever be the direction in which it traverses the crystal.

The extraordinary ray, however, does not obey the law of sines, and, therefore, the velocity for that ray is not the same for all directions of propagation. An incident ray A B gives rise to two refracted rays B O and B E, a second ray A′ B′ incident in a different direction produces two refracted rays B′ O′, B′ E′. The velocities in the two ordinary rays B O, B′ O′, are the same. Those in the two extraordinary rays B E, B′ E′, are different.

When the two rays O and E fall on a second rhomb of the crystal, each is in general resolved by it into two rays with vibrations in the directions for light falling at that angle on the rhombic face. If the incident vibration be already

in one of these two directions there is no need for resolution to take place; this will happen when the principal planes of the two rhombs are parallel or perpendicular. In the first case O will be vibrating at right angles to the principal plane of the second crystal, and will pass through it as an ordinary ray O_0, there will be no O_E, while E will be vibrating in the principal plane and will traverse the second rhomb as an extraordinary ray E_E. If the principal planes are at right angles in the two rhombs O becomes O_E, an extraordinary ray, while E gives rise to the ordinary ray E_0. According to our third assumption refraction through a crystal will give rise to waves in which the direction of vibration is fixed when we know the direction in which the wave traverses our crystal. This fixity of direction constitutes a difference between such a wave and common light, and enables us to explain the phenomena which occur when the wave is again refracted by the second rhomb.

The first hypothesis then follows as a consequence of the third, though we have not shown that this is the only difference between plane polarised and common light.

The light which emerges from the crystal is distinguished from common light by the fact that the direction of vibration remains fixed. Each ether particle disturbed by the wave vibrates in a straight line, and these straight lines are all parallel. In this case, at any rate, our plane polarised ray has the properties assumed for it in our first hypothesis.

Let us now consider what grounds we have on which to base our third assumption, which supposes that for each wave front within a crystal there are two, and only two, possible directions of vibration in that front consistent with the propagation of the wave. For this purpose we must consider the difference between an isotropic body such as glass or water, and a crystal such as Iceland spar or aragonite. If we start from a point O in the centre of a mass of water which is uniform in temperature, all the properties of the water are the same in whatever direction we travel. Take any two lines

O P and O P′ meeting at O, then light would travel at the same rate along O P and O P′. So, too, would sound and electricity, and if we were to apply heat at O, so as to cause our body to expand, the expansion would take place at the same rate in all directions. All the physical properties of the body are the same whatever be the direction. But with a crystal this is not the case. If O P, O P′, be two lines drawn from a point O, light we find travels at a different rate along O P and O P′, so too does sound; and if O become a source of heat, the linear expansion produced in the body is different in the two directions O P and O P′.

The same, too, is true of compression. If I cut a sphere of glass and subject it to a uniform pressure over the whole surface, it will remain a sphere; but a sphere of crystal changes its shape under the same circumstances. The elasticity of glass is the same in every direction, that of the crystal depends upon the direction considered. There is one point of importance, however, in which glass and crystal media resemble each other. If O_1 be a second point of the medium the properties round O_1 are identical with those round O for both glass and crystal. If we draw $O_1 P_1$ parallel to O P, light, heat, and sound each travel respectively at the same rate along O P and $O_1 P_1$. If I cut out small equal cubes of the media about O and O_1 in such a way that each edge of one is parallel to an edge of the other, I can interchange these cubes, still keeping the corresponding edges parallel, without altering the general properties of the medium in the least. Both media are homogeneous. In glass or water, it is true, I can do more. I can turn the cubes round into any other position, there is no need to keep corresponding edges parallel. Now it follows from this that if a particle of glass or water, or any isotropic substance be displaced from its position of rest through a given distance, the force of restitution which tends to bring it back is the same whatever be the direction in which the displacement is made; but if the medium be a

crystal this is no longer true. The force produced by a given displacement depends on the direction of that displacement. I displace a particle for a given small distance along O P. The elasticity of the medium produces a certain force of restitution. If the displacement had been made along O P′ the force of restitution would have been different. In neither case probably would the forces have acted along O P or O P′, but in directions related to them in a certain definite manner. In any crystal, however, we can show that there are three fixed directions at right angles such that the force of restitution produced by a displacement in either of these directions is parallel to the displacement. These three directions are called the axes of the crystal. If a particle of the crystal be displaced parallel to one of these axes, the force produced by the displacement will also be parallel to the axis and will tend to pull the particle back into its original position. If the impressed displacement had been in any direction which does not coincide with an axis, the force of restitution would not have been parallel to the displacement, but would have tended to pull the particle in a direction inclined to that of the displacement.

Let us now assume that the connection between matter and the ether is such that in a crystal the ether is crystalline in structure, so that its elasticity is different in different directions; and consider a wave of light traversing the crystal. We will take the most general form in which neither of the two refracted waves obeys the ordinary law, and suppose that the ether has crystalline axes parallel to those of the crystal.

In the first case, when the displacement of the ether is parallel to an axis, each particle can continue to vibrate parallel to that axis.

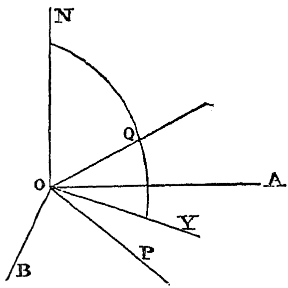
This second case we shall have to consider further.

Let us suppose the plane of the paper represents a wave front traversing our crystal, and that all the ether particles in it are displaced a distance O P (fig. 130) in that plane. The

direction of the force of restitution will be the same for each particle, but it will be along O Q some line not in the plane of the paper. This force can be resolved into two, one along O Y in the plane of the paper, the other perpendicular to that plane.

Suppose we consider the effect of the force along O Y only, assuming that that perpendicular to the plane of the wave cannot affect the motion which constitutes light. Our displacement is along O P, and the force along O Y; this force, it is clear, will not tend to pull P back to O and so set the medium into a state of regular vibration, for O Y and O P do not coincide. It can be shown, however, that if, as supposed, we do neglect the effect of the force perpendicular to the paper, there are two directions O A and O B at right angles such that if the displacement take place along either of these the force of restitution is in the same direction.

FIG. 130.

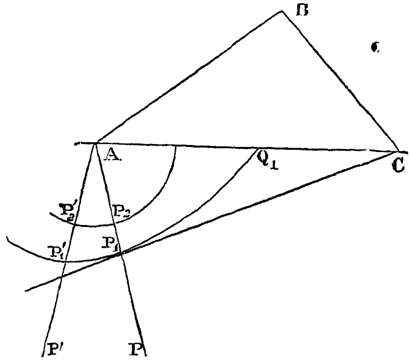


If the particle be displaced along O A it will, therefore, continue to vibrate along O A, and if it be displaced along O B it will continue to vibrate along O B. Moreover, we can show that the waves set up by these vibrations will traverse the crystal with different velocities. A displacement such as O P incident on the crystal cannot traverse it, but is resolved into two along O A and O B, which travel at different rates.

Let us suppose that the forces of restitution produced by a displacement of length p along each of the axes respectively are $a^2 p$, $b^2 p$, and $c^2 p$, and construct an ellipsoid with its centre in the wave-front, and its axes equal to $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ respectively, in the direction of the crystallographic axis.

The wave-front we have been considering will cut this ellipsoid in an ellipse, and the possible directions of vibration in this front can be shown to be the axes of this ellipse. Moreover, if O A, O B be these axes, the velocity of propagation of a wave in which the displacement is along O A will be 1/O A, while for a wave in which the vibrations are parallel to O B the velocity will be 1/O B. These propositions were proved by Fresnel and form the basis of his theory of double refraction. We must remember that they are only true on the assumption that we may neglect altogether the effect of the force normal to the wave.

FIG. 131.



Thus we see that any given disturbance falling on the crystal breaks up into two, polarised in different planes, travelling with different velocities, and therefore refracted according to different laws.

Let us turn now to consider the refraction of an incident plane wave on the undulatory theory. We have already proved a construction which will give us the position of the refracted wave. Let the plane of the paper be the plane of incidence, A B (fig. 131) the trace of the incident wave, A C the trace of the face of incidence. Draw B C perpendicular to A B and let t be the time which it will take the disturbance to travel from

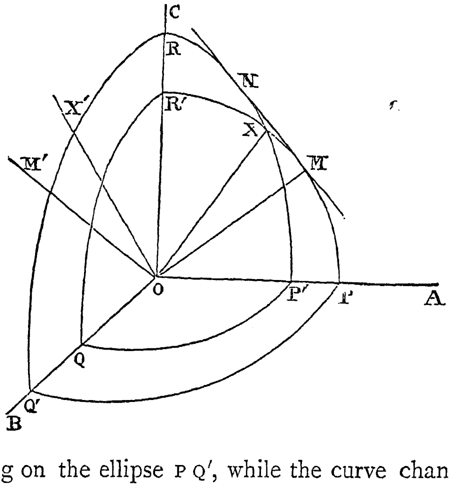
c. With A as centre describe in the second medium the e surface $P_1 Q_1$ of that medium for a time t. Draw a plane ugh C perpendicular to the plane of incidence to touch surface; this plane is the refracted wave. This con- ction is, as we have seen, perfectly general, but there es the question: In the case of a crystal what is the ı of the wave surface? A disturbance is originated at A he second medium and spreads outwards. The wave ace for the time t is the locus of the points which the urbance has reached in the time. For an isotropic lium this surface is a sphere, and if V be the velocity of light in the medium the radius of the sphere is V t. But ı crystal any incident vibration splits into two, which el with different velocities. If we draw any line A P in second medium and enquire what point along A P the urbance has reached, we find that there are two, P_1, P_2, esponding to the two velocities possible for a wave elling in the direction A P. If we take any other line we get two other points P_1', P_2', and the wave surface is locus of all these points; it is a surface with two separate ets or portions, and the intersection of this surface by the ıe of the paper will be two separate curves $P_1 P_1'$, $P_2 P_2'$. e exact form of the surface requires mathematical analysis its determination; we may, however, state some of the ılts.

Let O A, O B, O C (fig. 132) be the three axes of the crystal, . in O A take two points P, P', such that O P $= b$, O P' $= c$. O B two points Q, Q' such that O Q $= c$, O Q' $= a$. In O C two nts R, R', such that O R $= a$, O R' $= b$. Suppose a, b, c are lescending order of magnitude. Join R' P by a circle of ius b, P'Q by a circle of radius c, and Q' R by a circle of ıus a in the planes A O C, A O B, and B O C respectively, l in these same planes draw ellipses R P', P Q', and Q R', h their axes along O A, O B, O C.

Then on Fresnel's theory these circles and ellipses re- sent the sections of the wave surface by these planes,

which are known as the principal planes of the crystal. The circle and ellipse in the plane A O C will intersect, let X be one of the points in which they cut, then the two sheets of the wave surface meet in X. If we suppose the curve R′ X P′ to move round O R′ so that P′ always remains on the ellipse P′ Q, while the curve alters in form as it turns according to a certain definite law, until at last it coincides with R′ Q, then this moving curve will trace out the inner sheet of the wave surface, and if R X P move round the same axis O R, P always

FIG. 132.



remaining on the ellipse P Q′, while the curve changes until at length it coincides with R Q′, the surface thus traced will be the outer sheet of the wave surface. Any disturbance originated at O will at once split up into two; after a time *t* one of these will be spread over the inner sheet, the other over the outer sheet of the wave surface. These two sheets meet at X and at three other points symmetrical with X in the plane A O C. Suppose we consider a wave-front travelling in the crystal in a given direction. At time *t* it will be a tangent to this wave surface, and since we can draw two

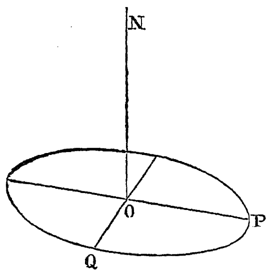
parallel tangent planes, one to each sheet, there are two possible positions for the front and two possible velocities. It may happen, however, that these two parallel wave-fronts coincide. The circle and ellipse in the plane A O C have a common tangent line, and a plane through this line perpendicular to A O C touches the surface, and is therefore a possible position for the wave-front. If the wave normal in the crystal be perpendicular to this plane, there is only one wave and one velocity of wave propagation. The direction of the wave normal for which this is the case is said to be an optic axis of the crystal. The optic axis is a line such that all waves normal to it traverse the crystal with the same wave velocity. We may therefore call it the axis of single *wave velocity.* For this direction of wave normal there is only one wave, for any other there are two. In the figure, if M N touch both the circle R′ P and the ellipse R P′ in M and N respectively, O M is an optic axis, and if O M′ be drawn on the other side of O C, so that the angle M′ O C may be equal to C O M, O M′ is an optic axis also. The crystal has two optic axes and is said to be biaxal. We must distinguish carefully between the optic axes and the principal axes of the crystal. Of these latter there are three defined by the property that a displacement parallel to either of them produces a force in that same direction. The forces produced by unit displacement in directions O A, O B, O C are a^2, b^2, c^2 respectively; a, b, c are also the velocities with which waves of displacements in these three directions respectively traverse the medium. In the figure a, b, c are the radii of the three circles in planes perpendicular to O A, O B, and O C.

The foregoing is an outline of Fresnel's theory of double refraction. As we shall see afterwards, the form of the wave surface to which it leads represents with a remarkable degree of exactness the direct consequences of experiment. The basis on which the theory rests is, however, dynamically unsound, for if we consider the effect of the force of restitution

normal to the wave-front produced when a particle of ether is disturbed, we are led to the conclusion that it must exercise very considerable influence on the result, and ought by no means to be neglected. There are, moreover, other objections to the theory which can hardly be fully discussed here.

We shall, however, treat it as the result of experiment that the form of the wave surface in a biaxal crystal is Fresnel's surface, and that the laws arrived at by his theory connecting the velocity with directions of displacement and wave propagation are true. The laws are simply these. Let a, b, c be the wave velocities for displacements at right angles to the crystallographic axes. Construct an ellipsoid, with its axes equal to $1/a$, $1/b$, $1/c$, in these directions. Any wave-front through the centre of this ellipsoid will cut it in an ellipse, draw the two axes of this ellipse. Then the directions of vibration in the wave are along these two axes, and the velocity with which a wave of displacement parallel to either axis travels through the medium is equal to the reciprocal of that axis.

FIG. 133.



In the figure (fig. 133) P O Q is the central section of the ellipsoid by the plane of the wave, O P, O Q are the axes of the section. O N is the normal to the wave. The ether particles in the wave can vibrate along O P or O Q, and the wave travels along O N, always remaining parallel to itself. If the displacement be along O P the velocity of propagation is $1/$O P, if the displacement be along O Q, the velocity is $1/$O Q.

In the first case, according to Fresnel, the plane Q O N, which is perpendicular to O P, is the plane of polarisation, while, when the displacement is along O Q, P O N is the plane of polarisation. The velocity for a given displacement

depends on the axis of the ellipse in the direction of the displacement. A similar construction has been arrived at by somewhat different reasoning for the form of the wave surface by Neumann and MacCullagh, but on their theory the velocity of propagation is measured by the reciprocal of that axis of the ellipsoid which is perpendicular to the displacement. For a displacement along O P the velocity is $1/OQ$, and *vice versâ*. The plane of polarisation on both theories is perpendicular to the axis of the ellipse which determines the velocity, but with them the direction of vibration lies in the plane of polarisation, while with Fresnel the two are at right angles.

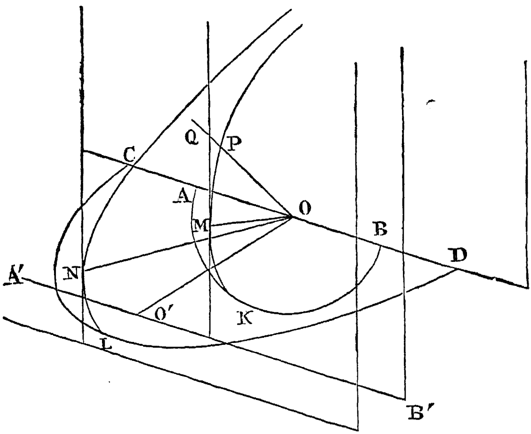
In each principal plane the section of the wave surface is an ellipse and a circle. The circle whose radius is b, the mean velocity, must cut the ellipse whose axes are a and c, the greatest and least. The two lines through the centre at right angles to the common tangent lines are the optic axes. If 2θ be the angle between them we can show that $\tan^2\theta = (a^2 - b^2)/(b^2 - c^2)$. If then we know a, b, and c, the velocities along the crystallographic axes, we can find 2θ, the angle between the optic axes. a, b and c are called the principal velocities.

Suppose now that, knowing the position of the wave-front at a given epoch, we wish to find it after a time t. We have to describe the wave surface for time t corresponding to all points on one wave-front, then as we have seen, the envelope of these wave surfaces will be the new wave-front, and this envelope will be a plane touching all the wave surfaces and parallel to the original wave-front.

Let this plane touch in M (fig. 134) a wave surface whose centre is O, and join O M. Then, as before, the part of the original wave to which the disturbance at M is due is that just round O. If we place an obstacle at O so as to block out a part of our original wave, there will be darkness at M. The shadow of the obstacle will be thrown on the new wave-front at M; O M is the direction in which the ray from O travels. In

the cases already considered when the medium is isotropic and the wave surfaces spheres O M is perpendicular to the wave-front, the directions of the ray and the wave normal coincide. For a crystalline medium this is not the case. Suppose the wave-front to be perpendicular to the paper and to cut it in A B. Let A K B, C L D be the traces of the section of the wave surface whose centre is O by the plane of the paper. Two planes can be drawn parallel to the original wave, each touching one sheet of the wave surface, and if M N be the points

FIG. 134.



of contact, O M and O N are the directions of the rays within the crystal which correspond to the given wave front A B. In general M and N will not be in the plane of the paper, and the ray will be inclined to the wave normal. The wave normal at O is at right angles to the wave-front, the ray is the line joining O to the point in which the wave surface centre O is touched by a plane parallel to A B. The effective portions of the disturbance at O travel along the rays O M, O N. The wave travels parallel to itself along the wave normal. The velocity of wave propagation is the rate at which the wave

front moves, thus if p be the perpendicular distance O O′ between the two wave-fronts A B and A′ B′, and t the time for which the wave surface has been drawn, the ratio p/t measures the wave velocity. It is this wave velocity which is given by Fresnel's construction.

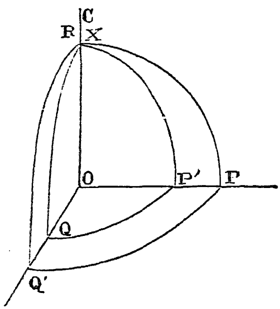
The ray velocity in a crystal is different; it is the rate at which the disturbance travels along the ray O M and will be measured by the ratio O M$/t$. If we draw a line O P Q cutting the surface in P and Q, and draw tangent planes at P and Q, O P is the ray corresponding to the plane touching the surface at P, while O Q is the ray for a plane touching at Q. There are two ray velocities in the direction O P Q, one for each of these waves. If, however, O P Q is in the direction O X (fig. 132), so that P and Q coincide at X, the two ray velocities become the same; O X is the axis of single *ray-velocity*; O X′ is also an axis of single ray-velocity. We must distinguish between these axes and the optic axes, or axes of single *wave-velocity*. The axes of single ray-velocity join the centre of the surface to the points of intersection of the two sheets; the optic axes are perpendicular to the common tangent lines of the elliptic and circular sections in the plane A O C. The wave-velocity is the same along the optic axes; the ray velocity is the same along the axes of single ray velocity for all waves. In fig. 132, O M is an optic axis, O X an axis of single ray-velocity.

The above account of Fresnel's theory is general; let us turn now to the case of crystals, in which one ray is refracted according to the ordinary law. Of these, Iceland spar may be taken as the type. In general, we have three principal axes in a crystal and three principal wave velocities, a, b, c. Let us consider the modification which takes place when two of these wave-velocities, a and b suppose, became equal. The wave-surface changes its shape.

R and R′ (fig. 135) coincide, and O P becomes equal to O Q′, so that the ellipses P Q′ P R′ become circles of radius a or b; and the two ellipses R P′ and R′ Q become equal;

and X the intersection of the ellipse R P′ and the circle R′ P coincides with R and R′; the two optic axes have also closed up together and coincide with O C. The crystal is no longer biaxal, but uniaxal, and O C is its optic axis. One sheet of the wave surface has its three principal axes, O X, O P and O Q′ (fig. 135), each equal to a, and can be proved to be a sphere. The other sheet has for its principal sections a circle P′ Q, and two equal ellipses X Q and X P′.

FIG. 135.



We may show that it is a spheroid, and that O X is its axis. In a uniaxal crystal, then, the form of the wave surface is a sphere and a spheroid. The optic axis is the axis of revolution of the spheroid, and the two sheets touch at the point where they are cut by the optic axis. The sections of the wave surface by a central plane at right angles to the optic axis are two circles of radii a and c. Any two lines in this section at right angles to each other may be treated as axes of the crystal, and any section through the optic axis will be a principal section. In the figure, a is supposed to be greater than c, and the spheroid is prolate, that is to say, it is generated by the revolution of an ellipse about its major axis.

In Iceland spar the radius a of the spherical sheet is less than c, and the spheroid is oblate, being generated by the revolution of an ellipse axes, a and c, about its minor axis a. The spheroidal sheet lies outside the spherical, while the optic axis coincides in position with the line already defined as the axis of the crystal.

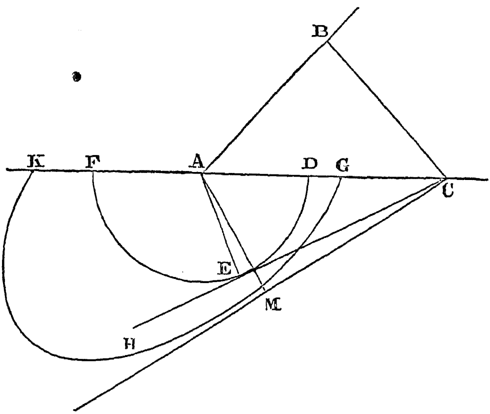
Any central section of the wave surface will be a circle of radius a, and an ellipse lying outside the circle. If the section be a principal one, that is, if it contain the optic

axis, the circle and the ellipse touch. If it be at right angles to the optic axis, the ellipse becomes a circle of radius c.

Let us consider a wave refracted obliquely at the surface of a piece of uniaxal crystal. Let the plane of the paper be the plane of incidence; A C (fig. 136) the trace of the face of incidence; A B that of the incident wave, which is at right angles to the plane of the paper.

Let D E F be the trace on the plane of the paper of the spherical sheet of the wave surface, centre A, G H K that of

FIG. 136.



the spheroidal, t being the time for which this surface is drawn. Let B C be perpendicular to the incident wave, and suppose t is also the time taken by the displacement at B to reach C.

The refracted waves touch the two sheets, D E F, G H K, are perpendicular to the paper, and pass through C. The point of contact of one refracted wave with the spherical sheet, whose section is D E F, will be the point in which a tangent from C to the circle D E F meets the circle, let it be E; C E is the trace of one refracted wave. The point of

contact of the second refracted wave and the spheroidal sheet, section G H K, will in general not be in the plane of the paper, and if C M be the trace on that plane of the second refracted wave, C M does not touch the ellipse G H K, but a plane through C M, perpendicular to the paper does touch the spheroid, of which G H K is a section.

The line A E is the direction of one of the refracted rays; the other is the line joining A to the point of contact of the plane through C M and the spheroidal sheet. Draw A M perpendicular to C M. Then A E and A M are perpendicular to the refracted waves. They are the refracted wave normals.

For the one wave C E, the directions of the ray and of the wave normal coincide. For the other they are different. We shall show that C E is refracted according to the ordinary law of sines, it is the ordinary wave; C M is the extraordinary wave. For let ϕ be the angle between the incident wave A B and the face of incidence ϕ', ϕ'' the angles between the ordinary and extraordinary refracted waves and the same face.

Then $BCA = \phi$, $ECA = \phi'$, $MCA = \phi''$.

Let v be the velocity of light in air, v', v'' the wave velocities in the ordinary and extraordinary waves.

Then we have $BC = vt$, $AE = v't$, $AM = v''t$;

Also $\sin\phi : \sin\phi' = CB : AE = v : v'$;

and $\sin\phi : \sin\phi'' = CB : AM = v : v''$,

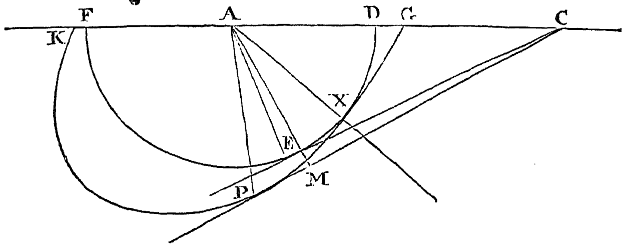
but A E is a radius of the spherical sheet of the wave surface for time t, and is, therefore, the same for all positions of the ordinary refracted wave; and this radius is equal to $a\,t$. Thus $v' = a$, and is therefore constant. Thus for the ordinary wave, $\sin\phi/\sin\phi' = v/a$, a constant; or the ordinary wave obeys the law of sines.

For the extraordinary wave, we have $\sin\phi/\sin\phi'' = v/v''$, and v'' depends on the direction of the extraordinary wave normal. The extraordinary wave touches the spheroidal sheet of the surface, and if p be the length of the perpen-

dicular from A on this tangent plane to the spheroid, then $v'' = p/t$. Analytical geometry enables us to determine the value of p in terms of the position of the wave-front with reference to the optic axes and the principal velocities. Thus for this wave the ratio $\sin\phi/\sin\phi''$ is not constant.

One or two special cases of refraction at the surface of a uniaxal crystal require separate notice. Suppose the axis of the crystal lies in the plane of incidence so that this plane is a principal plane of the crystal. Let A X (fig. 137) be the direction of the axis; then the spheroidal and spherical sheets of the wave surface touch at X, and the section of the spheroidal sheet, being a principal section, is an ellipse with axes, a and c, along and perpendicular to A X.

FIG. 137.



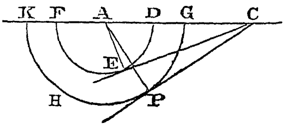
The point of contact of the extraordinary wave C M with the spheroidal sheet lies in the plane of the paper; let it be P, then A P is the extraordinary refracted ray.

For one position of the incident wave, P and E coincide with X; the tangent planes to the two sheets are the same, and touch them at X; there is one refracted wave and one refracted ray coinciding with the axis A X. If light be incident on a piece of the crystal so that the refracted ray coincides with the optic axis, the crystal is for that angle of incidence singly refracting. We have said already that in Iceland spar the optic axis is equally inclined to those three edges which meet in three obtuse angles. If, then, we polish a face on a crystal of the spar so that light can traverse

the crystal in that direction, we get only one refracted ray. If, for example, we cut off the obtuse angles by planes at right angles to the optic axis, and, polishing these two artificial parallel faces, look through them normally at a dot or pinhole, we only see one image.

We may consider one other special case of refraction. Let the optic axis of the crystal be perpendicular to the plane of incidence. The section of the wave surface by the principal plane will then be two circles of radii, $a\,t$ and $c\,t$. The point of contact of the extraordinary wave with the spheroidal sheet will lie on the circle G H K (fig. 138); let it be P. Then since G H K is a circle of radius c and centre A, and C P touches it, C P A is a right angle. A P, the extraordinary ray, coincides with the extraordinary wave normal. Also $\mathrm{AP} = c\,t$. Thus $\sin\phi/\sin\phi'' = \mathrm{BC}/\mathrm{AP} = \mathrm{V}/c$, which is constant for all angles of incidence in this plane. Thus if the plane of incidence be at right angles to the optic axis, the extraordinary wave also obeys the law of sines. c is the velocity of the extraordinary wave when it is travelling at right angles to the optic axis; V/c is known as the extraordinary refractive index for light of the wave-length considered; V/a is the ordinary refractive index. The fact that the extraordinary wave surface in Iceland spar is a spheroid was stated and experimentally proved by Huyghens in his 'Traité de Lumière,' published in 1690, 130 years before the time of Fresnel. And we make this the basis of our explanations of the phenomena of double refraction in Iceland spar quite independently of Fresnel's theory.

FIG. 138.



Let us see, however, what the theory has to tell us as to the directions of the vibrations in the ordinary and extraordinary waves.

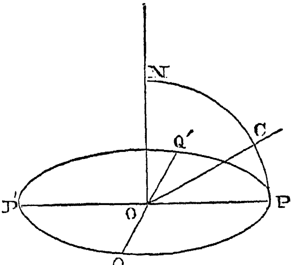
These were found by constructing an ellipsoid with axes $1/a$, $1/b$, and $1/c$, and considering the central section

of this ellipsoid by the wave-front. For a uniaxal crystal $a = b$, so that our ellipsoid becomes a spheroid with semi-axes $1/a$ and $1/c$.

Moreover, the axis $1/c$ coincides with the optic axis of the crystal, which is, therefore, the axis of revolution of the spheroid. The ellipsoid of elasticity, as it is called, is, in this case then, a spheroid of revolution about the optic axis of the crystal.

Let P O Q (fig. 139) be the central section of this spheroid by the wave-front, and let a plane through the optic axis perpendicular to the wave cut it in O P. Draw O Q also in the wave-front perpendicular to O P. O P and O Q are clearly the axes of the elliptic section of the spheroid made by the wave-front. They are, therefore, the directions of vibration; and the velocities of wave propagation for these directions respectively are $1/\text{OP}$ and $1/\text{OQ}$. But the planes C O P and P O Q are at right angles, and O Q is at right angles to O P; therefore O Q is at right angles to O C. Now the section of our spheroid by a central plane at right angles to O C is a circle of radius $1/a$. Hence O Q is equal to $1/a$, and the velocity of wave propagation for vibrations parallel to O Q is a. Again, C O P is the principal plane of the wave-front. Hence a disturbance in which the vibrations are at right angles to the principal plane passes through with the velocity a, which is the same for all positions of the wave-front. Light polarised in the principal plane, therefore, travels at the same rate in any direction in the crystal.

FIG. 139.



The ordinary wave, therefore, consists of light polarised in the principal plane, the vibrations in which are at right angles to the optic axis, and travels at the same rate in all

directions. This is what we should naturally expect if we assume with Fresnel that the velocity depends on the elasticity of the ether in the direction of vibration, for from symmetry it follows that the ether is equally elastic in all directions perpendicular to the optic axis, and therefore that the velocity of propagation for all waves in which the vibrations are at right angles to the axis will be the same.

Turning now to the extraordinary ray, in which the vibration is along O P, that is, in the principal plane, the wave normal O N will lie in the plane P O C, and this plane will cut the spheroid of elasticity in an ellipse with axes $1/c$ and $1/a$ along, and perpendicular to, the optic axis; O N will be one radius vector of this ellipse, and O P another at right angles to it. To find, then, the velocity of the extraordinary wave, draw in the principal plane an ellipse axes $1/c$ and $1/a$ along, and perpendicular to, the optic axis respectively, and take the radius vector of this ellipse which is at right angles to the wave normal. The reciprocal of this radius vector is the velocity required.

The optical effects then produced by a crystal on light can be calculated if we know the position of the wave-front with reference to the principal axes and the values of the principal velocities. The principal axes, we assume, coincide with the crystallographic axes of our medium, and are known from its form. If we have an artificial face cut on a crystal, we can determine by means of the goniometer the position of this face with reference to two of the faces of the crystal, and hence can deduce, by the aid of some spherical triangles, its position with reference to the axes. To find the values of one of the principal velocities, a suppose, in the case of a biaxal crystal we cut a prism from the crystal, with its edge parallel to the axis O A, and pass a ray of homogeneous light through the prism in a plane at right angles to the edge.

The section of the wave surface by this plane is a principal section, and consists of an ellipse axes b and c along

the principal axes, and a circle of radius a. One ray, then, is ordinarily refracted obeying the law of sines, and the refractive index for this ray may be found as for an isotropic medium by turning the prism round an axis parallel to its edge until the deviation is a minimum, and observing it; then if D be this minimum deviation, i the angle of the prism, and V the velocity of light in air, we have seen that the refractive index is $\sin \frac{D+i}{2} \big/ \sin \frac{i}{2}$. But the refractive index is V/a. Therefore $a = V. \sin \frac{i}{2} \big/ \sin \frac{D+i}{2}$.

To make sure which ray is ordinarily refracted, observe the minimum deviation D_1 and D_2 for each wave separately, and at the same time measure the angles of incidence ϕ_1 and ϕ_2. Then for a wave obeying the ordinary law we have $2\phi = D + i$, ϕ being the angle of incidence, D the corresponding minimum deviation. If, then, $2\phi_1 = D_1 + i$, we know that the wave to which D_1 and ϕ_1 belong is the ordinary wave. If this equation does not hold, we shall have $2\phi_2 = D_2 + i$, and ϕ_2 and D_2 belong to the ordinary wave. One of these equations must hold; it is just possible that they may both be found true; this would be the case if the axes of the crystal in the principal plane of the prism were parallel to the bisectors of the angle of the prism, and then the expression $V \sin \frac{i}{2} \big/ \sin \frac{D_2+i}{2}$ will give the value of either b or c. To determine these directly it is simplest to obtain when possible two more prisms of the crystal with their edges parallel to the other two crystallographic axes, and proceed in the same manner.

For a uniaxal crystal, one prism with its edge parallel to the optic axis is sufficient to give us both a and c; for in that case, since the axis of the prism is at right angles to the plane of incidence both waves will be refracted according to the ordinary law, and if D_1, D_2 are the minimum deviations observed, we shall have

$$a = \text{v} \sin \frac{i}{2} \Big/ \sin \frac{\text{D}_1 + i}{2}$$

$$c = \text{v} \sin \frac{i}{2} \Big/ \sin \frac{\text{D}_2 + i}{2}$$

The values of a, b, and c depend, of course, on the nature of the light used, being different for different colours. They are found also to differ slightly in different specimens of the same substance.

The ratios v/a, v/b, and v/c, are the principal refractive indices of the medium.

The following tables give their values for aragonite and topaz as determined by Rudberg for the principal Fraunhofer lines.

Aragonite.

Rays	$\frac{\text{v}}{a}$	$\frac{\text{v}}{b}$	$\frac{\text{v}}{c}$
B	1·52749	1·67631	1·68061
C	1·52820	1·67779	1·68203
D	1·53013	1·68157	1·68589
E	1·53264	1·68634	1·69084
F	1·53479	1·69053	1·69515
G	1·53882	1·69836	1·70318
H	1·54226	1·70509	1·71011

Topaz.

B	1·60840	1·61049	1·61791
C	1·60935	1·61144	1·61880
D	1·61161	1·61375	1·62109
E	1·61452	1·61668	1·62408
F	1·61701	1·61914	1·62652
G	1·62154	1·62365	1·63123
H	1·62539	1·62745	1·63506

Rudberg and Mascart have observed the values of v/a and v/c for Iceland spar and quartz, two uniaxal crystals; v/a is the ordinary refractive index, v/c is the extraordinary. The table gives Rudberg's values.

Table.

	Spar		Quartz	
Rays	V/*a*	V/*c*	V/*a*	V/*c*
B	1·65308	1·48391	1·54090	1·54990
C	1·65452	1·48455	1·54181	1·55085
D	1·65850	1·48635	1·54418	1·55328
E	1·66360	1·48868	1·54711	1·55631
F	1·66802	1·49075	1·54965	1·55894
G	1·67617	1·49453	1·55425	1·56365
H	1·68330	1·49780	1·55817	1·56772

On looking at this table we observe that the difference between the values of the two indices for any given ray is greater in the spar than in quartz. The separation, therefore, of the emergent rays is greater in the spar.

Moreover, the refractive index of spar for the ordinary ray is greater than that for the extraordinary. Thus in spar the velocity of the ordinary wave is less than that of the extraordinary. The spherical sheet of the wave surface lies inside the spheroidal. Iceland spar is in consequence called a negative crystal. In quartz, on the contrary, the spherical sheet is the exterior, the ordinary wave travels more quickly than the extraordinary; and quartz is called a positive crystal. Among other positive uniaxal crystals may be named zircon, titanite, boracite, apophyllite, and ice, while the list of negative uniaxal crystals contains besides Iceland spar, tourmaline, rubellite, corundum, sapphire, ruby, emerald, beryl, apatite, and others.

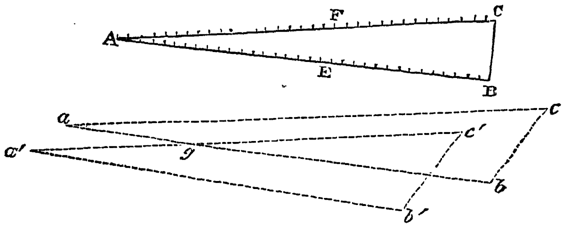
In most biaxal crystals it is found that two of the principal indices are nearly equal, and differ, it may be considerably, from the third. One sheet of the wave surface, therefore, is much more nearly spherical than the other. If this sheet is the outer, the crystal is positive; if it be the inner, the crystal is negative.

Thus in aragonite v/*b* and v/*c* are nearly equal to one

another and considerably greater than v/a, so that b and c are less than a, and it is the inner sheet of the wave surface which is most nearly spherical; aragonite, therefore, is a negative crystal. In topaz, the difference between a and b is less than that between b and c, and a and b are greater than c. The outer sheet, then, is most nearly spherical and topaz is a positive crystal.

Other positive biaxal crystals are borax, the sulphates of baryta, nickel, zinc, ammonia, strontium, potassium, and calcium, also cyanite and citric acid, while carbonates of lead, lime (in the form of aragonite), baryta, strontium, sodium, and ammonia are negative, together with mica, talc, sugar, felspar, and nitre. Crystallised carbonate of lime

FIG. 140.



takes the form of Iceland spar (calcite) or aragonite, according to its temperature at the time of crystallisation and other circumstances.

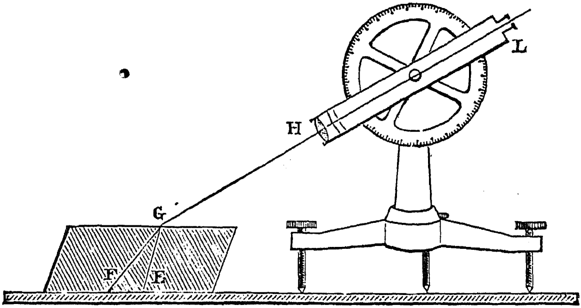
It remains now to consider the experimental facts on which the forms of the wave surface in crystals really rest. Huyghens, who first stated the law of extraordinary refraction, verified his construction only in some special cases. In 1802, Wollaston made a series of observations on the angle at which the extraordinary ray was totally reflected at the inner surface of a piece of Iceland spar, and arrived at conclusions in accordance with theory. Further measurements were made by Malus and Biot. In Malus' experiments two scales, AB, AC (fig. 140), were engraved on a plate of

polished steel, inclined to each other at a small angle and divided into small equal parts. A plate of the crystal, with parallel faces, was laid on the scales, and they were viewed through a telescope mounted on a graduated vertical circle.

Let us suppose that the plate is horizontal.

Two images of each of the scales will be seen—let them be $a\,b$, $a\,c$, $a'\,b'$, $a'\,c'$; generally $a\,b$ will cut $a'\,c''$ as at g; suppose that the axis of the telescope H L (fig. 141) is directed to view this point, and let it cut the surface of the crystal in G. We can determine the position of G with reference to the

FIG. 141.



scales. Let E, F (fig. 141) be the points on the scales which appear to coincide at G.

The divisions of the scales give us at once the lengths $a\,g$ and $a'\,g$ (fig. 141), that is, they give us the positions of the two points, E and F, on our scales.

Now a ray from E is refracted into the crystal along E G, and refracted out along G H, while one from F being refracted in along F G emerges also along G H.

If we know the thickness of the crystal our observations of the positions of G, E, and F are sufficient to give us the two angles of refraction, while the angle of incidence is the angle which the axis of the telescope makes with the vertical, and is given by the graduated circle; we have, there-

fore, sufficient data to compare theory and experiment. Biot's method was a modification of this, using a prism instead of the plate.

Perhaps the simplest method of proving that one ray in Iceland spar obeys the ordinary law is that adopted by Brewster. He took two prisms of spar having the same refracting angle with their faces differently inclined to the axis, and placed one vertically above the other, so that their edges were in the same straight line, and their faces continuous. They were cemented together in this position, and their faces repolished so as to ensure exact equality in the refracting angle.

If we place such a combination on the table of a spectroscope, and look at a source of homogeneous light, the ordinary images of the slit after refraction through the two portions of the prism are seen, one vertically below the other, in the same straight line. The extraordinary images, however, are separated by a considerable interval; the image formed by the upper prism not being vertically above that formed by the lower. The ordinary rays are refracted in exactly the same manner by the two portions, and therefore the refractive index does not depend on the direction in which the light traverses the crystal.

Until quite recently Fresnel alone had attempted to verify his theory for biaxal crystals. His experiments consisted in a modification of the fundamental interference experiment described in Chapter V.

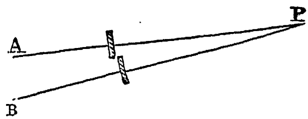
If A and B (fig. 142) be two exactly similar and equal small sources of light close together, we have seen that interference fringes are formed on a screen placed so as to receive the light from these. If P be a point on the screen, P will be dark if the time taken by the light in travelling from A to P differs from that occupied in travelling from B to P by an odd multiple of half a period.

Suppose we place in the path of the pencil A P a thin plate of glass or other transparent substance, then since

light travels at a different rate in glass and air, this produces a shifting of the bands of interference, which depends on the difference in the times taken by the light to traverse the glass and an equal thickness of air. By measuring the distance through which the bands have been shifted and the thickness of the glass, we can calculate the velocity of the light in the glass.

If, now, we take two pieces of glass of exactly the same thickness, and place one in the path of each pencil, since the times taken by light to traverse the pieces are the same for both, no shifting of the bands is observed. But, if instead of glass, we take two pieces of crystal, cut in different directions from the same specimen, and worked so as to be of exactly the same thickness, and place one in the path of each pencil as before, the interference bands are immediately displaced. Light travels at different rates through the two plates, and therefore the difference in phase between the two interfering rays at P is no longer the same as it was before the insertion of the plates. The amount through which the bands have been moved depends on the difference of velocity in the two plates, and that difference can be calculated from the displacement of the bands and the thickness of the plates.

FIG. 142.



But if we know the principal velocities for the crystal employed, and also the position of the faces of the plates with reference to the axes, we can calculate the velocity in each plate, and find the amount of displacement which would be produced if Fresnel's theory be true. If this agrees with the amount actually observed, we infer the truth of our theory within the limits of the errors of our observations. In Fresnel's experiments the agreement was fairly close, but his measurements were not exact enough to

decide between his theory and several others, which have been proposed.

In 1862, Professor Stokes suggested a method for determining the velocity of wave propagation in different directions in a crystal. We will give his account of it taken from the Report of the British Association.

Let the crystal to be examined be cut, unless natural faces or cleavage planes answer the purpose, so as to have two planes inclined at an angle suitable for the measure of refractions, there being at least two natural faces or cleavage planes left undestroyed, so as to permit of the exact measure of the directions of any artificial faces. The prism thus formed having been mounted as usual (on the table of a spectrometer), and placed in any azimuth, let the angle of incidence be measured by observing the light reflected from the surface, and likewise the deviation for several standard fixed lines in the spectrum. Each observation furnishes us with an angle of incidence, and the corresponding angle of emergence, the angle of the prism being known. For if ϕ be the angle of incidence, D the deviation, ψ the angle of emergence, and i the angle of the prism,

$$\mathrm{D} = \phi + \psi - i,$$
$$\therefore \psi = \mathrm{D} + i - \phi.$$

Now if ϕ', ψ' be the angles which the refracted wave normal makes with the normals to the faces of incidence and emergence, V the velocity of light in air, v the wave velocity in the crystal, we have seen that it follows from Huyghen's principle of the superposition of small motions, that

$$\mathrm{V}/v = \sin\phi/\sin\phi'$$
$$= \sin\psi/\sin\psi' \text{ in a similar manner.}$$

Hence $\quad v \sin\phi = \mathrm{V}\sin\phi' \quad . \quad . \quad . \quad . \quad (1),$

$\quad v \sin\psi = \mathrm{V}\sin\psi' \quad . \quad . \quad . \quad . \quad (2),$

also $\quad \phi' + \psi' = i \quad . \quad . \quad . \quad . \quad . \quad (3).$

Adding and subtracting (1) and (2) we get, remembering (3),

$$v \sin\frac{\phi + \psi}{2} \cos\frac{\phi - \psi}{2} = \mathrm{v} \sin\frac{i}{2} \cos\frac{\phi' - \psi'}{2}$$

$$v \cos\frac{\phi + \psi}{2} \sin\frac{\phi - \psi}{2} = \mathrm{v} \cos\frac{i}{2} \sin\frac{\phi' - \psi'}{2}$$

By division

$$\tan\frac{\phi + \psi}{2} \cot\frac{\phi - \psi}{2} = \tan\frac{i}{2} \cot\frac{\phi' - \psi'}{2}$$

or
$$\tan\frac{\phi' - \psi'}{2} = \tan\frac{\phi - \psi}{2} \tan\frac{i}{2} \cot\frac{\phi + \psi}{2}$$

From this equation we can find $\frac{\phi' - \psi'}{2}$, for i, ϕ and ψ are known, but (3) gives us $\phi' + \psi'$, and hence we get ϕ' and ψ'.

Then we can find the ratio v/v from the equation

$$\mathrm{v}/v = \sin\phi/\sin\phi';$$

v is the velocity of wave propagation, and v/v we may call the refractive index of the medium in the direction of the wave normal considered.

Hence we can find the velocity of propagation of a wave, the normal to which lies in a plane perpendicular to the faces of the prism, and makes a known angle with these faces, and therefore with the axes of the crystal. But Fresnel's construction enables us, if we know the principal velocities, to calculate the wave velocity in any other direction, and hence to compare theory and experiment.

Professor Stokes has himself measured by this method the refractive indices in different directions in Iceland spar, and compared the results with Huyghen's construction.

The greatest differences between two corresponding values, one given by theory, the other by experiment, only amount to ·0001.

The indices observed lie between 1·65850 and 1·48635, the values of the ordinary and extraordinary refractive indices for the line D, so that Professor Stokes' results indicate that the differences between Huyghen's theory and experiment in the case of Iceland spar do not amount to more than 1 in 15,000.[1]

Within the last few years, I have made two series of experiments by this method, the one on aragonite, the other on Iceland spar. The results are contained in two papers printed in the 'Philosophical Transactions' of the Royal Society for 1879 and 1880.

The first paper relates to aragonite. Two specimens were used: one was cut so as to allow me to determine the wave velocities in the neighbourhood of the optic axes, and also in a plane at right angles to that containing the optic axes, while with the other the observations extended over almost a quadrant of a section of the wave surface, made by a plane inclined at 60° on to that containing the optic axes, and passing very nearly through one of the principal axes of the crystal.

In the paper the values of the refractive indices in different directions, as given by theory and experiment, are tabulated. The agreement between the two is striking, but there are differences which appear to follow some regular law, and which are greater than can easily be accounted for by errors of experiment. Thus with the first prism, the greatest difference between theory and experiment is ·0005, or 5 in about 15,000, while experiments made at an interval of several months agreed to about ·00005, or 5 in 150,000. With the second piece of the crystal the differences were greater, amounting to ·0009, or 9 in 15,000. These differences, of course, are exceedingly small, and we may say

[1] These results have been more recently confirmed by the observations of Professor Hastings.

with certainty that Fresnel's wave surface is an exceedingly close approximation to the truth, while yet it seems probable that it is only an approximation.

The experiments on Iceland spar were made with the view of seeing, if these same small differences could be observed, how far they depended on the refrangibility of the light used.

The observations, therefore, were made for the three bright lines of hydrogen, C, F, and *g*, as well as for the sodium flame, which alone was used in the case of the aragonite. Four prisms were cut from the same block of spar, and the experiments extended over a complete quadrant of a section of the wave surface, made by a plane passing through the optic axis.

The differences observed were exceedingly small; in only eight out of about sixty measurements recorded for the line C, were they as great as ·0001, or about 1 in 15,000, while the mean difference, irrespective of sign, was ·000055, or about 5 in 150,000. For the lines F and *g* the differences were perhaps a little greater, but it is shown that a small alteration in the focusing of the observing telescope would account for this.

Huyghen's construction, therefore, for the extraordinary wave surface, represents its true form to within about 1 in 30,000.

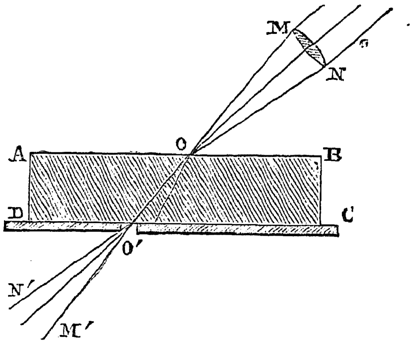
The experimental discovery of the phenomena of external and internal conical refraction by Lloyd, after they had been predicted by Hamilton as a consequence of Fresnel's theory, has always been looked upon as a striking verification of that theory. Professor Stokes, however, has shown that almost any form of wave surface that could arise from a theory of undulations traversing a medium, would have characteristics resembling closely those of Fresnel's surface near the optic axes and would give rise to conical refraction.

We will explain what these phenomena are, and how they are to be accounted for.

Let us suppose a ray of light is incident on a plate of a

biaxal crystal. In general there will be two wave-fronts in the crystal, which will each touch one sheet of the wave surface; for one particular angle of incidence, however, these two fronts coincide, and we have a single wave which touches both sheets of the wave surface, and in this case the wave normal coincides with an optic axis. In this position it is found that, instead of two refracted rays, we have an infinite number within the crystal, forming a cone with the point of incidence for its vertex, and on an elliptic base in the plane of emergence; and on emergence these rays form a hollow cylinder of light. If we look through the crystal in

FIG. 143.



the proper direction at a bright point, instead of seeing two images, we have a bright ring elliptical in shape. This is interior conical refraction.

Again, it is found that a ray travelling along the axis of single ray velocity in the crystal may be refracted on emergence in any of an infinite number of directions, which form a cone with its vertex at the point of emergence; so that if the incident pencil be a cone of the proper angle, with its vertex at the point of incidence, and with its axis in the proper direction, one set of refracted rays in the crystal will travel along this line, and, on emergence, will form a

divergent conical pencil. This is called exterior conical refraction.

The axes of single ray velocity in aragonite are inclined to each other at about 18° 10′.

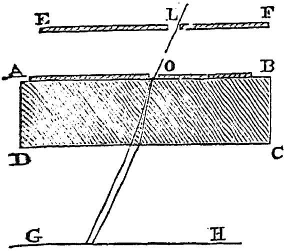
Dr. Lloyd took a plate, A B C D (fig. 143), of the crystal, with its faces at right angles to the line bisecting this angle, and made the rays of the sun converge to a point O on the surface A B by means of a lens placed at the distance of its own focal length from O. A metallic plate perforated with a small hole O′ was placed on the second surface. For one position of this hole, he found that the emergent pencil consisted of a hollow cone of rays with O′ as vertex, with a vertical angle of about 3°. When this conical pencil was formed, he found that O O′ was parallel to the axis of single ray velocity in the crystal.

Let M′ O′ N′ in the figure represent this emergent conical pencil. Corresponding to our emergent cone M′ O′ N′, we shall have a cone M O N in the incident pencil, with its vertex at O, of the same angle, and with its axis in the same direction as that of M′ O′ N′. Any ray M O of this cone falling on the crystal at O is doubly refracted; one of the refracted rays travels along O O′, the other in some different direction. The first of these emerges at O′ and is refracted along O′ M′ parallel to M O, the second is stopped by the plate; while O O′ is the common direction of refraction for one of the two rays into which each ray in the incident cone, M O N, is divided by the crystal. All these rays, and these only, can emerge at O′, and are refracted parallel to their original directions, forming the conical pencil M′ O′ N′.

In investigating interior conical refraction the same plate of crystal was used. The upper surface was covered with a thin metallic plate pierced with a small hole O (fig. 144). At a small distance in front of the crystal was another thin metallic screen, E F, with a second small hole L in it, and the light of a lamp or of the sun passing through the two holes, L and O, fell on the crystal in a definite direction at O.

By moving either plate parallel to itself the angle of incidence could be varied and, in general, two spots of light were seen on a screen G H, placed to receive the emergent rays. For one particular angle of incidence a brilliant annulus of light appeared on the screen. In this position there was only one refracted wave in the crystal, and the wave normal was parallel to the optic axis. When the aperture O in the second plate was ever so slightly shifted, the phenomenon rapidly changed, resolving itself into two separate pencils.

FIG. 144.

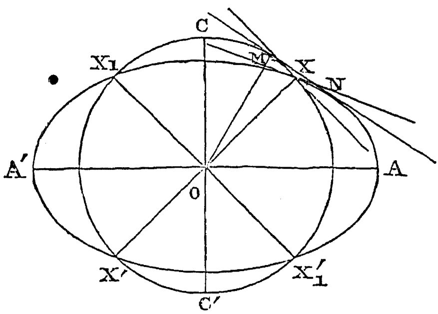


The angle of incidence at which this ring of light was formed was 15° 40′, and the vertical angle of the cone of rays formed in the crystal was found to be 1° 50′. According to Fresnel's theory, these angles should have been 15° 19′ and 1° 55′.

Let us consider how Fresnel's theory accounts for these phenomena. The sections of Fresnel's wave surface by the three principal planes are ellipses and circles. Let us suppose, as before, that the three principal velocities, a, b, c, are in order of magnitude, and consider the section perpendicular to the axis of b—the mean axis of elasticity, as it is called. In this plane the radius of the circle is intermediate between the axes of the elliptic section, and the two cut in four points, x, x', x_1, x_1' (fig. 145), at the extremities of the axes of single ray velocity. Any tangent plane to the wave surface is a possible position for a wave-front in the crystal, and the radius vector joining the centre of the surface to the point of contact of the tangent plane is the direction of the ray corresponding to the wave-front At x two tangent lines to the surface can be drawn in the plane of the paper, and through these lines two planes can be drawn touching the

surface. The same is the case for any section we take passing through the axis of single ray velocity, so that at the extremity of such an axis there are an infinite number of wave-fronts touching the wave surface, and O X is the direction of the ray within the crystal for each of these fronts. The shape of the wave surface near the extremity of one of these axes seen from outside is like the vertex of a hollow cone looked at from within. There is there a kind of small pit or crater—a conical point, as it is called. Each of the waves which touch the surface at X is differently refracted

FIG. 145.



on emergence from the crystal, and the emergent rays form a cone.

If, then, we reverse the direction of each of the rays in this conical pencil, the refracted wave-fronts will all touch the wave surface at X, and the refracted ray will in each case coincide with the axis of single ray velocity. On emergence from the other face of the crystal they will again form a cone of the same vertical angle, and with its axis in the same direction as that of the incident cone; we have external conical refraction. An incident hollow cone of rays gives rise to a single ray in the crystal and to a second hollow cone on emergence. In Dr. Lloyd's experiment a solid cone of rays fell on the crystal. Among all these, the

rays which lay on a certain conical surface, M O N (fig. 143), were incident at such an angle that one of the two refracted waves produced in the crystal touched the wave surface at the extremity of an axis of single ray velocity. This axis was the direction of one of the two refracted rays for each ray in the hollow cone, M O N. The rays which traversed the crystal in this direction emerged through the aperture O′ in the second plate; all others were stopped, and thus the emergent pencil was the hollow cone M′ O′ N′ with O′ as its vertex. Each emergent ray is polarised, but the planes of polarisation of different rays are distinct.

For the phenomena of internal conical refraction we turn again to the section of the wave surface by a plane perpendicular to the mean axis of elasticity. The circle and ellipse in this plane of section have in each quadrant a common tangent line (fig. 145). Let M N be this tangent line, M and N being the points of contact with the circle and ellipse respectively, and the plane of the paper the plane of section. A plane through M N at right angles to the paper touches both sheets of the wave surface at M and N. Analysis shows us that M and N are not the only points of contact, but that the plane and surface touch along a curve and that this curve is a circle. Not only is there, as it were, a pit or crater in the surface, but the edge of the pit is such that we can lay a plane flat on it, and cover it completely in. If we look on the wave surface as a sort of spherical elastic ball filled with air, and suppose that one point on the ball is pressed inwards, forming a kind of dimple, and held down from the underside, we can clearly lay a plane down on the surface touching the edges of this dimple all round. The lowest point of the dimple corresponds to the extremity of the axis of single ray velocity; the plane which touches it is like the plane through M N in the figure. This plane is a position of the wave-front in the crystal, and O M is the wave normal; for M N touches in M a circle whose centre is O, and therefore O M is at right angles to M N.

O M is an optic axis of the crystal. But this wave-front touches the surface along a circle, that is to say, there are an infinite number of points of contact, and the line joining O to each of these points is a ray. Thus there are an infinite number of ray directions corresponding to this wave-front, and if an incident wave be such that the refracted wave in the crystal is parallel to M N, the one incident ray will give rise to a cone of rays in the crystal.

Each of these rays will produce a refracted ray parallel to the incident one, and the refracted pencil will be a hollow cylinder. Thus, internal conical refraction occurs when the incident wave is such that there is but one refracted wave in the crystal. To this refracted wave there corresponds a cone of rays, which becomes a cylinder on emergence.

The line O M, which is at right angles to this single wave-front, is the optic axis of the crystal. Internal conical refraction, then, is related to the optic axes, or axes of single *wave* velocity, which are sometimes called the axes of internal conical refraction. External conical refraction depends on the axes of single *ray* velocity, and these are often known as the axes of external conical refraction.

It follows, also, as we have said from Fresnel's theory, that the direction of vibration in any given wave-front is the projection on the wave-front of the ray corresponding to it.

Thus each ray, both in internal and external conical refraction, is plane polarised, though the planes of polarisation are different for the different rays.

Taking the case of internal conical refraction, let M P N P′ (fig. 146) be the circle as seen in projection, in which the wave in the crystal touches the wave surface, O M the optic axis; O M is normal to the wave. If we consider the ray O P coming to a point P on the surface, M P will be the direction of vibration for that ray, and the plane of polarisation will be perpendicular to M P. The directions of vibration all pass through M, the foot of the common wave normal.

For external conical refraction the direction of all the rays

within the crystal is the same, coinciding with O X, the axis of single ray velocity. If O Y be perpendicular on any wave-front, then X Y is the projection of O X, and is therefore the direction of vibration in that wave.

If we do not know the direction of the ray corresponding to a given wave-front in a crystal we may use the

FIG. 146.

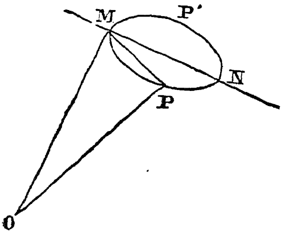
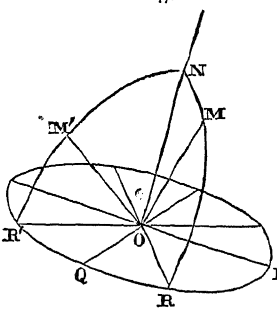


FIG 147.



following construction, which may be proved by the aid of co-ordinate geometry, to find the directions of vibration.

Let O N (fig. 147) be the wave normal, O M, O M′ the optic axes. Through O M and O N draw a plane cutting the wave in O R, and through O M′ and O N draw a second plane, cutting the wave-front in O R′. Let O Q bisect the angle R O R′, and O P be perpendicular to O Q. O P and O Q are the possible directions of vibration in this front.

We are now in a position to describe the experiment by which Fresnel showed that the vibrations which constitute light are in the wave-front.

Let us turn again to the fundamental interference experiments in which we have two identical sources of light—two parallel slits—giving bands of interference on a screen, and suppose we polarise the light from each slit and observe the effect. We can accomplish this polarisation most easily by the use of two pieces of tourmaline of the same thickness.

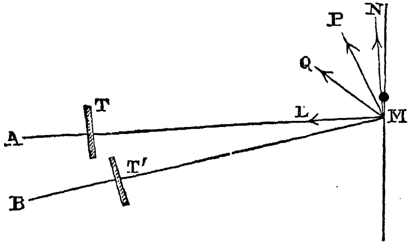
This crystal has the property of absorbing completely, even when quite small thicknesses are used, one of the two rays into which a pencil of ordinary light is divided by it. If we take a plate of tourmaline cut parallel to the axis, and allow ordinary light to fall on it, the emergent light is plane polarised at right angles to the principal plane; only the extraordinary pencil can pass through; the vibrations in the light after passing the tourmaline are all parallel to its axis. Let us take, then, two plates cut from the same tourmaline which have been carefully worked to the same thickness, and place them before our two sources of light, A and B, in such a way that the light passes normally through each plate. If the axes of the two tourmalines be parallel, the interference bands are unaffected; but, suppose we turn one of the tourmalines round the direction of the incident light, then the bands get fainter, until at last, when the axes of the two tourmalines are at right angles, no trace of interference is left. The light from A and B is polarised in planes at right angles, and we see that rays polarised in two planes at right angles cannot interfere.

It follows from this that the directions of vibration in these two rays are at right angles to each other, for if O P and O Q are two directions of vibration not at right angles, we can resolve the displacement along O Q into one component along O P and a second at right angles to O P. The component along O P will interfere with the displacement along O P, and variations in intensity will occur, depending on the difference of phase between these two. If we cannot trace any variations in intensity, we infer that O Q has no component along O P—that is, that O P and O Q are at right angles.

Suppose, now, that the two wave-fronts from A (fig. 148) and B which interfere at the point M on the screen are at right angles to the paper. Let the axis of the tourmaline at A be in the paper, then the vibration in the wave from A is also in the paper. If possible, let it not be in the wave-front at right angles to A M, but in some direction, M P, inclined to

it. The axis of the tourmaline at B is at right angles to the paper, and if we consider a plane through M B at right angles to the paper, the direction of vibration in the wave from B will be in this plane. Let it be in direction M Q which we will suppose is not in the wave front at B. Then M P and M Q are not at right angles to each other, and a displacement along M Q will have a component parallel to M P, which will produce interference effects at M; but no such effects can be observed, hence we infer that M P and

FIG. 148.



M Q are at right angles. Thus, B Q is at right angles to the paper—that is, it is in the wave-front. So, too, M P must be in the wave-front.

Thus, the vibrations which constitute polarised light are in the wave-front.

We may put the point in a somewhat different manner, as follows. Suppose we consider a wave travelling along A M, and let the displacement be in the direction M P—not at right angles to A M. We can resolve this displacement into M L along A M, and M N at right angles to it. The theory of elastic solids shows that these two vibrations will traverse the medium with very different velocities. The disturbance along M P will split up into two waves, one consisting of longitudinal vibrations in the direction A M—such as constitute sound; the other of transverse vibrations at right angles to A M. It is these last vibrations which we

consider in the theory of light. The others, if they exist at all, do not affect our eyes.

A Nicol's prism has been mentioned in Chapter X. as an instrument for determining whether light be plane polarised or not. It is also a convenient apparatus for giving us a plane polarised ray.

If we pass light from an aperture through a rhomb of Iceland spar, we get two images of the aperture and two beams of light; but, unless the rhomb is very thick and the aperture small, these two beams will overlap, and the light in the part common to the two will not be plane polarised. Nicol succeeded in getting rid entirely of the ordinary ray from the emergent beam by means of total reflection at the surface of a film of Canada balsam.

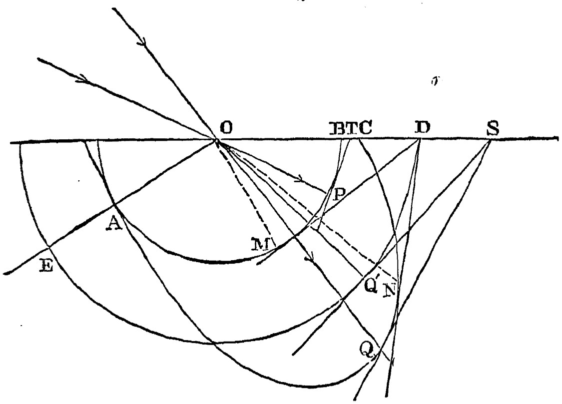
Canada balsam is a transparent substance whose refractive index lies between the ordinary and extraordinary indices of Iceland spar. Let us consider what happens when two waves, ordinary and extraordinary, pass from Iceland spar into Canada balsam.

Let us suppose, for simplicity's sake, that the plane of the paper, which we shall take as the plane of incidence, is a principal plane of the spar. Let O B (fig. 149) be the trace of the common surface of the two media, O A the optic axis, A B and A C the traces of the wave surface for the spar drawn in the Canada balsam, D E the trace of the wave surface in the Canada balsam. A B is a circle of radius a, D E another circle of radius greater than a, for the refractive index for Canada balsam is less than the ordinary index for spar. Consider now only an ordinary wave in the spar. Let O P be the direction of the ordinary ray cutting the wave surface in P, and draw P T a tangent to the circle A B, cutting the face of incidence in T. The ordinary wave-front in the spar is perpendicular to the paper and parallel to P T. From T draw a tangent to the circle D E, and through it pass a plane also perpendicular to the paper. This plane will give us the position of the refracted wave

in the balsam. This tangent is not drawn in the figure. But since the refractive index of the spar is greater than that of the balsam, the incident light may be totally reflected at the surface. This will be the case if T lies between B and D, and therefore within the wave surface for balsam, as in fig. 149. Thus no tangent could be drawn; there would be no refracted wave; the light is totally reflected at the surface of the balsam.

Consider now an extraordinary wave falling on the balsam. The section of the wave surface will be an ellipse,

FIG. 149.



A C suppose, cutting the face of incidence at C, and lying in part outside the circle D E. And if O Q be the direction of the extraordinary ray, Q S a tangent at Q meeting the face of incidence at S, and S Q′ a tangent to D E from S, the refracted wave will pass through S Q′, and O Q′ will be the direction of the refracted ray in the balsam.

Clearly, as before, S may lie between C and D, and the extraordinary ray also be totally reflected. Let us suppose that C lies between O and D, and from D draw D M, touching the ordinary wave-front, and D N touching the extraordinary.

Then it is clear that if P lie between B and M the ordinary ray will be totally reflected, and if Q lie between C and N the same will be true for the extraordinary ray; also the limits for total reflection for the extraordinary ray are less than those for the ordinary. If, then, we have two rays, an ordinary and extraordinary, traversing the crystal in such a

FIG. 150.

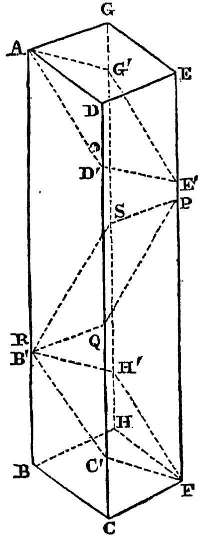
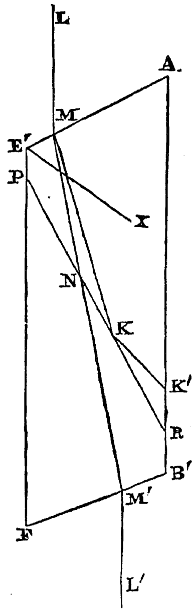


FIG. 151.



way that the direction of both lies within the angle M O N, the ordinary ray is totally reflected at the surface of the balsam, the extraordinary ray is refracted into it in the usual manner.

To construct a Nicol's prism, a rhomb of spar is taken with its length about double of its thickness. Let A D E G

B C F H (fig. 150) be the rhomb, E and B being the solid angles, which are each contained by three obtuse angles. The faces A D E G and B C F H are inclined to the blunt edges H B and E F respectively at angles of about 71°. These two faces are cut off, and two new faces, A D′ E′ G′, B′ C′ F H′ formed, inclined at 68° to the above edges. The whole block is then divided into two by a plane at right angles to the principal section A E′ F B′, inclined at an angle of about 22° to the same edges—that is, at right angles to the two new faces A D′ E′ and B′ C′ F. In the figure, P Q R S represents this plane of section. The two surfaces thus formed are then polished and cemented together again with Canada balsam. Figure 151 gives a section by a plane through A E′ F B′; P R is the trace of the plane of section. The optic axis lies in the plane of the paper and in the direction E′ X, making an angle of about 48° 30′ with the line A E′.

A ray of light L M falling on the face, A E′, in a direction nearly parallel to A B′ is there refracted into two, M N and M K, which meet the surface of the balsam at N and K. The angle of incidence of these two rays is such that, as already explained, the ordinary ray M K is totally reflected by the balsam along K K′, while the other, the extraordinary ray, M N, is refracted into the thin film of balsam, again refracted at the second surface into the spar, passes along the spar in the direction K M′, and emerges at M′ along M′ L′ parallel to L M. This ray is plane polarised at right angles to the principal plane. The vibrations which compose it are all in the plane of the paper and at right angles to L′ M′.

Foucault's prism is constructed in a similar manner, only the Canada balsam is done away with, and a thin layer of air is left between the two parts of the prism. The angles at which the various faces are inclined to each other are somewhat different, and the prism is rather shorter.

Polarising prisms have been constructed in which the faces on which the light falls are at right angles to the

length of the prism, instead of being inclined to it at 68°, as in Nicol's prism. For many purposes these are more useful.

CHAPTER XII.

REFLEXION AND REFRACTION OF POLARISED LIGHT.

REFRACTION through a crystal is not the only means we possess of producing plane polarised light. This was discovered by Malus, who found, while examining with a prism of Iceland spar the light of the sun reflected from one of the windows of the Luxembourg palace in Paris, that for one position of the prism one of the two images of the sun formed by the prism disappeared. On turning the prism round the line of sight this came into view again, and when the prism had been turned through 90° the other image vanished in its place. Let us go back to the rhomb of spar described in our first account of polarisation, and suppose that instead of looking at the sun directly through it and a pinhole in a sheet of paper, we look at the light reflected from a plate of unsilvered glass.

Then we shall find that for one particular angle of incidence, if the principal plane of our prism be parallel to the plane of reflexion from the glass, only the ordinary ray emerges from the spar, while if we turn the rhomb round through 90°, so that the principal plane is at right angles to the plane of reflexion, only the extraordinary image of our pinhole is seen. The light reflected at this angle from the glass is related to the plane of reflexion in the same manner as the ordinary ray emerging from our first rhomb of spar is related to the principal plane of that rhomb.

The reflected light is polarised in the plane of reflexion; its vibrations are at right angles to that plane. The particular angle of incidence at which this occurs when crown glass is the reflecting medium is 54° 35′.

Brewster was led by experiment to infer that light could

be polarised completely by reflexion from any transparent surface.

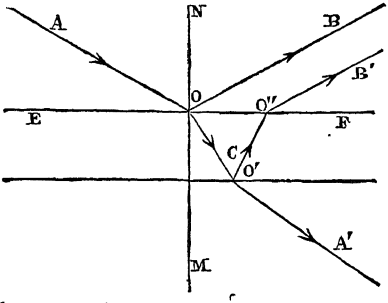
The particular angle of incidence at which this took place he called the polarising angle, and he found from his experiments that the polarising angle was defined by the equation $\tan\phi = \mu$, where μ is the refractive index and ϕ the polarising angle.

This law may be expressed also by the equation $\phi + \phi' = 90°$, if ϕ' be the angle of refraction corresponding to the polarising angle ϕ; for $\mu = \sin\phi/\sin\phi'$, so that the equation $\tan\phi = \mu$ becomes

$$\tan\phi = \sin\phi/\sin\phi'$$
$$\therefore \cos\phi = \sin\phi'$$
$$\phi = 90° - \phi'$$
$$\phi + \phi' = 90°.$$

Let A O, O B, O C (fig. 152), be the incident, reflected, and refracted rays, the angle of incidence being the polarising

FIG. 152.



angle, and let N O M be the normal to the reflecting surface. Then

$$\text{NOA} = \text{NOB} = \phi$$
$$\text{MOC} = \phi'$$
$$\therefore \text{NOB} + \text{MOC} = 90°$$
$$\therefore \text{BOC} = 90°,$$

or the reflected and refracted rays are at right angles.

Thus, according to Brewster, if light fall at the polarising angle on any transparent surface the reflected and refracted rays are at right angles; or, again, if the angle of incidence be such that the reflected and refracted rays are at right angles, the reflected light is plane polarised in the plane of reflexion.

More delicate experiments, especially those of Jamin,[1] have shown that this law is not quite exact. Except in the case of media whose refractive indices are about 1·46, the light reflected at this particular angle of incidence is not quite plane polarised. In addition to the disturbance at right angles to the plane of reflexion, there is some disturbance in this plane, though this latter is very small in amount compared with the former. Moreover, these two vibrations differ in phase. The reflected light is said to be elliptically polarised. We shall return to this later. At present, we shall consider some of the consequences of Brewster's law which is true, at any rate, as a first approximation.

Let us suppose plane polarised light falls on a transparent reflecting surface. We can resolve the vibration in the incident wave into two—the one parallel to, the other perpendicular to, the plane of incidence,—and consider these separately.

From the vibration at right angles to the plane of incidence, we have both a reflected and refracted wave, in which the vibrations are in the same direction. The amplitudes of the disturbance in these two waves are of course different from the amplitude in the incident wave. Likewise, too, the disturbance in the plane of incidence produces a reflected and refracted wave, in which the vibrations are in the plane of incidence, and the amplitudes in these two waves differ from that in the incident one.

Thus considering the two components in the incident wave, the light in both the reflected and refracted waves is the resultant of two component vibrations perpendicular and

[1] See however page 386 for an explanation of Jamin's results.

ised in the plane of reflexion ; this emerges, still plane polarised, from the first surface. On placing a second plate behind the first, parallel to it, some of the light transmitted by the first is reflected from it at the polarising angle, and after refraction again through the first plate emerges from it plane polarised.

Thus if we allow light to fall at the polarising angle on a number of parallel plates of glass, all the reflected light is polarised in the plane of incidence.

Such a contrivance is called a pile of plates, and is a common method of producing polarised light. Again, consider the light transmitted by such a pile ; at each surface the whole of the vibration in the plane of incidence is transmitted, and only a small portion of that perpendicular to the plane of incidence. After passing through several plates, therefore, the latter vibration will have been so much weakened compared with the former, that the emergent beam will be practically plane polarised at right angles to the principal plane.

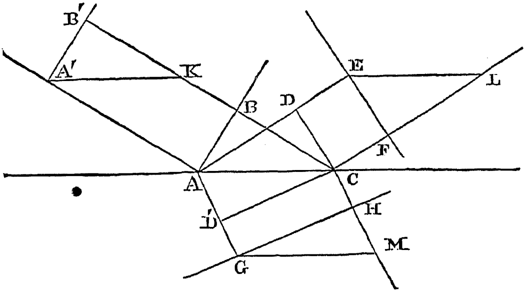
It is clear, as we have said, that when light falls on a refracting surface, and a reflected and refracted ray are produced, there must be some relation between the amplitudes in the incident, reflected and refracted rays.

Let A C (fig. 154) be a trace of the reflecting surface, A B that of an incident wave, the plane of the paper being that of incidence. Draw C B at right angles to A B to meet the surface in C. During any moment a certain amount of energy arrives in the incident wave at the surface A C, and is there transferred to the reflected and refracted waves.

The wave A B will of course give rise to a reflected and refracted wave in the two media; let us suppose that E F and G H are the positions of these two waves after a unit of time; and let A′ B′ be the wave-front which at that moment has arrived at the position A B. Draw A′ K, E L, and G M parallel to A C. Then during that unit of time the energy in the space A′ A C K has been transferred to the spaces A C L E and

A C M G. Let us suppose that ρ, ρ' are the densities of the ether in the two media, v, v' the velocities of light, λ, λ' the wave-lengths, ϕ, ϕ' the angles of incidence and refraction; let a, a_1, a' be the amplitudes of the incident reflected and refracted vibration. Then if x be the distance of the incident wave from some fixed point measured along A′ A, the

FIG. 154.



displacement in it is $a \sin \frac{2\pi}{\lambda}(v t - x)$, and the velocity with which any particle of ether in the wave is moving may be shown to be

$$\frac{2 \pi v}{\lambda} a \cos \frac{2 \pi}{\lambda}(v t - x).$$

If m be the mass of the particle, the kinetic energy of its motion is $\frac{1}{2} m \frac{4 \pi^2 v^2}{\lambda^2} a^2 \cos^2 \frac{2\pi}{\lambda}(v t - x)$.

The total energy of the particle is partly kinetic and partly potential, and the sum of the two is constant. Thus if T represent the kinetic, V the potential, and E the whole energy, T + V = E where E is constant.

Thus T = E − V and T is greatest when V = 0 (V cannot be negative), and that greatest value of T is the value of E. Now the greatest value of the kinetic energy is

$$\frac{1}{2} m \frac{4 \pi^2 v^2}{\lambda^2} a^2.$$

Thus the total energy of any particle in the incident wave is $\frac{2\, m\, \pi^2\, v^2\, a^2}{\lambda^2}$, and the whole energy in the space A′ K C A is $\Sigma\, \frac{2\, m\, \pi^2\, v^2\, a^2}{\lambda^2}$, where Σ indicates the result of adding together the separate amounts of energy of each particle. Now ρ being the density of the ether we have

$$\Sigma\, m = \rho \times \text{vol A}'\text{ A C K}.$$

Thus the energy required $= \frac{2\, \rho\, \pi^2\, v^2\, a^2}{\lambda^2} \times$ vol A′ A C K.

Now A′ A = space traversed by wave in unit of time $= v$, and A B = A C cos ϕ. Thus the energy

$$= \frac{2\, \rho\, \pi^2\, v^2\, a^2}{\lambda^2}\, v.\ \text{A C. cos}\, \phi.$$

Similarly for the energy in E A C L we get

$$\frac{2\, \rho\, \pi^2\, v^2\, a_1{}^2}{\lambda^2}\, v.\ \text{A C. cos}\, \phi\,;$$

and for that in A G M C $\frac{2\, \rho\, \pi^2\, v'^2\, a'^2}{\lambda'^2}\, v'$. A C. cos ϕ'. Thus, since the energy in A′ C = energy in E C + energy in A M, we have

$$\frac{\rho\, v^2\, a^2}{\lambda^2}\, v \cos \phi = \frac{\rho\, v^2\, a_1{}^2}{\lambda^2}\, v \cos \phi + \frac{\rho'\, v'^2\, a'^2}{\lambda'^2}\, v' \cos \phi\,;$$

but

$$\frac{v}{\lambda} = \frac{v'}{\lambda}$$

Hence $\rho\, a^2\, v \cos \phi = \rho\, a_1{}^2\, v \cos \phi + \rho'\, a'^2\, v' \cos \phi'$ . . (1)

We require now to make some assumption as to the ratio between the densities of the ether in the two media. Now in elastic media we can show that if e be the elasticity and ρ the density, the velocity of propagation is $\sqrt{\frac{e}{\rho}}$, so that if we had two media, and if e', ρ' were the corresponding quantities for the second, and v, v' the velocities of propagation, we should have $v = \sqrt{\frac{e}{\rho}}$; $v' = \sqrt{\frac{e'}{\rho'}}$

Fresnel, who was the first to consider the theory of the reflexion and refraction of light, assumed that these expressions held in the case of the ether, and that in addition the ether in all bodies was equally elastic, so that $e = e'$.

Thus $v/v' = \sqrt{\rho'/\rho}$, or $\rho/\rho' = v'^2/v^2 = \sin^2\phi'/\sin^2\phi$.

Hence, substituting in the above equation (1) for this ratio, and also for the value of v/v', we get

$$a^2 \sin\phi' \cos\phi = a_1{}^2 \sin\phi' \cos\phi + a'^2 \sin\phi \cos\phi' \quad . \quad (2)$$

We have thus arrived at one equation between a, a_1 and a'. It is important to remember that (1) has been obtained quite strictly from the principle of the conservation of energy, and holds, whatever be the relation between ρ and ρ'. Equation (2), which we shall proceed to use, is only true, of course, on certain assumptions.

In order to determine the values of a_1 and a' in terms of a, we require an additional equation. To obtain this, Fresnel supposes that the displacement resolved along the surface of separation must be the same in the two media, and at the same time that no alteration of phase is produced by the reflexion or refraction.

In the first place, let the light be polarised in the plane of incidence; the vibration is then perpendicular to that plane in both media, and parallel to the surface of separation of the two. The amplitude of the displacement in the first medium is $a + a_1$; in the second it is a'; and Fresnel's second principle gives us $a + a_1 = a'$.

Also from (2) $(a^2 - a_1{}^2)\sin\phi'\cos\phi = a'^2 \sin\phi\cos\phi'$.

Thus
$$a - a_1 = \frac{a' \sin\phi \cos\phi'}{\sin\phi' \cos\phi}$$

$$2a = \frac{a' \sin(\phi + \phi')}{\sin\phi' \cos\phi}$$

$$a' = \frac{2a \sin\phi' \cos\phi}{\sin(\phi + \phi')}$$

$$a_1 = a' - a$$

$$= -a\frac{\sin(\phi - \phi')}{\sin(\phi + \phi')}$$

We have thus obtained expressions for the amplitudes of the reflected and refracted waves when the incident light is polarised in the plane of incidence.

Now let the light be polarised perpendicular to this plane, and let a, a_1 and a' be amplitudes. The vibrations in all the waves are in the plane of incidence along A B, E F, and G H respectively, and these have to be resolved along A C. In the first medium, then, we have for the amplitudes of the displacements in the required direction $a \cos \phi$ and $- a_1 \cos \phi$. (To determine the sign to be attached to a_1 and a', suppose that the positive direction in the incident ray is to the right as we look along it, then the positive direction in the other two is to the right also as we look along them in the direction in which the light is going.) In the second medium the amplitude is $a' \cos \phi'$. Equating these we obtain $(a - a_1) \cos \phi = a' \cos \phi'$. Also from (2)

$$(a^2 - a_1{}^2) \cos \phi \sin \phi' = a' \cos \phi' \sin \phi.$$

Thus
$$a - a_1 = a' \frac{\cos \phi'}{\cos \phi}$$

$$a + a_1 = a' \frac{\sin \phi}{\sin \phi'}$$

$$2\, a = a' \left(\frac{\cos \phi'}{\cos \phi} + \frac{\sin \phi}{\sin \phi'} \right)$$

$$= \frac{a' \sin (\phi + \phi') \cos (\phi - \phi')}{\cos \phi \sin \phi'}$$

and $\therefore$
$$a' = \frac{2\, a \cos \phi \sin \phi'}{\sin (\phi + \phi') \cos (\phi - \phi')}$$

$$a_1 = a \frac{\tan (\phi - \phi')}{\tan (\phi + \phi')}$$

We have thus obtained expressions for the amplitudes of the refracted and reflected vibrations in this case also.

Let us now suppose that the incident light is plane polarised, in a plane inclined at an angle θ to the plane of

incidence, a being the amplitude. Then we can resolve the displacement into $a \cos \theta$ perpendicular to that plane, and $a \sin \theta$ in the plane. Each of these gives rise to a reflected and refracted wave with vibrations in these two directions; the two vibrations in the reflected wave combine to form one, and the reflected light is plane polarised in a plane making an angle θ_1, suppose, with the plane of incidence, while the refracted wave is plane polarised in a plane at an angle θ'.

We have then the following equations between a, a_1, a', θ, θ_1, and θ'.

From the displacements perpendicular to the plane of incidence we get

$$a' \cos \theta' = 2\, a \cos \theta \frac{\sin \phi' \cos \phi}{\sin (\phi + \phi')}$$

$$a_1 \cos \theta_1 = - a \cos \theta \frac{\sin (\phi - \phi')}{\sin (\phi + \phi')};$$

and from those in the plane of incidence

$$a' \sin \theta' = 2\, a \sin \theta \frac{\sin \phi' \cos \phi}{\sin (\phi + \phi') \cos (\phi - \phi')}$$

$$a_1 \sin \theta_1 = a \sin \theta \frac{\tan (\phi - \phi')}{\tan (\phi + \phi')};$$

and from these equations we can find a', a_1, θ' and θ_1.

From the above equations we see that if

$$\phi + \phi' = 90^\circ \text{ so that } \tan (\phi + \phi') = \infty$$
$$a_1 \cos \theta_1 = - a \cos \theta \sin (\phi - \phi')$$
$$a_1 \sin \theta_1 = 0.$$

Thus θ_1 is zero for all values of θ, or the reflected beam is polarised in the plane of reflexion. If we have ordinary light falling on the surface instead of plane polarised, we may resolve the ordinary beam into two, polarised in planes at right angles to each other, and consider the reflected beam as the resultant of the components arising from each of these; and since, if $\phi + \phi' = 90^\circ$, θ_1 is zero

for all values of θ, each of these two incident waves gives rise to one reflected wave polarised in the plane of reflexion. Thus when ordinary light falls on the surface at an angle of incidence, such that $\phi + \phi' = 90°$, the reflected beam is plane polarised in the plane of reflexion.

This, then, agrees with the law discovered experimentally by Brewster, and as we have seen that that law is only an approximate expression of the truth, we may infer that the above formulæ are only approximately true.

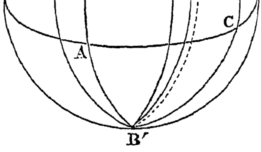
It will be useful to consider the values of θ' and θ_1 more fully.

By division we have from the above equations

$$\begin{aligned} \tan \theta' &= \tan \theta \sec (\phi - \phi') \\ \tan \theta_1 &= - \tan \theta \cos (\phi + \phi') \sec (\phi - \phi') \\ &= - \tan \theta' \cos (\phi + \phi') \end{aligned}$$

θ, θ' and θ_1 are the angles between the plane of incidence and the planes of polarisation of the incident, refracted and reflected waves. Now these waves all cut the refracting surface in the same straight line at right angles to the plane of incidence, and the direction of vibration in each is at right angles to the plane of polarisation. Thus the above angles are the angles between the common intersection of the three waves with the refracting surface and the directions of vibration in these waves respectively. Consider a sphere, centre O, and let the surface of separation be parallel to the plane O A B, cutting the sphere in A B (fig. 155); let O B be the intersection of the surface and the three waves, and let planes parallel to these waves meet the surface in B P, B P$_1'$ and P$_1'$ B P$_1$, and let O P, O P$'$, and O P$_1$ be parallel to the directions of vibration.

FIG. 155.



Then arc $BP = \theta$, arc $BP' = \theta'$, arc $BP_1 = \theta_1$. Also $ABP = \phi$, $ABP_1' = \phi$, and $ABP' = \phi'$.

$$\therefore PBP' = \phi - \phi'$$
$$P_1'BP' = \phi + \phi';$$

for BP_1' and BP' are on opposite sides of BA.

Make $BP_1' = BP_1$, and join PP' and $P_1'P'$ by arcs of great circles.

Then the formula $\tan \theta' = \tan \theta \sec (\phi - \phi')$ gives us $\tan BP' = \tan BP \sec PBP'$. From this it follows that the angle $P'PB$ is a right angle, or OP is the projection of OP' on the plane of the incident wave. Again, $BP_1 = \theta_1$ and BP_1' is equal to BP_1, but is drawn in the opposite direction from B thus, $BP_1' = -\theta_1$, and the formula

$$\tan \theta_1 = -\tan \theta' \cos (\phi + \phi')$$

gives us

$$\tan BP_1' = \tan BP' \cos P_1'BP'.$$

Thus $P'P_1'B$ is a right angle. Thus, to find the direction of vibration in the reflected wave we project on it the direction of vibration on the refracted wave, and then draw a line on the opposite side of OB, making the same angle with OB as this projection.

These expressions for θ' and θ_1 have been tested experimentally by Brewster, who found that they expressed the results of his observations fairly accurately. M. Jamin's experiments have shown that near the angle of total polarisation the formula for reflexion does not hold; some experiments of my own show that at low angles of incidence the formula for refraction is very nearly true, but when the angle of incidence is large, say from 40° to 80°, there are very considerable differences between the theory and observation. The observations were made by allowing light, polarised in a known azimuth θ, to pass obliquely through a plate of glass. If $\bar{\theta}$ be the azimuth of the plane of polarisation of the emergent light, and θ' that of the light in the glass, we have

$$\tan \theta' = \tan \theta \sec (\phi - \phi'),$$

and, from considering the refraction out again, we obtain

$$\tan \bar{\theta} = \tan \theta' \sec (\phi' - \phi);$$

so that

$$\tan \bar{\theta} = \tan \theta \sec^2 (\phi - \phi').$$

If the value of θ be known, and that of ϕ be observed, ϕ' is known from the formula $\sin \phi = \mu \sin \phi'$, μ being the refractive index. Thus the formula gives the value of $\bar{\theta}$, but this can be determined by experiment; for it is the azimuth of the plane of polarisation of the emergent light.

For angles of incidence up to about 40° the agreement is fairly close; when the angle of incidence is increased up to 75° the theoretical value of θ exceeds the experimental by about 1° 30′.

Returning now for a little to the assumptions on which these results have been obtained, let us consider the second, which supposes that the displacements along the surface of separation are the same in the two media. This is demanded by the continuity of the ether, for if it were not true, there would be a gap between the two adjacent particles of ether in the two media.

But then this principle demands equally that the displacement at right angles to the surface should be the same; now this would lead to the condition when the light is polarised at right angles to the plane of incidence,—remembering the rule already stated as to signs of a_1 and a'

$$(a + a_1) \sin \phi = a' \sin \phi';$$

while we already have seen that $(a - a_1) \cos \phi = a' \cos \phi'$. Thus

$$(a^2 - a_1^2) \sin \phi \cos \phi = a'^2 \sin \phi' \cos \phi'.$$

But the principle of the conservation of energy gives us

$$\rho \,(a^2 - a_1^2) \sin \phi \cos \phi = \rho' \, a'^2 \sin \phi' \cos \phi',$$

and these two equations are inconsistent unless $\rho = \rho'$. Thus Fresnel's assumption as to the relation between the densities is inconsistent with the continuity of the ether at

right angles to the surface of separation. If we take as our conditions at the surface the equations of continuity, then the conservation of energy demands that we should treat the density as the same in the two media, and if the light be polarised at right angles to the plane of incidence—that is, if the vibration be in the plane of incidence, we get as the equations of condition

$$(a + a_1) \sin \phi = a' \sin \phi'$$

$$(a - a_1) \cos \phi = a' \cos \phi'$$

$$2\, a = \frac{a' \sin (\phi + \phi')}{\sin \phi \cos \phi}$$

$$a' = \frac{2\, a \sin \phi \cos \phi}{\sin (\phi + \phi')};$$

$$\text{and } a_1 = -\, a \frac{\sin (\phi - \phi')}{\sin (\phi + \phi')}.$$

While for light in which the vibrations are at right angles to this plane we have $a + a_1 = a'$

$$(a^2 - a_1{}^2) \sin \phi \cos \phi = a'^2 \sin \phi' \cos \phi'$$

$$\therefore\ (a - a_1) \sin \phi \cos \phi = a' \sin \phi' \cos \phi'$$

$$2\, a = \frac{a' (\sin \phi \cos \phi + \sin \phi' \cos \phi')}{\sin \phi \cos \phi}$$

$$a' = \frac{2\, a \sin \phi \cos \phi}{\sin (\phi + \phi') \cos (\phi - \phi')}$$

Also

$$a_1 = a \frac{\tan (\phi - \phi')}{\tan (\phi + \phi')}$$

If we compare these equations with those deduced from Fresnel's hypothesis, we see that the two for the reflected ray have been reproduced with this difference, that Fresnel's expression for light vibrating in the plane of incidence is here the expression for light vibrating at right angles to that plane, and *vice versâ.*

The expressions for the refracted rays have been inter-

changed in the same manner, and at the same time altered in the ratio of $\sin\phi : \sin\phi'$.

The theory just given is that of Neumann and MacCullagh, and rests on the assumptions that the disturbance in each medium and all its effects are confined to the wave-front, and that the density of the ether is the same in the two media. On these assumptions the expressions satisfy the principles of the conservation of energy, and of the continuity of the motion in the two media.

They lead to the same two equations to determine θ_1 and θ' in terms of θ as Fresnel's—viz.,

$$\tan\theta' = \tan\theta \sec(\phi - \phi')$$
$$\tan\theta_1 = -\tan\theta \sec(\phi - \phi') \cos(\phi + \phi').$$

The same two mathematicians have considered the propagation of light in crystalline media, and according to their theory it follows that the direction of vibration coincides with the plane of polarisation instead of being perpendicular to it. Thus, combining this fact with the equations above, we conclude that the expressions given for the amplitudes of the reflected rays polarised in and at right angles to the plane of incidence are the same in the two theories; for the refracted rays they differ in the ratio of $\sin\phi : \sin\phi'$. The expression for the amplitude of light refracted through a plate, if polarised in the plane of incidence, will be, on Fresnel's theory,

$$\frac{4\,a \sin\phi' \cos\phi \sin\phi \cos\phi'}{\sin^2(\phi + \phi')}$$

or

$$\frac{a \sin 2\phi \sin 2\phi'}{\sin^2(\phi + \phi')}$$

and on MacCullagh's we get the same expression. Thus observations on the intensity of light refracted through or reflected from a plate will not enable us to decide between the two theories. MacCullagh's formulæ express the amplitudes of the reflected and refracted rays on the electro-

magnetic theory of light proposed by Clerk Maxwell. This we shall consider later.

We have had occasion to use the term elliptically polarised light. We proceed to explain more fully the meaning of the phrase. In ordinary light we have seen that the vibrations are in the wave front, and that we may in the most general case, consistent with the propagation of light of a definite wave length, treat each particle of the ether as if it were moving in an ellipse the axes of which are continually changing both in magnitude and position.

Let us suppose that the nature of the vibration is such that each particle moves always in a *fixed ellipse*, describing the ellipse in the same time, and that these ellipses are the same, both in magnitude and position, for all the positions of the wave-front; so that if A B C is any wave-front, P any point in it is describing an ellipse, and if A′ B′ C′ is any other position of the front, then P′ any point in it is describing an equal and similarly situated ellipse. The light in this case is said to be elliptically polarised.

If the axes of these ellipses become equal, so that the particles of ether all move in circles, the light is said to be circularly polarised. If, again, one of the axes of the ellipse is zero, so that the curve becomes a straight line coincident with the other axis, and all the particles move in parallel straight lines, the light is plane polarised.

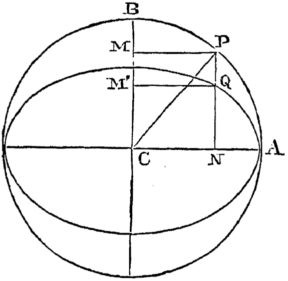
Light is polarised when all the particles of the ether in the ray describe equal and similarly situated curves, the curves remaining the same throughout the period considered. If the curves be ellipses the polarisation is elliptic, if they be circles the light is circularly polarised, and if they be straight lines the light is plane polarised.

Let us analyse this a little more closely.

Suppose P (fig. 156) is a point moving uniformly in a circle and describing the circle in time τ. Let C be the centre, C A, C B two radii at right angles, and let t be the time which has elapsed since the particle P left A. Since, in time τ,

the radius vector CP tracts out an angle 2π, and the motion is uniform, the angular velocity is $\frac{2\pi}{\tau}$; and the angle ACP has been described in time t therefore the angle $ACP = \frac{2\pi t}{\tau}$.

FIG. 156.



Let PM and PN be perpendicular on CB and CA respectively, and let a be the radius of the circle. Then

$$CM = PN = PC \sin PCA$$
$$= a \sin \frac{2\pi t}{\tau}$$
$$CN = PM = PC \cos PCA$$
$$= a \cos \frac{2\pi t}{\tau} = a \sin \left(\frac{2\pi t}{\tau} + \frac{\pi}{2}\right) = a \sin \frac{2\pi}{\tau}\left(t + \frac{\tau}{4}\right)$$

The motion of P is compounded of that of N and M.

Both these vibrate along CA and CB respectively, according to the harmonic law; but when N is at C, the centre of its swing, M is at B the extremity of its swing, and *vice versâ*. The two vibrations differ in phase by $\frac{\tau}{4}$, or one quarter of a complete period. The amplitude of the two vibrations is the same.

Thus, two vibrations of equal amplitudes in directions at right angles which differ in phase by a quarter period combine to produce a uniform motion in a circle; that is to say, two plane polarised rays of equal intensity, differing in phase by a quarter period, produce circularly polarised light; or a wave of circularly polarised light may be resolved into two plane polarised rays, polarised in planes at right angles, of equal amplitude, and differing in phase by a quarter period.

Now let Q be a point in PN, and, as P moves round the

circle, let Q move in such a way that the ratio Q N : P N remains constant.

Suppose that Q N : P N $= b : a$. Then geometry tells us that Q describes an ellipse, whose axes are a and b along C A and C B respectively; and, as P moves round the circle, Q describes this ellipse in the constant periodic time τ.

$$\text{Also P N} = a \sin \frac{2\pi t}{\tau} \text{ and Q N} = \text{P N} \times b/a.$$

$$\text{Thus Q N} = b \sin \frac{2\pi t}{\tau},$$

and if Q M′ be perpendicular on C B,

$$\text{C M}' = \text{Q N} = b \sin \frac{2\pi t}{\tau};$$

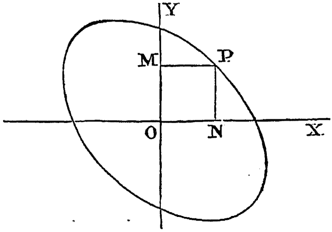
$$\text{while C N} = a \sin \frac{2\pi}{\tau}\left(t + \frac{\tau}{4}\right).$$

The motion of Q is compounded of those of N and M′.

That of N corresponds to a plane polarised ray of amplitude a, in direction C A; that of M′ to a plane polarised ray of amplitude b in direction C B, and these differ by a quarter period.

Elliptically polarised light is the resultant of two plane polarised vibrations along the axes of the ellipse. The amplitudes are equal to the axes of the ellipse, and the vibrations differ in phase by a quarter period. An elliptically polarised ray, then, may be resolved into these two components.

FIG. 157.



Analysis, however, tells us more than this. We can show that the resultant of any two plane polarised vibrations differing in phase by any given quantity is an elliptic vibration. This ellipse may, in

the extreme cases, be a circle or a straight line. We can prove this most easily by some analytical geometry. Let O X and O Y (fig. 157) be the two directions, and let the displacement in direction O X be

$$x = a \sin \frac{2\pi t}{\tau},$$

and in direction O Y.

$$y = b \sin \left(\frac{2\pi t}{\tau} + \delta\right).$$

δ is the difference in phase, a and b the amplitudes.

Let O N $= x$ at any moment and O M $= y$, and draw M P and N P parallel to O X and O Y respectively. Then P is the position of the particle when displaced by the two vibrations simultaneously.

$$\frac{x}{a} = \sin \frac{2\pi t}{\tau}$$

$$\frac{y}{b} = \sin \frac{2\pi t}{\tau} \cos \delta + \cos \frac{2\pi t}{\tau} \sin \delta.$$

$$\text{Thus, } \cos \frac{2\pi t}{\tau} = \frac{1}{\sin \delta}\left\{\frac{y}{b} - \frac{x \cos \delta}{a}\right\}.$$

$$\text{But } \sin^2 \frac{2\pi t}{\tau} + \cos^2 \frac{2\pi t}{\tau} = 1$$

$$\therefore \frac{x^2}{a^2} + \frac{1}{\sin^2 \delta}\left\{\frac{y}{b} - \frac{x \cos \delta}{a}\right\}^2 = 1,$$

$$\text{or } \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy \cos \delta}{ab} = \sin^2 \delta.$$

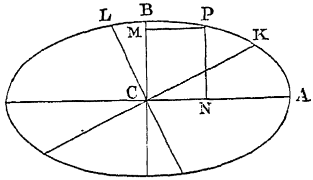
This equation, we know, represents an ellipse. Thus, P moves always on a certain fixed ellipse in a definite periodic time, τ. Any two vibrations differing in phase produce an elliptic vibration.

If $\delta = 0$, so that the difference of phase is zero, the ellipse becomes a straight line. Two plane polarised rays of the same phase compound to form a plane polarised ray.

We can prove easily, without any analytical geometry, that an elliptically polarised ray can be resolved into two plane polarised rays in directions at right angles with a certain difference in phase.

We have seen already that the elliptic vibration is equivalent to two vibrations—$a \cos \frac{2\pi t}{\tau}$ and $b \sin \frac{2\pi t}{\tau}$—along the axes. Let C K, C L (fig. 158) be the required directions at right angles, and let the angle $KCA = \alpha$. Then $LCB = \alpha$ also; resolve the two vibrations in the directions C L and C K. We have along C K,

FIG. 158.



$$a \cos \alpha \cos \frac{2\pi t}{\tau} + b \sin \alpha \sin \frac{2\pi t}{\tau},$$

and along C L,

$$-a \sin \alpha \cos \frac{2\pi t}{\tau} + b \cos \alpha \sin \frac{2\pi t}{\tau}.$$

Let us suppose that c and γ are two quantities such that $a \cos \alpha = c \sin \gamma$, $b \sin \alpha = c \cos \gamma$, and $c\ \gamma'$ two quantities such that $a \sin \alpha = c' \sin \gamma'$, $b \cos \alpha = c' \cos \gamma'$.

Then the vibrations along C L and C K are respectively

$$c \sin\left(\frac{2\pi t}{\tau} + \gamma\right), \text{ and } c' \sin\left(\frac{2\pi t}{\tau} - \gamma'\right).$$

Each of these vibrations is a plane polarised ray. The amplitudes are c and c' respectively, and the difference in phase $\gamma + \gamma'$; and these quantities are known in terms of a, b, and α.

Thus an elliptic vibration can be resolved into two plane polarised vibrations at right angles differing in phase; and, conversely, two plane polarised vibrations at right angles, of different phase, combine to constitute an elliptic vibration.

A similar method of proof is applicable when the plane polarised vibrations are not at right angles, but the expressions become very complicated.

We have measured the difference of phase in fractions of a complete period. It is sometimes more convenient to have it in fractions of the wave-length. Thus, since $\tau = \frac{\lambda}{v}$, v being the velocity of the light, we may write for the two components along the axes,

$$a \sin \frac{2\pi}{\tau}\left(t + \frac{\tau}{4}\right), \text{ and } b \sin \frac{2\pi t}{\tau},$$

$$a \sin \frac{2\pi}{\lambda}\left(vt + \frac{\lambda}{4}\right), \text{ and } b \sin \frac{2\pi}{\lambda} vt \text{ respectively;}$$

and similarly for the other expressions.

We shall use this notation for the future; so that when we say that the difference of phase is δ, we mean that the two vibrations may be represented by

$$a \sin \frac{2\pi}{\lambda}(vt), \text{ and } b \sin \frac{2\pi}{\lambda}(vt + \delta) \text{ respectively;}$$

or, it may be, by

$$a \sin \frac{2\pi}{\lambda}(vt - x), \text{ and } b \sin \frac{2\pi}{\lambda}(vt - x + \delta).$$

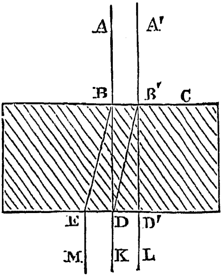
In each of these cases the difference of phase, measured as an angle in circular measure, is $\frac{2\pi\delta}{\lambda}$.

We will now describe some of the methods by which we can produce elliptically polarised light.

Let us consider a plane wave of plane polarised light incident normally on a plate of crystal. Suppose that the plate is perpendicular to the paper, and let B C D E (fig. 159) be a section of it. A ray A B incident at B, is divided by the plate into two—B D, the ordinary ray, and B E, the extraordinary; and these emerge parallel to A B from D and E respectively. Draw D B′ parallel to E B, meeting the first

surface of the plate in B′, and draw B′ D′ parallel to B D. A ray incident at B′ is also divided into two, and of these B′ D′ is the ordinary ray, B′ D the extraordinary. Thus D is the point of emergence of the ordinary ray from B and the extraordinary ray from B′, both of which, after traversing the crystal, travel along D K. These two rays are polarised in planes at right angles to each other. On entering the crystal at B and B′ the phase was the same in the two rays; but since they have travelled by slightly different paths and with different velocities through the crystal a difference of phase has been set up. Thus, at D, we have two rays polarised in perpendicular planes with a difference of phase between them. The light emerging from D, therefore, is elliptically polarised. Let us suppose that the faces of the plate are parallel to a principal plane of the crystal. Let a and c be the velocities of propagation for waves parallel to that plane travelling through the crystal; and let ϵ be the thickness of the crystal.

FIG. 159.



The directions of vibration are parallel to the axes of the crystal. Let us suppose that the displacements in the incident wave parallel to these axes may be represented by

$\alpha \sin \frac{2\pi}{\lambda} v t$, and $\beta \sin \frac{2\pi}{\lambda} v t$.

The first of these will traverse the crystal in time ϵ / a, and therefore the displacement in it on emergence will be

$$\alpha \sin \frac{2\pi}{\lambda} \left(v t + \frac{v}{a} \epsilon \right),$$

and in the other it will be

$$\beta \sin \frac{2\pi}{\lambda} \left(v t + \frac{v}{c} \epsilon \right).$$

The difference of phase between these, therefore, is

$v\,\epsilon\left(\frac{1}{a}-\frac{1}{c}\right)$, and if μ_1, μ_2 be the refractive indices of the medium for waves travelling with velocities a and c respectively, $\mu_1 = v/a$, $\mu_2 = v/c$, so that the difference of phase is $\epsilon\,(\mu_1 - \mu_2)$. By altering the thickness of the crystal we can make this difference of what we please. If we choose ϵ, so that $\epsilon\,(\mu_1 - \mu_2) = \frac{\lambda}{4}$, the axes of the elliptic vibration in the emergent light will be parallel to the axes of the crystal, and their lengths will be α and β.

Such a plate is known as a quarter undulation plate. If the amplitude of the vibration in the incident polarised beam be b, while the direction of vibration is inclined at an angle θ to the axis a of the crystal, we have $\alpha = b\cos\theta$, $\beta = b\sin\theta$. Hence, if θ be $45°$, or the plane of polarisation of the incident light bisect the angle between the axes of the quarter undulation plate, α is equal to β. The axes of the ellipse in the emergent vibration are equal, and the emergent light is circularly polarised.

Thus plane polarised light passed through any plate of a crystal or doubly refracting medium becomes elliptically polarised. If the plate be a quarter undulation plate, the axes of the emergent vibration are parallel to those of the crystal; while if the plane of polarisation of the incident light bisect the angle between these axes, the emergent beam is circularly polarised.

We should notice that the thickness of a quarter undulation plate depends on the wave-length of light, so that the same plate will not produce circular polarisation for all rays of the spectrum.

Let us now suppose that the difference of phase, $\epsilon\,(\mu_1 - \mu_2)$, is equal to $\frac{\lambda}{2}$. Put $\epsilon\,\mu_1 = x$, then $\epsilon\,\mu_2 = x - \frac{\lambda}{2}$, and we have for the emergent vibrations

$$b\cos\theta\sin\frac{2\pi}{\lambda}(vt + x),\text{ and } b\sin\theta\sin\frac{2\pi}{\lambda}\left(vt + x - \frac{\lambda}{2}\right)$$

This last vibration reduces to

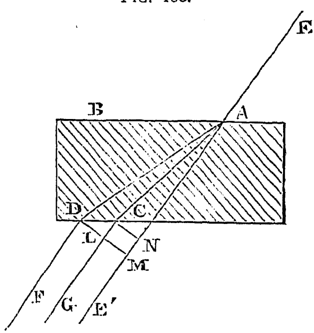
$$-b \sin \theta \sin \frac{2\pi}{\lambda}(v t + x).$$

These two vibrations combine to produce a plane polarised ray, and if the plane of polarisation make an angle θ_1 with the axis a, we have $\tan \theta_1 = -\tan \theta$ and $\theta_1 = -\theta$.

Thus the emergent beam is plane polarised, the plane of polarisation being on the opposite side of a to that of the incident beam and equally inclined to it.

It is often useful to have an expression for the difference in phase of two waves after traversing a crystal plate obliquely. Let the plane of the paper be the plane of incidence, and let A B C D (fig. 160) be a section of the plate. Let A E be the trace of the incident wave, A C and A D of the two refracted waves. Draw D F and C G parallel to A E. Then D F and C G are the traces of the emergent waves. Let ϕ, ϕ', ϕ'' be the angles of incidence and refraction, and v, v' and v'' the velocities of propagation in the three waves, and let ϵ be the thickness of the crystal. Produce E A to E′. Then, if the crystal had not been in the way, A E′ would have been the position of the wave-front at the instant at which the two refracted waves occupy respectively the positions D F and C G.

FIG. 160.



Each of these waves has been retarded by a different amount by its passage through the crystal. The difference of retardation is what we desire to find, and if D L be perpendicular on C G, the difference of phase between the two emergent beams is D L. Let D L meet A E′ in M, and draw C N perpendicular to A E′ to meet it in N.

D M is the retardation produced in the one wave, C N that in the other.

Now $\quad DAM = \phi - \phi'$,

and $\quad DM = DA \sin DAM$

$\quad = DA \sin (\phi - \phi')$.

Also $\quad DA = \epsilon \operatorname{cosec} \phi'$

$\therefore DM = \epsilon (\sin \phi \cot \phi' - \cos \phi)$.

Similarly $\quad CN = \epsilon (\sin \phi \cot \phi'' - \cos \phi)$.

Hence $\quad DL = \epsilon \sin \phi (\cot \phi' - \cot \phi'')$.

Thus the difference in phase in the two emergent waves is

$$\epsilon \sin \phi (\cot \phi' - \cot \phi'');$$

and since v' v'' are the velocities of wave propagation,

$$\sin \phi' = \frac{v'}{v} \sin \phi$$

$$\cot \phi' = \sqrt{\operatorname{cosec}^2 \phi' - 1} = \frac{\sqrt{(v^2 - v'^2 \sin^2 \phi)}}{v' \sin \phi}$$

and

$$\cot \phi'' = \frac{\sqrt{(v^2 - v''^2 \sin^2 \phi)}}{v'' \sin \phi}.$$

Thus, if we denote by V the difference of phase, we have

$$V = \epsilon \left\{ \frac{\sqrt{(v^2 - v'^2 \sin^2 \phi)}}{v'} - \frac{\sqrt{(v^2 - v''^2 \sin^2 \phi)}}{v''} \right\}.$$

The apparatus known as Fresnel's rhomb affords us another means of obtaining elliptically polarised light.

Let us suppose that light travelling in a dense medium is incident on the surface of one less dense. Then we know that if the angle of incidence exceed a certain value—the critical angle—there is no refracted wave; all the light is totally reflected. If we consider the expressions for the amplitudes of the reflected waves polarised either in or at right angles to the plane of incidence, we find that they involve the symbol $\sqrt{(-1)}$—they become imaginary, and our formulæ cease to hold. Now we nave assumed above—or rather it follows as a consequence from the assumptions there made—that the phase of the disturbance is the same

in the incident, reflected and refracted waves; or, to be somewhat more accurate, that the difference between them is a multiple of $\frac{\lambda}{2}$. If we represent the vibration in the incident wave by $a \sin \frac{2\pi}{\lambda}(vt - x)$, that in the reflected will be $a_1 \sin \frac{2\pi}{\lambda}(vt - x)$, and in the refracted

$$a' \sin \frac{2\pi}{\lambda}(vt - x),$$

and we see that in the case in which we have no refracted wave this assumption leads to impossible results.

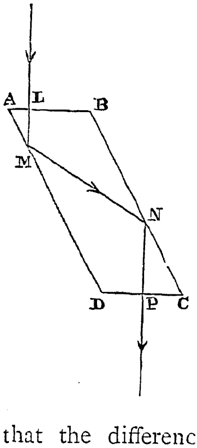
Let us suppose, therefore, that the reflected disturbance is represented by $a_1 \sin \frac{2\pi}{\lambda}(vt - x + \delta)$, so that there is a change in phase produced by the act of reflexion, and δ is this change in phase.

We can determine from the conditions at the reflecting surface the value of this quantity δ, and we find that it depends on the angle of incidence, and is different for light polarised in and perpendicular to the plane of incidence. If, then, we consider the two components in and perpendicular to the plane of incidence, the phase of each differs from that of the incident wave, but the difference is not the same for the two; so that these two components differ in phase. The reflected beam, then, is the resultant of two plane polarised waves polarised in planes at right angles, and differing in phase. It is, therefore, elliptically polarised. Thus plane polarised light becomes elliptically polarised by total reflexion. The difference of phase between the two components in and perpendicular to the plane of incidence depends on the nature of the reflecting medium and on the angle of incidence. If we call the change of phase for a vibration in the plane of incidence δ_1, and that for a vibration at right angles to the plane of incidence δ_2, the difference of phase between the two compo-

nents of the elliptic vibration in and perpendicular to the plane of incidence is $\delta_1 - \delta_2$. Now, the values obtained by Fresnel for these two quantities δ_1 and δ_2 lead to the conclusion that if light be reflected from the inner surface of St. Gobain glass for which the refractive index is 1·51, at an angle of $54° 37'$, the difference $\delta_1 - \delta_2$ is equal to $\frac{\lambda}{8}$, so that the difference of phase produced by two such reflexions is $\frac{\lambda}{4}$, and the axes of the resultant elliptic vibration are in and perpendicular to the plane of incidence.

Suppose, then, we construct a rhomb of this glass, two of whose faces are parallel to the paper, while the others are at right angles to it. Let A B C D (fig. 161) be a section of the rhomb, the angles at A and C each being $54° 37'$. Let L M N P be a ray of plane polarised light falling normally on the face A B at L. This is totally reflected at M by the face A D, again totally reflected at N by B C, and emerges normally at P from the face C D. The angles of incidence at M and N are $54° 37'$. No change of phase is produced by the direct refractions at L and P; but by each of the total reflexions at M and N a difference of $\frac{\lambda}{8}$ is produced between the components in and perpendicular to the paper, so that the difference in phase of these components in the emergent light is $\frac{\lambda}{4}$, and the axes of resultant elliptic vibration are in and perpendicular to the paper. If, again, the original plane of polarisation be inclined at 45° to the paper, the amplitudes of the incident, and therefore, also, of the emergent, vibrations in and perpendicular

FIG 161.



to the paper, are equal, so that the emergent light is circularly polarised. A rhomb of glass cut in this manner is called a Fresnel's rhomb.

Metallic reflexion is another method of producing elliptically polarised light. If light polarised in the plane of incidence fall on a polished metallic surface, the reflected beam is plane polarised also in the plane of incidence, but the reflexion produces a change in the phase of the light. The same, too, is the case if the light be polarised at right angles to the plane of incidence, but the change of phase produced in this latter case is less than in the former. If, therefore, plane polarised light polarised in a given azimuth fall on a metallic reflecting surface, there will be a difference of phase produced between the components of the reflected beam polarised in and perpendicular to the plane of incidence, the former of these being accelerated relatively to the latter, so that the reflected beam is elliptically polarised. This difference of phase is zero for normal incidence, and as the angle of incidence is increased increases gradually up to the value $\frac{\lambda}{2}$ when the incident ray just grazes the surface.

For one particular value of the angle of incidence, the difference of phase between the two components is $\frac{\lambda}{4}$, and in this case the axes of the resulting elliptic vibration are in and perpendicular to the plane of incidence.

This particular angle of incidence is known as the angle of maximum polarisation, for it is found that if ordinary light be allowed to fall on the surface, the reflected beam is always partially polarised, and if the angle of incidence be that of maximum polarisation, the amount of plane polarised light in the reflected beam is greater than under any other circumstance. A number of very careful observations on the light reflected from metals have been made by M. Jamin. He finds that for polished steel the angle of maxi-

mum polarisation is 76°, while the difference of phase between the two components decreases from the value $\cdot 4 \times \lambda$ when the angle of incidence is 84° to $\cdot 035 \times \lambda$, when it is 39° For the angle of incidence 76°, the difference of phase, of course, is $\cdot 25 \times \lambda$.

We have said already that the statement that plane polarised light remains plane polarised after reflexion from transparent media is only true approximately.

M. Jamin has also investigated this question. He finds that some few substances only, for which the refractive index is about 1·46, polarise light completely by reflexion at the angle ($\tan^{-1}\mu$). In all other cases, the component vibration which lies in the plane of incidence is never completely destroyed, though it becomes very small compared with the vibration at right angles to that plane. There is an angle of *maximum* polarisation given by the formula $\tan\phi = \mu$, instead of an angle of *total* polarisation.

Moreover, a difference of phase between these two components is introduced by the reflexion. This difference of phase is exceedingly small, except for angles of incidence nearly equal to that of maximum polarisation.

If the refractive index is greater than 1·46, the phase of the vibration in the plane of incidence is accelerated, compared with that at right angles to that plane; if the index be less than 1·46, the reverse is the case.

According to Fresnel's formulæ for light polarised at right angles to the plane of incidence, we have:

$$a_1 = a \frac{\tan(\phi - \phi')}{\tan(\phi + \phi')}$$

Thus if $\phi + \phi'$ is less than 90° a and a_1 have the same sign, when $\phi + \phi' = 90$ a_1 is zero, and when $\phi + \phi'$ is greater than 90°, a_1 is of the opposite sign to a. Thus on passing through the angle of maximum polarisation, the value of a_1 changes sign, and this is equivalent to an alteration of $\frac{\lambda}{2}$ in

Phase of the reflected vibration. No such alteration es place in the component perpendicular to the plane of idence. Thus according to Fresnel's theory we may say t the difference in phase between the two components the reflected wave is zero from normal incidence up to angle of total polarisation. At that angle it changes ldenly to $\frac{\lambda}{2}$, and remains $\frac{\lambda}{2}$ till we reach grazing inci-nce. M. Jamin's results show that this change is not lden. The difference of phase is very small—practically o—from normal incidence to an angle somewhat less than t of maximum polarisation; it then changes gradually but idly from 0 to $\frac{\lambda}{2}$, passing through the value $\frac{\lambda}{4}$ at the angle maximum polarisation, and becoming very nearly equal $\frac{\lambda}{2}$ for an angle not much greater than this. Thus for mond, for which the angle of maximum polarisation is 40′, we have, according to M. Jamin's tables, the follow-results :—

$\mu = 2{\cdot}434$. Azimuth of plane of primitive polarisation,

Angle of incidence.	Difference of phase as a fraction of $\frac{\lambda}{2}$
60°	·034
65°	·097
67°	·286
67°30′	·441
67°55′	·538
70°	·897
75°	·970

For flint, $\mu = 1{\cdot}714$. Azimuth of plane of primitive arisation, 77° 30′.

Angle of incidence.	Difference of phase as a fraction of $\frac{\lambda}{2}$
53°	·026
58°	·100
59°30′	·382
60°	·623
65°15′	·965

For flint, the angle of maximum polarisation is 59° 44′. In the case of fluorine, for which μ is 1·441, that is, less than 1·46, the differences of phase are of opposite sign, the vibration in the plane of incidence being retarded relatively to *the other*. Between the limits within which this difference of phase is changing, the reflected light is elliptically polarised, and these limits are wider apart the greater the difference between the refractive index of the substance and the value 1·46. Beyond these limits, plane polarised light remains plane polarised after reflexion from the surface.

These various measurements were made by M. Jamin, by the aid of a piece of apparutus which we will describe later.

The subject of these researches of M. Jamin has received a good deal of attention in recent times. Fresnel's theory of reflexion assumes a perfectly sudden transition between the properties of the ether in the two media at the confines of which reflexion is taking place. In the case of any substance actually experimented with there are various reasons why this sudden transition should not take place. Thus a substance such as glass requires polishing, and traces of the material used for polishing are almost certain to be left on the surface. There will, therefore, be a thin film of foreign matter between the glass and the air; or, again, it may be that forces are exerted between the matter of the glass or air and the ether. The ether in the neighbourhood of the surface is therefore in a different condition from that in the interior of either the glass or the air which surrounds it. In its case the forces are due to the combined action of the air and glass particles; the ether in the glass is subject to the action of glass-matter only, that outside is acted on by air-matter only. Thus there may be a film of ether over the surface having properties intermediate between those of the air-ether and the glass-ether, although there is no foreign matter present. And, thirdly, many substances

have the power of condensing moisture or gases over their surface, and this condensed gas may constitute such a film.

Now such a film, provided it were very thin—of thickness, say, comparable with that of the black spot in Newton's rings—would produce the Jamin effect. Near the polarising angle a small amount of light polarised at right angles to the plane of incidence would be reflected, and elliptic polarisation would be produced.

In consequence experiments have been made with a view of discovering how far the Jamin effect can be modified by altering the surface conditions slightly.

Wernicke has devised an ingenious method of cleaning the surface from foreign matter; he finds that as the cleaning proceeds the Jamin effect is diminished, though he was never able to get rid of it entirely.

Drude has investigated the effect on freshly cleaved faces of rock-salt, and finds that it is at first very small, but increases somewhat rapidly as the face is exposed to the air; while Lord Rayleigh has examined the light reflected from distilled water, using a method devised by himself to clean the surface of the water from impurity. He finds that in the case of the liquids examined the ellipticity is very much less than was supposed by Jamin, whose results for water and aqueous solutions were almost certainly vitiated by the presence of greasy contamination. Thus the intensity of reflexion from clean water is not much more than the 1/1000 part of that given by Jamin.

There is no experimental evidence against the rigorous applicability of Fresnel's formulæ to the ideal case of an abrupt transition between two uniform transparent media. Lord Rayleigh has also experimented on the amount of light reflected at normal incidence from water. He finds the reflexion to be about $1\frac{1}{2}$ per cent. greater than that given by Fresnel's formulæ, and concludes that while this disagreement may be real, it is too small a foundation on which to build with confidence.

Thus, within the limits of experimental error, Fresnel's formulæ express the facts.

The results of Jamin and other investigators, if this explanation be accepted, were due to the want of sudden transition between the two media experimented on. In this case, then, Fresnel's tangent formula is strictly true. Light can be polarised completely by reflexion from a transparent surface at an angle whose tangent is equal to the refractive index.

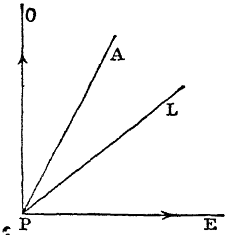
Now, Fresnel's formula can be deduced strictly either from the electro-magnetic theory of light, or from a theory of the constitution of the ether recently suggested by Lord Kelvin, and developed as to some of its consequences by myself.

CHAPTER XIII.

INTERFERENCE OF POLARISED LIGHT.

WE come next to consider the effects produced when plane polarised light is allowed to pass through a plate of crystal and examined by an analyser, such as a Nicol's prism, a tourmaline, or a pile of plates placed so as to receive the light at the polarising angle.

FIG. 162.



Let us suppose at first that the incident beam is a parallel pencil which falls normally on the plate, and that the analyser is a Nicol's prism. The polarisation of the incident beam may have been produced either by reflexion from a pile of plates or in any other manner. The light coming from any point of the second surface of the crystal, as P (fig. 162), will consist, we have seen, of two rays polarised in planes

at right angles and differing in phase; let P O and P E be the directions of vibration in these two rays, and let V be the difference in phase. If the vibrations were in the same direction they would interfere, being at right angles they produce in general elliptically polarised light. Let us consider one of them at a time, and take first the vibration along P O. This falls on the analysing Nicol, and is resolved into two, one of which is totally reflected. The emergent beam consists of vibrations in the principal plane of the Nicol; let this be parallel to P A. Then the effect of the Nicol on the vibration P O is to resolve it into two, one along P A and the other perpendicular to it, and to quench the latter. Similarly its effect on the vibration P E is to resolve it into two, in these same two directions along and perpendicular to P A, and quench the latter.

The disturbance, then, that emerges from the Nicol consists of the two vibrations parallel to P A, one being the resolved part of P O, the other of P E in the direction P A. But the vibrations along P O and P E differ in phase by the quantity V; so too, therefore, do their resolved parts. We have then to consider the resultant of two vibrations in the same direction, differing in phase; that is of two vibrations in a condition to interfere. Interference effects, therefore, are produced, depending on the value of V, the amplitudes of the two vibrations in directions O and E, and the position of the principal plane of the analyser. If we call k and k' the amplitudes of the two resolved disturbances in the direction P A, the intensity in the emergent light is, we know from our previous investigation, $k^2 + k'^2 + 2\,k\,k' \cos \frac{2\pi V}{\lambda}$.

If the light used be homogeneous, k and k' may be so related that no light will emerge from the analyser; in this case, of course, the difference of phase V will be an odd multiple of $\frac{\lambda}{2}$, and the amplitudes of the two resolved vibrations in the direction P A will be equal. If the analysing

Nicol be rotated, the light will pass through again. If white light be used, then, in any position of the Nicol, light of definite wave-lengths, that is of definite colours only, can be extinguished; the others will be transmitted, and the field of view will appear coloured. On turning the Nicol round, the tint of the field will change, and for two positions of the Nicol at right angles to each other the colours will be complementary.

Let us investigate a little more fully the effects when a plate of crystal, cut parallel to a principal plane, is placed between two Nicols.

Let PO and PE be the directions of the axes of the crystal plate, PL the direction of the vibration in the incident light, and PA the principal plane of the analysing Nicol, that is, the direction of vibration in the emergent light.

Then we have seen that if μ_1 μ_2 be the refractive indices of the plate for vibrations in directions PO and PE and ϵ its thickness, the difference in phase V between these two vibrations is $\epsilon(\mu_1 - \mu_2)$.

Let $OPL = \alpha$, $OPA = \beta$, and let ρ be the amplitude of the vibration in the incident light in direction PL; this vibration is resolved, on entering the crystal, into $\rho \cos \alpha$ along PO, and $\rho \sin \alpha$ along PE; and if we denote the emergent vibration along PO by $\rho \cos \alpha \sin \frac{2\pi}{\lambda}(vt - x)$, that along PE will be $\rho \sin \alpha \sin \frac{2\pi}{\lambda}(vt - x + \text{V})$.

Each of these is resolved by the action of the analyser along PA, and we get in this direction

$$\rho \cos \alpha \cos \beta \sin \frac{2\pi}{\lambda}(vt - x);$$

and

$$\rho \sin \alpha \sin \beta \sin \frac{2\pi}{\lambda}(vt - x + \text{V}).$$

The intensity of the light in the emergent beam is therefore, $\rho^2 (\cos^2 \alpha \cos^2 \beta + \sin^2 \alpha \sin^2 \beta +$

$2 \sin \alpha \sin \beta \cos \alpha \cos \beta \cos \frac{2 \pi v}{\lambda}\Big)$. Calling this I, we may show by adding and subtracting $2 \sin \alpha \sin \beta \cos \alpha \cos \beta$, that

$$I = \rho^2 \left\{ \cos^2 (\alpha - \beta) - \sin 2 \alpha \sin 2 \beta \sin^2 \frac{\pi v}{\lambda} \right\}$$

$\alpha - \beta$ is the angle between the directions of vibrations in the polariser and analyser respectively, or, which comes to the same thing, the angle between the planes of polarisation and analysation, α and β, are the angles between these planes and one of the axes of the crystal.

Let us suppose, first that α is equal to β, or that the principal planes of the polarising and analysing Nicols are parallel, then $I = \rho^2 \left\{ 1 - \sin^2 2 \alpha \sin^2 \frac{\pi v}{\lambda} \right\}$.

If $\alpha = 0$ or $90°$, so that the principal plane of the Nicol is also a principal plane in the crystal plate, $I = \rho^2$. All the incident light gets through and the plate produces no effect. For a plate of given thickness and homogeneous light, I is least when $\sin^2 2 \alpha = 1$, or when $\alpha = \frac{\pi}{4}$ or $\frac{3\pi}{4}$, that is to say, when the principal plane of the Nicol bisects either of the angles between the principal planes of the plate. For any given position of the plate, I is greatest when $\sin^2 \frac{\pi v}{\lambda} = 0$, or when $v = n \lambda$; and is least when

$$\sin^2 \frac{\pi v}{\lambda} = 1, \text{ or } v = \frac{2n+1}{2} \lambda.$$

The greatest value of I is ρ^2, and the least is $\rho^2 (1 - \sin^2 2\alpha)$. If, then, the thickness of the plate be such that

$$v = \frac{2n+1}{2}\lambda, \text{ and } \alpha = \frac{\pi}{4} \text{ or } \frac{3\pi}{4}, \text{ \&c.,}$$

no light of that wave-length is transmitted. This is the

case whenever $\varepsilon(\mu_1 - \mu_2) = \frac{2n+1}{2}\lambda$. If, however,

$$\varepsilon(\mu_1 - \mu_2) = n\lambda,$$

the plate is without effect.

If, again, $\alpha - \beta = \frac{\pi}{2}$, so that the Nicols are crossed, that is, have their principal planes at right angles,

$$I = \rho^2 \sin^2 2\alpha \sin^2 \frac{\pi V}{\lambda}.$$

If we call this I_2, and if I_1 be the intensity in the first position, we see that $I_1 + I_2 = \rho^2$, so that the appearances now presented are complementary to those described above.

The intensity is zero when $\alpha = 0$ or $\frac{\pi}{2}$, that is when the principal planes of the plate are parallel to those of the Nicol. It is zero, too, for all values of α if $V = n\lambda$, and in these two cases, as before, the plate is without effect; if, however, $V = \frac{2n+1}{2}\lambda$, I_2 has a maximum value for light of that wave-length.

In the general case denote $\alpha - \beta$ by γ, then γ is the angle between the principal planes of the two Nicols, and $I = \rho^2 \left\{\cos^2\gamma - 2\sin 2\alpha \sin 2(\alpha - \gamma)\sin^2\frac{\pi V}{\lambda}\right\}$. If we turn the analyser through 90°, we increase γ by 90°, and we get as the value of I

$I' = \rho^2\left\{\sin^2\gamma + 2\sin 2\alpha \sin 2(\alpha - \gamma)\sin^2\frac{\pi V}{\lambda}\right\}$; and $I + I' = \rho^2$.

Thus the effects in the two positions are complementary. For a given value of γ, and a given thickness of plate, the intensity is greatest when $2\alpha = 0$, or $2(\alpha - \gamma) = 0$, that is when the principal planes of the plate are parallel or perpendicular to the principal planes of the polariser or analyser. If, then, the plate be turned round its own normal there

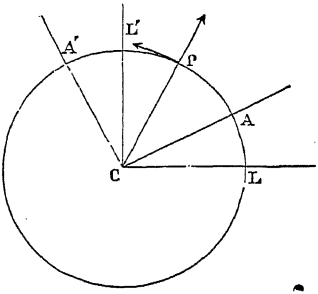
will be four positions of maximum illumination, and of course four positions of minimum at right angles to these. For plates of various thickness, the intensity is a maximum where $\sin^2 \frac{\pi v}{\lambda} = 0$, or $v = n\lambda$; and a minimum when $v = \frac{2n+1}{2}\lambda$.

Similar results exactly hold when a plane wave traverses obliquely a plate of the crystal, only the value of v is no longer given by $\epsilon(\mu_1 - \mu_2)$, but by the more complicated formula of page 364, and depends on the direction of the wave normals within the crystal with reference to the crystallographic axes.

If, instead of a parallel beam of rays, we use a conical pencil, the angle of incidence and the direction of the light at different points on our plate are different, and v varies, therefore, from point to point, so also do α and β. If, however, at each point of the crystal, v, α and β have the meanings attached to them above, the expression in the formula gives the intensity of the emergent beam at that point.

This intensity, of course, varies from point to point. A curve drawn on the surface of the crystal such that the intensity is the same at all points lying on the curve, is called an isochromatic line. Let us suppose our plate is a section of a uniaxal crystal by planes perpendicular to the axis; let the plane of the paper be the surface of emergence, and let the axis of the conical pencil cut the plate in C (fig. 163). We suppose that this axis passes normally through the plate and coincides, therefore, with the optic axis; let P be any point in the plate, let C L be parallel to

FIG. 163.



the principal plane of the polariser, C A to that of the analyser. A plane, perpendicular to the paper, passing through C P, is the principal plane of the plate at P. A vibration, parallel to C L, falling on any point of the under side of the plate, is in general resolved by it into two, an ordinary and extraordinary. Consider those which emerge from P; the vibration in the ordinary ray is perpendicular to C P, that in the extraordinary ray is along C P. These two differ in phase by a quantity V, and falling on the analyser, are resolved by it in the direction C A and interfere. For given positions of the planes of polarisation and analysation, the intensity of the resultant disturbances depends on the value of V, and this again on the angle of incidence and direction of the refracted wave normals in the spar with reference to the optic axis.

Again, let C L′ be perpendicular to C L, and C A′ perpendicular to C A. If P be on C L, the vibration at P, incident on the crystal plate, is in the direction C L; that is, in the principal plane at P. This passes unaltered through the plate as an extraordinary wave; and is resolved by the analyser into two components along and perpendicular to C A. The first of these alone can pass through the analyser; thus all points in C L appear equally bright, and the brightness depends on the angle L C A. If this angle be zero, so that the principal planes of the polariser and analyser coincide, all the incident disturbance at any point of C L traverses the analyser. If the angle be a right angle, the disturbance at all points in C L is at right angles to the principal plane of the analyser and C L appears as a dark line across the field.

Similar effects hold for all points in C L′. The incident disturbance passes unaltered through it as an ordinary wave, and is resolved by the analyser into two components along and perpendicular to C A, the first of which alone passes the analyser. Thus C L and C L′ appear as two bands crossing the field—bright, if the polariser and analyser are parallel;

black, if they are crossed, and of intermediate intensity for other positions. The same kind of reasoning applies to points on C A and C A′. If P lies on C A, the incident vibration is resolved into two along and perpendicular to C A, and of these the former only gets through the analyser; while if P be on C A′, the resolved part of the incident vibration perpendicular to C A′ is alone transmitted. Thus C A and C A′ also appear as bands crossing the field. Hence when a thin plate of a uniaxal crystal, cut perpendicular to the axis, is viewed between a polariser and analyser in convergent light, a series of light and dark rings is seen. If the polariser and analyser be neither crossed or parallel, these rings are intersected by two brushes of uniform intensity, with their arms parallel and perpendicular to the principal planes of the polariser and analyser. If the polariser and analyser be parallel, there is one bright brush; if they be crossed, there is a dark brush.

We can find a mathematical expression for the intensity of the disturbance at any point P. Let P C L $= \theta$, and A C L $= \gamma$, and let ρ be the amplitude of the incident displacement—parallel to C L,—which emerges at P as an ordinary vibration perpendicular to C P and an extraordinary parallel to C P. The amplitudes of these will be $\rho \sin \theta$, and $\rho \cos \theta$, respectively; and if V be the retardation produced by the plate at P, the disturbances will be $\rho \sin \theta \sin \frac{2\pi}{\lambda}(vt - x)$, and $\rho \cos \theta \sin \frac{2\pi}{\lambda}(vt - x + \text{V})$.

Each of these is resolved by the analyser along and perpendicular to C A, and the first component alone gets through. Thus the displacement in the emergent light is

$$\rho \sin \theta \sin (\theta - \gamma) \sin \frac{2\pi}{\lambda}(vt - x).$$

$$+ \rho \cos \theta \cos (\theta - \gamma) \sin \frac{2\pi}{\lambda}(vt - x + \text{V}),$$

and the intensity is

$$I = \rho^2 \left\{ \cos^2 \gamma - \sin 2\theta \sin 2(\theta - \gamma) \sin^2 \frac{\pi V}{\lambda} \right\}.$$

Thus if $\theta = 0$, or $90°$; or $\theta - \gamma = 0$, or $90°$, the intensity is constant and equal to $\rho^2 \cos^2 \gamma$. Now $\theta = 0$, for all points on CL, and $\theta = 90°$ for all points on CL′; $(\theta - \gamma) = 0$ along CA, and $(\theta - \gamma) = 90°$ along CA′. Thus the intensity is the same along CL, CL′, CA, and CA′, and we have two rectangular brushes along which the intensity is $\rho^2 \cos^2 \gamma$ intersecting the field. If $\gamma = 0$ or the polariser and analyser are parallel, the intensity is that of the incident light; if $\gamma = 90°$, or they are crossed, the brushes are black.

For points between these brushes, if $\sin 2\theta \sin 2(\theta - \gamma)$ is positive, the intensity is greatest when $\sin^2 \frac{\pi V}{\lambda} = 0$, or when $V = n\lambda$, and least when $\sin^2 \frac{\pi V}{\lambda} = 1$, or $V = \frac{2n+1}{2}\lambda$.

If $\sin 2\theta \sin 2(\theta - \gamma)$ is negative, the intensity is least when $V = n\lambda$, and greatest when $V = \frac{2n+1}{2}\lambda$.

It remains then to determine V. Let a and c be the principal velocities in the crystal, let ϕ be the angle of incidence, ϕ', ϕ'' be those of refraction, v, v', v'', the corresponding velocities, and ϵ the thickness of the plate. Then the crystal being uniaxal $v' = a$, and since ϕ'' is the angle between the optic axis and the wave normal for which the velocity is v'',

$$v''^2 = a^2 \cos^2 \phi'' + c^2 \sin^2 \phi'',$$

$$\text{also } v''^2 = \frac{v^2 \sin^2 \phi''}{\sin^2 \phi}.$$

$$\therefore \sin^2 \phi'' \left\{ \frac{v^2}{\sin^2 \phi} + (a^2 - c^2) \right\} = a^2.$$

$$\sin^2 \phi'' = \frac{a^2 \sin^2 \phi}{v^2 + (a^2 - c^2) \sin^2 \phi},$$

$$\cos^2 \phi'' = \frac{v^2 - c^2 \sin^2 \phi}{v^2 + (a^2 - c^2) \sin^2 \phi}.$$

$$\therefore \cot \phi'' = \frac{\sqrt{(v^2 - c^2 \sin^2 \phi)}}{a \sin \phi},$$

ilarly,

$$\cot \phi' = \frac{\sqrt{(v^2 - a^2 \sin^2 \phi)}}{a \sin \phi},$$

But we have seen that $\text{v} = \varepsilon \sin \phi \{\cot \phi' - \cot \phi''\}$, erefore

$$\text{v} = \frac{\varepsilon}{a} \left\{ \sqrt{(v^2 - a^2 \sin^2 \phi)} - \sqrt{(v^2 - c^2 \sin^2 \phi)} \right\}.$$

Now let us suppose that ϕ is so small that we may lect $\sin^4 \phi$ and higher powers, then expanding the square ts, we get

$$\text{v} = \frac{\varepsilon}{2\,a\,v} (c^2 - a^2) \sin^2 \phi.$$

This, then, expresses the retardation in the case before Again, let r be the distance C P, and let d be the distance from the vertex of the conical pencil, then

$$\frac{r}{d} = \tan \phi = \sin \phi = \phi,$$

is so small that we may neglect ϕ^3 and higher powers.

$$\text{Thus v} = \frac{\varepsilon}{2\,a\,v} \; \frac{(c^2 - a^2) r^2}{d^2}.$$

Now let us suppose that the polariser and analyser are llel then $\gamma = 0$, and

$$\text{I} = \rho^2 \left\{ 1 - \sin^2 2\,\theta \sin^2 \frac{\pi \text{ V}}{\lambda} \right\}.$$

rings are brightest when $\text{v} = n \lambda$, or when

$$r = d \sqrt{\left\{ \frac{2\,a\,v\,\lambda}{\varepsilon (c^2 - a^2)} \right\}},$$

the intensity then is that of the incident light.

Thus the radii of the rings of maximum brightness vary as $\sqrt{\lambda}$ (if we treat $\frac{2av}{c^2 - a^2}$ as independent of λ, which is not strictly the case).

For a given value of θ the intensity is least when $v = \frac{2n+1}{2}\lambda$, so that points of maximum darkness on any line drawn through C are given by

$$r = d\sqrt{\left\{\frac{(2n+1)av\lambda}{\epsilon(c^2 - a^2)}\right\}},$$

the intensity at these points is $\rho^2(1 - \sin^2 2\theta)$, and as this is not constant for all values of θ, the rings of equal darkness are not circles, but are the curves given by

$$\epsilon\left\{1 - \sin^2 2\theta \sin^2\left(\frac{\pi\epsilon}{2av\lambda}(c^2 - a^2)\frac{r^2}{d^2}\right)\right\} = \text{a constant.}$$

Thus when the polariser and analyser are parallel, the curves of maximum brightness are a series of circles whose radii are proportional to the square roots of the even numbers.

The curves of uniform intensity lie between consecutive circles, but are not circles themselves. Between each two consecutive circles there are four points of maximum blackness, and the field is intersected by a white brush parallel and perpendicular to the plane of polarisation. The radii of the bright circles depend on λ, so that if white light be used, the field of view will appear to consist of a series of coloured rings, and after a time, owing to the superposition of the rings, all trace of colour will be lost. The radii of the rings vary inversely as the square root of the thickness of the plate. The rings, therefore, will be further apart in a thin plate than in a thick one.

If the Nicols had been crossed, we should have had $\gamma = 90°$, and $I = \rho^2 \sin^2 2\theta \sin^2 \frac{\pi V}{\lambda}$. The intensity is zero when $\theta = 0$ or $90°$, and we get a dark brush, it is also zero

when $\sin^2 \frac{\pi V}{\lambda} = 0$, or $V = n\lambda$, that is, for points given by the equation

$$r = d\sqrt{\left\{\frac{2 n a v \lambda}{\varepsilon(c - a^2)}\right\}}.$$

Thus there are a series of black circles, the squares of whose radii vary directly as the even numbers and inversely as the thickness of the plate.

For a given value of θ, that is for points lying on any straight line through C, the intensity is greatest when

$$\sin^2 \frac{\pi V}{\lambda} = 1, \text{ or } V = \frac{2n+1}{2}\lambda,$$

$$\text{or } r = d\sqrt{\left\{\frac{(2n+1)\, a v \lambda}{\varepsilon(c^2 - a^2)}\right\}}.$$

The isochromatic rings are not circles but the curves given by

$$\sin^2 2\theta \sin^2 \left\{\frac{\pi\varepsilon}{2 a v \lambda}(c^2 - a^2)\frac{r^2}{d^2}\right\} = \text{a constant.}$$

If γ be neither zero nor 90°, we have

$$I = \rho^2\left\{\cos^2\gamma - \sin 2\theta \sin 2(\theta - \gamma)\sin^2\frac{\pi V}{\lambda}\right\}.$$

Describe a circle (fig. 163), with C as centre, and let C A, &c., cut it in L, A, L′, A′. If P be between L and A, θ is positive, and less than 90°; $\theta - \gamma$ is negative,

$$\therefore \sin 2\theta \sin 2(\theta - \gamma) \text{ is negative,}$$

and the intensity is least when $\sin\frac{\pi V}{\lambda} = 0$, or when

$$r = d\sqrt{\frac{2 n a v \lambda}{\varepsilon(c^2 - a^2)}}.$$

If P be between A and L′, θ is positive and less than 90°, so also is $\theta - \gamma$.

Thus $\sin 2\theta \sin 2(\theta - \gamma)$ is positive, and the intensity is greatest when $\sin \frac{\pi V}{\lambda} = 0$, or

$$r = d\sqrt{\left\{\frac{2 n a v \lambda}{\varepsilon(c^2 - a^2}\right\}}.$$

If P is between L′ and A′, $\sin 2\theta$ is negative, $\sin 2(\theta - \gamma)$ is positive. Thus $\sin 2\theta \sin 2(\theta - \gamma)$ is negative, and

$$r = d\sqrt{\left\{\frac{2 n a v \lambda}{\varepsilon(c^2 - a^2)}\right\}}$$

gives us circles of minimum intensity. In both these cases the intensity is $\rho^2 \cos^2 \gamma$, or the same as that of the two brushes. Thus we have two brushes and a series of circles of uniform intensity; between these circles we have in the four portions of the field such as A C L, brighter spaces and in the four corresponding to A C L′ darker spaces.

We can investigate in a similar manner the rings and brushes seen when the crystal plate is cut in any direction instead of being perpendicular to the axis.

We can show that if the plate be cut parallel to the axis, the isochromatic lines are hyperbolas. If, again, we take two plates of the same thickness cut in the same manner from a uniaxal crystal, and superpose them so that the principal planes are at right angles, the isochromatic lines are straight lines bisecting the angle between the principal planes.

For biaxal crystals the problem is considerably more complicated.

The formula

$$I = \rho^2\left(\cos^2 \gamma - \sin 2\alpha \sin 2(\alpha - \gamma) \sin^2 \frac{\pi V}{\lambda}\right)$$

still holds, γ being the angle between the planes of polarisation and analysation, and α the angle between the direction of vibration of the incident wave and of one of the possible directions of vibration at P, but the expression for V becomes very complicated.

Let us suppose that the plate of crystal is cut at right angles to that axis which bisects the acute angle between the optic axes, and that the optic axes are close together; this is the case with a plate of aragonite cut at right angles to the least axis of elasticity; and let θ, θ' be the angles between the optic axes and the wave normal in the crystal. Then we may show that the difference of phase v is proportional to $\sin\theta \sin\theta'$, and the isochromatic lines are given by $\sin\theta \sin\theta' =$ a constant; also if r and r' be the distances between the point P and the extremities of the optic axes O, O′, $\sin\theta$ is proportional to r approximately, $\sin\theta'$ to r', so that the lines are given by $rr' =$ a constant. These curves are called lemniscates, and are represented in fig. 164. The curves are crossed by two dark brushes. We have seen that the plane

FIG. 164.

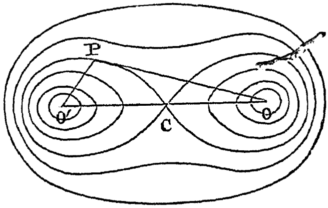
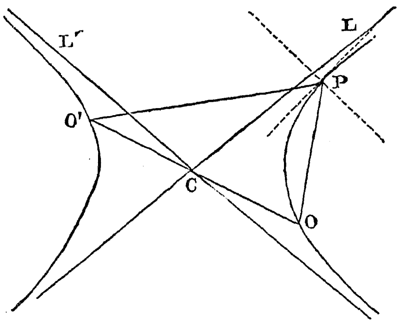


FIG. 165.



of polarisation at P bisects the angle O P O′. Now clearly the light is of the same intensity for all points at which the plane of polarisation in the crystal is parallel or perpendicular to the planes of polarisation or analysation. Let C L (fig. 165) be

e direction of vibration of the incident light. Then if the sector of the angle O P O′ is parallel or perpendicular to C L, is a point on one of these brushes; the brush itself is the cus of points such that the bisector of the angle O P O′ is rallel to a fixed line. Consider now a rectangular hyperola passing through O, P and O′ with C for centre and C L r an asymptote; O C O′ is a diameter and P is on the curve, erefore P O, P O′ are parallel to conjugate diameters, and njugate diameters are equally inclined to the asymptotes. nce the bisector of the angle O P O′ is parallel to C L, and e plane of polarisation at P is parallel or perpendicular to e original plane of polarisation, so that the hyperbola thus escribed is a dark or light brush. Similarly we have anher brush of uniform intensity in the form of an hyperbola rough O and O′, with its asymptotes parallel and perpencular to the plane of analysation. If the polariser and nalyser are parallel, the brushes are of the same intensity as e incident light; if they be crossed, the brushes are black.

Moreover, if O O′ be parallel or perpendicular to the ane of polarisation or analysation, the corresponding yperbolas become two straight lines along and perpenicular to O O′.

If white light be used and the position of the optic axes f the crystal for differently coloured rays be not very ifferent, the brushes retain nearly the same position for all olours; the lemniscates, however, of maximum brightness re different in size for different colours, and so we get these rushes of uniform intensity crossing a series of coloured urves, more or less resembling elongated figures of eight.

For many biaxal crystals the optic axes for different olours are widely separated, and the appearances are in onsequence considerably modified.

If, again, the optic axes form with the normal to the late angles which are nearly 90°, the isochromatic curves nay be shown to be nearly hyperbolas.

Again, let us return to the plate of uniaxal crystal cut at

right angles to the axis, and suppose that the incident light is circularly polarised. We may resolve it at each point into two vibrations of equal intensity in any two directions at right angles, and these vibrations will differ in phase by a quarter period. Thus at any point P (fig. 165) we have incident on the crystal a vibration $\rho \sin \frac{2\pi}{\lambda}(vt - x)$ perpendicular to C P, and $\rho \cos \frac{2\pi}{\lambda}(vt - x)$ along C P.

On emergence, these may be represented by

$$\rho \sin \frac{2\pi}{\lambda}(vt - x), \text{ and } \rho \cos \frac{2\pi}{\lambda}(vt - x + \text{V}),$$

Let β be the angle between C P and C A, the principal plane of the analyser; these two vibrations are resolved by the analyser each into two along and perpendicular to C A, and the first components only emerge.

Thus the disturbance in the emergent wave is

$$\rho \sin \beta \sin \frac{2\pi}{\lambda}(vt - x) + \rho \cos \beta \cos \frac{2\pi}{\lambda}(vt - x + \text{V}),$$

and compounding these as before, we have for the intensity

$$\rho^2\left(1 - \sin 2\beta \sin \frac{2\pi \text{V}}{\lambda}\right).$$

The intensity of the incident light is $2\rho^2$. V has the value already found, so that

$$\text{V} = \frac{\epsilon}{2av}(c^2 - a^2)\frac{r^2}{d^2}.$$

If $\beta = 0$, or $90°$, the intensity is ρ^2—half that in the incident light—thus there are two brushes along and perpendicular to the principal plane of the analyser. If $\sin 2\beta$ is positive, that is, if β is between 0 and $90°$, or $180°$ and $270°$, the intensity is a maximum when $\sin \frac{2\pi \text{V}}{\lambda} = 0$, that is if $\text{V} = \frac{n\lambda}{2}$; and it is then equal to ρ^2.

sin 2β is negative, that is, in the other two quadrants, value of v gives us curves of minimum intensity also to ρ^2. Thus we have a rectangular cross and a series cles whose radii are given by

$$r = d\sqrt{\left\{\frac{a v n \lambda}{\varepsilon(c^2 - a^2)}\right\}},$$

the same intensity ρ^2. In one pair of opposite quad- the spaces between these circles are brighter than the es, in the other pair they are darker. As we rotate the ser the appearances rotate with it. The same effects be obtained by analysing the light circularly instead of ising it so. To obtain the circularly polarised light we ld use either a quarter undulation plate with its axes ned at 45° to the plane of the polariser, or a Fresnel's b.

t remains now to explain why the rings and brushes are formed when ordinary light is viewed through a Nicol it has traversed a plate of crystal. We may divide the ary light into two plane polarised rays of equal inten- polarised in planes at right angles. Each of these rays ately will give rise to rings and brushes, but the system g from the one is complementary to that produced by ther, so that when the two are superposed the colour ts are destroyed.

he forms of the rings and brushes seen when thin s of crystal are examined under the microscope between Nicols are of great assistance to the mineralogist in mining the characters of the crystal.

he isochromatic curves formed when polarised light is ed through a plate of crystal and analysed, form a most ate test of the presence of polarisation in a beam of

avart's polariscope is an apparatus adapted for this. A plate from a uniaxal crystal is cut somewhat obliquely

to the axis. This is cut into two halves, and these are superposed so as to form a plate of double the thickness. One of the two is then turned through 90°, so that the principal planes are at right angles, and the whole is mounted in a small tube before a Nicol's prism ; if a convergent beam of plane polarised light fall on the plate, the isochromatic lines are, we have said, a series of straight lines. If, then, a beam of light coming from any object be polarised, or partially polarised, the object when viewed through the Nicol and plate appears crossed with coloured straight lines. The polarisation of light reflected from the sky is easily detected by means of the instrument. When the bands are seen most distinctly, they are inclined at 45° to the plane of polarisation.

The fringes formed by passing plane polarised light through two plates of quartz cut in a certain manner were used by Jamin in his investigations on elliptically polarised light. The apparatus is known as Jamin's or Babinet's compensator.

Two wedges or prisms of the same small angle are cut from a plate of quartz, one face in each prism being parallel to the optic axis. In one prism the optic axis is parallel to the edge, in the other it is at right angles to the edge. The two are put together so as to form a plate, the angles of the wedges being turned in opposite directions, while the optic axis in both crystals is parallel to the faces of the plate.

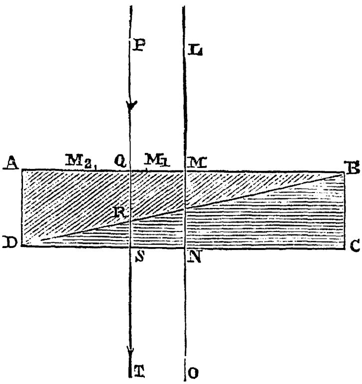
Let the figure A B C D (fig. 166) represent a section of the plate by a plane at right angles to the edge of each prism, A B D being one prism, B D C the other. In the first, the optic axis is parallel to A B. In the second it is at right angles to the plane of the paper.

Let us suppose further for simplicity that the plane of the paper is horizontal.

Consider now a ray P Q R S T falling normally on the

plate at Q, and suppose that the ray is polarised in a plane at 45° to the paper, so that the vibrations in it are also at 45° to the paper at Q; this disturbance is resolved into two of equal amplitude by the crystal. One of these, the ordinary ray, vibrates at right angles to the plane of the paper, while the other vibrates in that plane. The two traverse the distance Q R with different velocities, and a difference of phase which is proportional to Q R is established. In quartz the velocity of the ordinary ray is the greater, so that the vibration in the plane of the paper is retarded behind the other by a quantity proportional to Q R. Let us call this quantity e. On entering the second prism in which the axis is at right angles to the paper, this vibration becomes an ordinary one and travels through the remaining distance R S with greater velocity than the other, it is therefore accelerated in phase compared with that other by a quantity proportional to R S, let this be e'.

FIG. 166.



Then the total retardation of the disturbance in the plane of the paper behind the disturbance at right angles to the paper is $e - e'$, and this is proportional to QR − RS. Thus if we have a plane wave of light incident normally on the compensator polarised in a plane at 45° to the axes of the quartz, the emergent beam consists of two plane polarised beams, polarised in planes parallel to the axes of the quartz, of equal intensity, but differing in phase by a quantity $e - e'$ which is different for different parts of the compensator.

Thus, if L M N O is a line at right angles to the plate bisecting A B, e is equal to e', for the point M and the difference in phase is zero, the emergent light is polarised in the same plane as the incident. As we move to the left from M the difference $e - e'$ increases and the emergent light is elliptically polarised; after travelling a certain distance, a suppose, depending on the angle of the quartz wedges, the difference of phase has become equal to $\frac{\lambda}{2}$, and the emergent beam is plane polarised at right angles to the incident beam.

Let M_1 be this point. If M_2 be another point such that $M_1 M_2 = M M_1$, the difference of phase at M_2, since it is proportional to the difference in the thickness of the two quartz plates, is clearly twice that at M_1. Thus at M_2 the difference of phase is λ, and the light is plane polarised parallel to the incident beam.

Thus, in general, the light at any point Q is elliptically polarised after passing the plate, and the difference of phase of the two components is proportional to the difference of thickness at Q, and this again is proportional to the distance M Q, while for a series of points at distances 0, a, $2a$, $3a$, &c., on either side of M the polarisation is plane, the plane of polarisation being alternately parallel and perpendicular to that of the incident light. Thus, if we view the emergent beam through a Nicol with its principal plane parallel to the plane of polarisation of the incident light, the vibrations at M, M_2, M_4, &c., will be perpendicular to the principal plane of the Nicol and will be extinguished by it; at all other points of the field more or less light will get through; so that if the light be homogeneous the field will appear to be crossed by a series of dark vertical bars corresponding to the points M, M_2, M_4, &c. If we wish to determine the difference of phase between the vertical and horizontal components in the emergent light at any other point, we can place a fine fibre to coincide with the central black band, the fibre being attached

to a micrometer screw by means of which it can be moved parallel to itself across the field. Move the fibre until it coincides with the next dark band and measure the distance through which it has been displaced; this will be $2a$, and corresponds, as we have seen, to a difference of phase of λ. If now we have to move the fibre a distance x from its original position to bring it over the point at which the difference of phase is required, the difference of phase will be to λ in the ratio of x to $2a$. Thus the difference required is $\frac{\lambda x}{2a}$.

In M. Jamin's apparatus the fibre is fixed, but one of the wedges can move parallel to its face, and the distance through which it is displaced can be measured by a micrometer screw. Adjust the screw until the central band appears under the fibre. The difference of phase there is zero, the thicknesses of the two plates being equal. On moving the screw the second plate slides parallel to itself over the first, and the central band moves from under the fibre, while the next band moves towards it. The difference of phase in this next band is λ. Let us suppose that in order to bring it under the fibre we have to displace the plate a distance c. Then a difference of phase of λ corresponds to a displacement c, so that if we have to displace the plate a distance x to bring any given point under the central fibre the difference of phase at that point is $\frac{\lambda x}{c}$. Clearly c is four times as great as a, for in the figure, as the fibre moves, the thickness of the one plate increases and that of the other decreases at the same rate. If, however, one plate be displaced over the other, the thickness of the first under the fibre remains the same, so that to produce the same difference of thickness that exists at M_2 under the fibre, we must move the plate through $2MM_2$, or through $4a$; thus $c = 4a$. If the plane of polarisation of the incident light be inclined at an angle α to the horizon, the planes

of polarisation at the points M, M_1, &c., will not be at right angles. The direction of vibration at M_1 will be equally inclined to the vertical with that at M, but on opposite sides of it; the planes of polarisation at M and M_1 will be inclined at an angle 2α. To obtain blackness, then, in the bands, the principal planes of the polarising and analysing Nicols must not be crossed, but inclined at an angle 2α.

If we suppose that the incident light is elliptically polarised, we can resolve it at each point of the compensator into two components polarised parallel to the axes of the quartz and differing in phase by the same quantity v suppose. Let us suppose that v is the retardation of the component in the horizontal plane behind the other. Then in addition to the difference of phase $e - e'$ produced by the compensator we have this difference v. The total difference of phase at each point is $v + e - e'$, and under the central dark band $v + e - e'$ must be zero.

Suppose the apparatus is adjusted so that when plane polarised light is incident, the difference of phase under the fibre is zero, that is, so that the central band coincides with the fibre. When the light becomes elliptically polarised the central band will be displaced and the difference of phase beneath the fibre will be v. Move the screw until the central band is again brought under the fibre and let x be the displacement. The difference of phase under the fibre has been reduced from v to zero by a displacement x, and a displacement c changes this difference from zero to λ. Thus $v = -\frac{\lambda x}{c}$. The signs of x and c are, of course, the same, if the displacements take place in the same direction, and in this case v is negative, for the displacement x alters the difference of phase from v to zero, while c changes it from zero to λ. Instead of a single fibre we may use two close together and adjust the micrometer screw until the central band is exactly between them.

Since the distance between the dark bands depends on the wave-length, if white light be used, as was the case generally in M. Jamin's experiments, the field is crossed by a series of rainbow coloured bands, the central band alone is perfectly black.

It is difficult to get a homogeneous light of sufficient intensity to work with. I have, however, found that excellent results may be obtained by blowing a fine jet of oxygen into the flame of a Bunsen burner in which a bead of some sodium salt is placed.

The compensator will also enable us to determine the position of the axes of the elliptic vibrations. For we know that the components of the vibrations along the axes differ in phase by $\frac{\lambda}{4}$. Set the compensator, so that the central band is under the fibre when plane polarised light falls on it, and then displace the plate through a distance $-\frac{c}{4}$. If elliptically polarised light falls on the compensator and brings the band back under the fibre, we know that the difference in phase between the components of the elliptically polarised light parallel to the axes of the quartz plates must be $\frac{\lambda}{4}$, so that the axes of the quartz plates will then be parallel to the axes of the ellipse. Allow, therefore, the elliptically polarised light to fall on the compensator. The black central band will generally not be under the fibre, but on turning the compensator round an axis parallel to the direction of the incident light, the band moves across the field, and for certain positions of the compensator can be brought under the fibre. When this is so, the axes of the elliptic vibration are parallel to those of the quartz plate, for the difference of phase between the components in these directions is $\frac{\lambda}{4}$.

Perhaps, however, the simplest mode of analysing elliptically

polarised light is to reduce it by means of a quarter undulation plate to plane polarised. Let the elliptically polarised light fall normally on a quarter undulation plate, and turn the plate round an axis perpendicular to its plane until the axes of the plate are parallel to those of the ellipse. The difference of phase between the components in the incident beam parallel to these axes is $\frac{\lambda}{4}$, to this is added the difference $\pm \frac{\lambda}{4}$ produced by the plate. The light emerging from the plate consists, therefore, of two components differing in phase either by zero, or $\frac{\lambda}{2}$. It is plane polarised, and can be quenched by a Nicol's prism. If we do not know the position of the axes of the incident vibration, we must adjust first the plate and then the Nicol by trial until there is no emergent beam.

CHAPTER XIV.

CIRCULAR POLARISATION.

FOR many purposes we require to know the position of the plane of polarisation of a beam of light. We can do this most directly, though not with any great accuracy, by means of a Nicol, the position of whose principal plane we know, or a plate of tourmaline for which the axis has been determined.

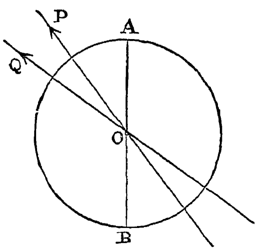
We have only to mount the Nicol or tourmaline in a graduated circle which can turn round an axis perpendicular to its plane, and note the position of the principal plane of the Nicol or of the axis of the tourmaline with reference to the zero of the graduations. Then turning the circle until no light passes the Nicol or the tourmaline, as the case may be, we know that the plane of polarisation of the incident light

is parallel to the principal plane of the Nicol, or the axis of the tourmaline.

But it is by no means easy to determine the exact position at which the light is quenched. For a considerable angle on either side of the required position so very little light passes the analyser that large errors may be made in the determinations. Various instruments have been devised to overcome this difficulty. One class of these depends on the fact that it is very much more easy to say if two similar objects placed side by side are equally illuminated, especially if the illumination is not very strong, than it is to decide when one of them is absolutely black.

Suppose now that the field of view is divided into two parts by a line A O B (fig. 167), and that in these two parts the planes of polarisation are inclined at a small angle, so that the direction of vibration in one is along O P, and the other along O Q. If we look at this through a Nicol's prism when the principal plane of the Nicol is perpendicular to O P, one half is black, and when it is perpendicular to O Q the other half is black. For a position intermediate between these, the two halves will appear equally illuminated, and then the principal plane of the Nicol will bisect the exterior angle P O Q.

FIG. 167.



Now let us suppose that we reverse the direction of the light and look at plane polarised light through some apparatus which produces a small difference P O Q in the planes of polarisation of the two halves of the beam which is transmitted through it.

Then if the principal plane of the Nicol be parallel to the exterior bisector of the angle P O Q, the two halves of the field are equally illuminated but not otherwise. If then we turn the apparatus round the direction of the light until the two

halves of the field are equally illuminated, we know that the plane of polarisation of the incident light bisects the angle P O Q.

This is the principle of Jellett's and Laurent's analysers. In the former a long rhomb of Iceland spar is taken, and the ends are sawn off at right angles to its length. Let A B C D (fig. 168) be the rhomb, and let A C be the longer diagonal of the end. If a wave of light falls normally on the end A B C D, the ordinary wave passes through undeviated and its plane of polarisation is perpendicular to A C, A C is parallel

FIG. 168.

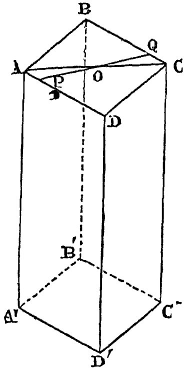
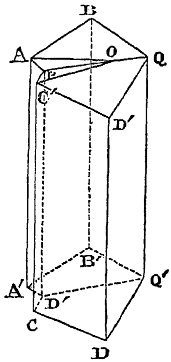


FIG. 169.



to the direction of vibration. On entering the rhomb the extraordinary pencil is bent to one side, and if the rhomb be of sufficient length and the breadth of the incident pencil not too great, the two are separated on emergence. Let P O Q be a line through O, the centre of the face A B C D, making a small angle α with A C, and suppose the rhomb be cut in two by a plane parallel to its length passing through P Q.

Let the common surface of the two portions thus formed be polished, and let one of them P D C Q be turned end for end, and then let the two halves be cemented together again. The end of the prism will then be as in fig. 169. This

is obtained from fig. 168, by supposing P D C Q to be rotated through two right angles about an axis perpendicular to P Q.

To use a mineralogical term the crystals P A B Q, P C′ D′ Q, are twinned about the plane P Q. The direction of vibration for an ordinary ray in the half P D′ Q is O C′, and in the half P A B Q it is O A, and these two are inclined to each other at the small angle 2α.

In the half P A B Q the extraordinary ray is thrown off in one direction, in the half P C′ D′ Q it is thrown in the other, and by placing a diaphragm with a circular hole in the centre between the analyser and the eye, the two extraordinary rays can be prevented from reaching the eye. If, then, we look through this analyser at a small aperture illuminated by a parallel beam of plane polarised light, so that one half of the aperture is seen through one half of the analyser, and the other through the other, for one position of the analyser one half of the field appears black, for a second inclined to the former at an angle 2α, the other half is black, and for a position midway between these the two halves are equally dark. In this position the plane of polarisation of the incident light is parallel to P Q.

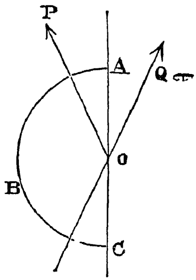
The essential portion of Laurent's analyser is a plate of quartz or gypsum cut parallel to its axis, and of such a thickness that the difference of phase it produces in the two waves which can traverse it is $\frac{\lambda}{2}$. This covers one half of the field of view, the other being occupied by a plate of glass or other transparent material of such a thickness as to absorb the rays of wave-length λ equally with the quartz. Let A B C (fig. 170) be the semicircular piece of quartz, and suppose A C is the axis.

Let light polarised at right angles to the direction O P fall on the quartz, and let O P make an angle α with A C.

The vibration in direction O P is resolved by the quartz into two respectively along and perpendicular to A C, and

of these the one in direction A C is retarded more than the other by the quartz, and a difference of phase of $\frac{\lambda}{2}$ is produced in the emergent pencil. It, therefore, is plane polarised, and the direction of vibration is inclined at an angle α to A C on the side remote from O P. If, then, O Q be drawn so that Q O A is equal to α, O Q is the direction of vibration in the pencil emerging from the quartz plate. The planes of polarisation in the two halves of the field are inclined to each other at an angle 2α. If we fix a Nicol before the plate, with its principal plane at an angle α to A C, and allowing a parallel pencil of plane polarised light to illuminate the plate, look at it through the Nicol, the two halves of the field will appear of the same intensity when the plane of polarisation of the incident beam is parallel to A C.

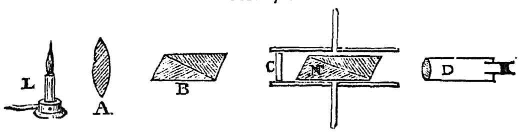
FIG. 170.



Since the thickness of a half wave plate depends on λ, Laurent's analyser can only be used for homogeneous light. Jellett's, on the other hand, can be used with the sun's rays.

Fig. 171 gives a general idea of the arrangement of the pieces. A is a convex lens arranged to produce parallel rays

FIG. 171.



from the source of light L which is placed at its focus. B the polarising Nicol. C is the analysing plate to which a second Nicol N is attached, or the Jellett analyser. This second Nicol is fixed to a graduated circle, which can rotate about an axis parallel to the incident light. D is a telescope

or eye-piece focused on the plate. In Laurent's instruments as made by him, a plate of bichromate of potash is placed between A and B to absorb all but the yellow rays of the source of light—a Bunsen burner with a sodium bead—and the plate of quartz is attached to B, the polarising Nicol, instead of to A the analysing; the theory is the same as that described above. There is also an arrangement by which the angle α can be varied, thus altering the sensitiveness of the apparatus. Jellett's apparatus can be used equally well as a polariser, and in many cases this is the most convenient arrangement. A still simpler application of the principle of these two instruments has been suggested lately by Professor Poynting, of Birmingham, but as it depends on the phenomena presented when plane polarised light is allowed to pass through a plate of quartz in a direction parallel to its axis, we must defer its description till these have been considered.

It is found that if plane polarised light is allowed to pass normally through a plate of quartz cut at right angles to its axis, the emergent beam is plane polarised; but its plane of polarisation does not coincide with that of the incident light. The passage through the quartz in this direction has rotated the plane of polarisation of the light.

This property is common to quartz and many other natural substances. Among crystals, it is shown by cinnabar and the chlorate, bromate, iodate and periodate of sodium, by hyposulphate of lead, and possibly also by some other hyposulphates, while solutions of sugar, morphia, and quinine, with the essences of turpentine, lemon, aniseed, lavender, and sassafras, also produce it. The rotation varies as the length of the substance through which the light passes; by doubling the length the rotation is doubled. It differs very considerably for different media in amount, and also in direction.

Let us suppose we are looking in the direction in which the light is travelling, then some media turn the plane of

polarisation in the direction in which the hands of a clock appear to move as we look at the face. The rotation is in this case said to be right-handed. The directions of rotation and wave propagation are related to each other in the same way as the directions of rotation and translation of an ordinary screw. In other media, the rotation is in the opposite direction, and is said to be left-handed. The rotation, moreover, depends on the wave-length of the light used, rays of different wave-length being turned through very different amounts.

The following table, taken from Verdet, gives the rotation of a decimètre of each of the substances, measured for the red rays transmitted by a glass coloured with oxide of copper. Right-handed rotation is considered positive, left-handed negative.

Essence of Turpentine	. . . .	$-29°\cdot6$
,,	Lemon	$+55°\cdot3$
,,	Aniseed	$-0°\cdot7$
,,	Bigarade	$+78°\cdot94$
,,	Carraway	$+65°\,79$
Solution of Sugar 50 %	. . . .	$+33°\,64$
,,	Quinine 6 % in alcohol . .	$-30°$

The rotation produced by a plate of quartz one millimètre thick for rays of about the same wave-length is 17° 15′; so that all the substances in the above list are much less powerful than quartz in producing this effect. The rotation, we have seen, is different for waves of different lengths. Experiment has shown that approximately, at any rate, the rotation varies inversely as the square of the wave-length. Thus, M. Broch gives the following table for the rotation produced by a plate of quartz, one millimètre thick, on the different rays. The third column gives the product of the rotation multiplied by λ^2, and since it is nearly constant, we infer that the rotation varies nearly inversely as λ^2.

Rays	Rotations	Rotation × λ^2
B	15°18′	7238
C	17°15′	7429
D	21°40′	7511
E	27°28′	7596
F	32°30′	7622
G	42°12′	7842

The products in the last column increase gradually, showing that the law is only approximate.

If a plane polarised ray of white light pass through such a plate, the planes of polarisation of the different coloured waves of which it is composed are very different. If, then, we examine the emergent beam with a Nicol, for any position of the Nicol within certain limits, light of one colour will be entirely prevented from passing through, while the others will be present in different proportions, and the quartz plate will appear coloured; the colour will alter very considerably as we turn the Nicol round. If the thickness of the quartz be not greater than five millimètres, for one position of the analysing Nicol the plate assumes a greyish violet colour, which is easily recognised. If the Nicol be turned a little from this position in one direction, the plate becomes red, and if it be rotated a little in the opposite direction, the plate appears blue. On passing through this position, the colour changes almost suddenly from red to blue or blue to red.

This peculiar tint is called the tint of passage, and it is found that when the quartz plate shows the tint of passage, the principal plane of the Nicol is parallel to the plane of polarisation of the yellow-green rays near the line E after they emerge from the quartz; these rays, therefore, are quenched by the Nicol.

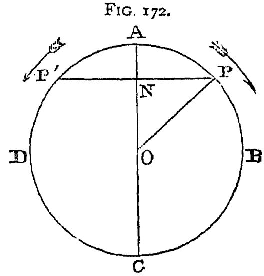
The simplest way to test what rays are absent from the light after it has passed the Nicol is to examine it by means of a spectroscope. A dark band, corresponding to the missing rays, will cross the spectrum.

A complete theoretical explanation of these various phenomena has not been given; we can, however, with advantage analyse them a little further.

We have explained the nature of circular polarisation, and have seen how circularly polarised light can be considered as the resultant of two plane polarised beams, differing in phase by $\frac{\lambda}{4}$, polarised in perpendicular planes. We shall now show that a plane-polarised ray can be considered as the resultant of two circularly polarised rays of the same intensity, in which the directions of vibration are opposite.

Let the circle, A B C D (fig. 172), represent the path of a particle in each of these rays, O being its centre; O is the position of rest of the particle. Consider the right-handed vibration first, and let P be the position after an interval t of a particle, which initially was at A. Draw P N perpendicular to A C, cutting it in N. Then, owing to this vibration, the ether particle is displaced from its position of rest a distance O N along C A and P N at right angles to C A; and if we produce P N to cut the circle again in P′, P′ will be the position of the particle in the other wave at the same moment. Its displacement parallel to C A is O N, and its displacement at right angles to C A is N P′, equal and opposite to P N. If, then, both these circular vibrations affect the particle, the displacements perpendicular to A C will destroy each other, and the whole displacement will take place along A C, and will be 2 O N. The particle will vibrate backwards and forwards along the line A C; the ray will be plane-polarised. If a be the radius of the circle, ω the angular velocity of the particle, and τ the time of a complete revolution,

FIG. 172.



$$\omega\,\tau = 2\,\pi \qquad \omega = \frac{2\,\pi}{\tau}$$

$$\text{Angle } A\,O\,P = \omega\,t = \frac{2\,\pi\,t}{\tau}$$

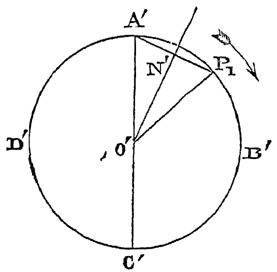
$$O\,N = a \cos A\,O\,P = a \cos \frac{2\,\pi\,t}{\tau},$$

and the displacement of the ether particle along A C is

$$2\,a \cos \frac{2\,\pi\,t}{\tau}.$$

Let us now suppose that the plane-polarised ray falling on the quartz is replaced in this way by two circularly polarised rays; and let us assume that the rate at which these two rays traverse the crystal is different, being less for the left-handed than for the right-handed ray; the particle P, as the light emerges from the quartz, will be retarded relatively to P′.

FIG. 173.



Let the thickness of the plate be such that the left-handed ray, which travels with the least velocity, just passes through in the periodic time τ, and consider a particle at the second face of the plate after an interval τ. Let the fig. 173 represent the displacements of this particle, O′ A′, O′ B′ being parallel to O A and O B respectively of fig. 172. Then, so far as the left-handed wave is concerned, the particle will be at A′.

Consider now the effect of the right-handed wave. We know that in any case of wave-motion the displacement of a particle at any epoch is the same as that which at a time x/v previously affected a particle at a distance x behind; x/v being the time taken by the wave to traverse with velocity v the distance x; let t'' be the time taken by the right-handed wave to traverse the plate. The displacement, then, at the second face, due to the right-handed wave,

will be the same as that at the first face at a time t'' before the epoch considered.

We can easily see what this time, t, is, for if λ' is the wave-length of the left-handed wave in the crystal, then, since the plate is traversed in time τ, its thickness is λ', so that $t = \frac{\lambda'}{v}$, and we require the displacement in the right-handed wave at a time λ'/v before the left-handed wave reaches the second surface, that is, at a time $\tau - \lambda'/v$ after the ether particle in either wave leaves A. Let P (fig. 172) be the position of the particle in the right-handed wave on the first face at this moment, and draw O′P$_1$ (fig. 173) parallel to O P; then P$_1$ is the position of the ether particle in the second face, at the epoch t, so far at least as it depends on the right-handed wave; and to find the actual displacement in this wave, we have to compound O′ A′ and O′ P$_1$; join A′ P$_1$, and draw O′ N′ to bisect it at right angles. The displacement O′ A′ is compounded of O′ N′ and A′ N′, O′ P$_1$ is compounded of O′ N′ and P$_1$N′; P$_1$ N′ and A′ N′ are equal and opposite, and the resulting displacement is twice O′ N′. Thus the emergent wave is plane polarised, and the plane of polarisation has been turned by the plate through the angle A′ O′N.′ Also,

$$A'O'N' = \frac{1}{2} AOP.$$

Now P (fig. 172) is the position of the ether particle in the right-handed wave at a time $\tau - \lambda'/v$. And if v' be the velocity of the left-hand wave, $\lambda' = v'/\tau$. Thus this time is $= \left(1 - \frac{v'}{v}\right)$. The angular velocity in the ether wave is $2\pi/\tau$. Therefore, the angle A O P $= \frac{2\pi(v - v')}{v}$, and a length λ' of the crystal rotates the plane of polarisation through $\pi \frac{(v - v')}{v}$. Hence a length c will produce a rotation of

$$\frac{\pi c(v - v')}{v\lambda'}$$

And if λ be the wave-length, v the velocity in air $\frac{\lambda'}{v'} = \frac{\lambda}{\mathrm{v}}$. Hence the rotation is $\frac{\pi c \mathrm{v}}{\lambda v v'}(v - v')$. Or, if μ, μ' be the refractive indices for the two circularly polarised waves, since $\mu = \frac{\mathrm{v}}{v}$, $\mu' = \frac{\mathrm{v}}{v'}$, the rotation $= \frac{\pi c}{\lambda}(\mu' - \mu)$.

The experimental fact that the rotation varies nearly inversely as λ^2, leads us to conclude that the difference $\mu' - \mu$ is inversely proportionate to λ.

We have thus been able to give a geometrical account of the action which we may suppose takes place when the plane of polarisation of light is rotated by a plate of any medium.

The amount of rotation produced by a given length of a substance is an important quantity, and we must consider, therefore, the various means we have of determining it. Jellett's prism and Laurent's analyser were constructed for this purpose. We first determine the position of the plane of polarisation of the light coming from the polariser; then introduce between the polariser and analyser a column of the substance we wish to examine, and again determine the position of the plane of polarisation of the light after passing through this column. If the substance be a liquid, it must be put in a tube, the ends of which are closed by pieces of plane parallel-sided glass.

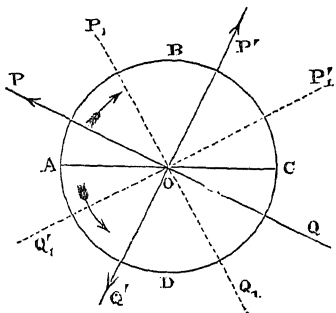
By using homogeneous light with the Jellett analyser, we can determine the rotation for light of a known wave-length.

The biquartz affords us another simple and fairly accurate method of making this determination. It is found that in some specimens of quartz the rotation is right-handed, and in others it is left. This difference is connected with the crystalline form. A biquartz consists of two semicircular plates of quartz, ABC, ADC (fig. 174), having a common diameter AC, one of which, ABC, is right-handed, while the other is left; the two plates are of the same thickness, and,

therefore, produce the same amount of rotation, though in opposite directions, in any given ray. If, now, plane polarised white light passes through the biquartz, the rays of difference refrangibilities are differently rotated ; and if the emergent light be analysed by a Nicol, the two halves of the biquartz will appear of different colours.

A position of the Nicol, however, can generally be found in which the colour is the same in the two. For, consider the wave which is turned through a right angle by the biquartz. If P O Q be the direction of vibration in the incident wave, and P′ O Q′ be at right angles to P O Q, then O P′ will be the direction of vibration in the wave emerging from A B C, and O Q′ in that which emerges from A D C; for A B C has turned the plane of polarisation in a right-handed direction through 90°, while A D C turns it through the same angle in a left-handed direction. Thus if the principal plane of the analyser be parallel to P O Q, light of the same wave-length is absent from the two halves of the field ; the other rays also are present in the same proportion in the two halves, so that they appear of the same colour.

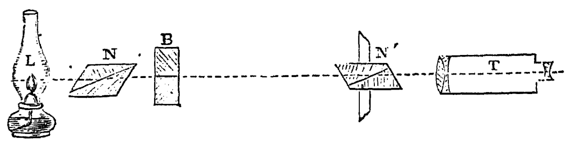
FIG. 174.



Now, if the thickness of the biquartz be about 3·3 mm., the rays which are turned through 90° are those in the neighbourhood of E, the brightest part of the spectrum ; and the common colour of the two halves is the tint of passage. A very slight rotation of the analyser in one direction renders the upper half red and the lower blue, while if the rotation be in the opposite direction, the upper half becomes blue, the lower red. Thus the position of the plane of polarisation of the light falling on the biquartz can be

determined with considerable exactness. If now we introduce a rotating medium, either between the polariser and biquartz, or between the biquartz and analyser, in the first case the direction of the ray which is turned through 90° by the biquartz will be in some position such as OP_1, and we shall have to turn the analyser until its principal plane coincides with OP_1 to restore uniformity of tint in the two halves, and the angle through which the analyser is turned gives us the rotation of the medium for light of that definite wave-length. When the rotating medium is placed between the biquartz and the analyser, the direction of vibration in both halves is turned from the position $P'OQ'$ to some such as $P_1'OQ_1'$, and to restore equality of tint, the polariser must be turned till its principal plane is perpendicular to $P_1'OQ_1'$.

FIG. 175.

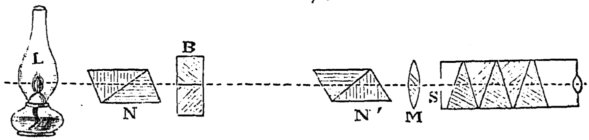


For success in practice, it is necessary that the biquartz should be cut perpendicularly to the axis of the quartz, and that the light should fall on the face of the quartz normally. To use the apparatus, we require a source of white light, as the sun or a good lamp L (fig. 175), a Nicol's prism, N, to polarise the light, the biquartz B, a second Nicol, N′, mounted in a graduated circle so that we can measure the angle through which the Nicol is turned, and a lens or small telescope, T, focused so as to see the biquartz distinctly. There will be two positions of the Nicol differing by about 180°, for which the two halves of the field appear of the same colour. The readings of the circle in both these positions should be observed and the mean taken in the result. By this we eliminate the greater part of the error which arises from the

fact that the axis of rotation of our Nicol is probably not exactly parallel to the light which traverses it.

Instead of observing the colour of the biquartz directly, we may make a spectrum of the light which passes it, and examine this. For this purpose, we remove the telescope T, and allow the light to fall on the slit of a spectroscope, S (fig. 176). A lens, M, is placed between B and S, and adjusted so as to form a real image of the biquartz on the slit. When this is done, two spectra, one above the other, are seen in the field, one from each half of the biquartz. Each of these spectra is crossed by a dark band, indicating the light from the two halves of the biquartz respectively that is quenched by the analysing Nicol. On turning the Nicol, these two bands move across the field in opposite directions, and can

FIG. 176.



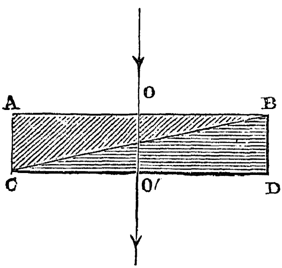
be brought one above the other. When this is the case, light of the same refrangibility has been destroyed in each spectrum by the analyser, and, as before, the principal plane of the Nicol is parallel to the direction of vibration of the incident light. It is more easy to decide when these two bands coincide than when the two halves of the biquartz are of the same colour; hence the method of analysis by the spectroscope is more delicate than the other. We may apply the spectroscope in a somewhat different manner, using only a single quartz plate instead of the biquartz, if we have a source of light, such as the sun, in the spectrum of which there are definite dark lines. For as before, we get a dark band in the spectrum, and can bring it into coincidence with any of the known Fraunhofer lines; on introducing our rotatory substance the dark band moves

across the field, and the analysing Nicol must be turned through an angle equal to the rotation produced by our substance to bring about the coincidence again.

I have found a small modification of this apparatus convenient. Supposing the polarising Nicol is mounted like the analysing in a graduated circle, we can remove the analyser entirely if we use a double image-prism of spar or quartz to form our spectrum; such a prism acts as its own analyser. We have merely to turn the polariser until the dark bands in the two spectra appear one vertically above the other, and read the circle; then introduce the active substance, turn the polariser until coincidence is again established, and take the reading.

Soleil's compensator is another apparatus designed for this purpose; A B C, B C D (fig. 177) are two wedges of quartz, as in Jamin's compensator, one of which can slide over the other. The faces A B and C D are, however, perpendicular to the axis in both wedges, but A B C is right-handed, B C D left-handed quartz. Consider a ray passing through the centre of the two wedges, along O O′; it is rotated to the right by A B C, and to the left by B C D, and since the thickness of the two wedges which it traverses is the same, no effect is produced. If, however, the movable wedge, B C D, be displaced to the right, a ray passing along O O′ will be rotated on the whole, by the plate to the right; and if B C D be displaced to the left, the ray on emerging will be rotated to the left, and the amount of rotation, depending as it does on the difference between the thicknesses traversed in the two wedges, will be proportional to the displacement. Suppose, now, this double wedge is introduced between a polariser and analyser, and adjusted to give the

FIG. 177.



tint of passage; if a column of an active substance be placed in the line of sight, the colour changes, but can be restored by displacing the movable plate, and the displacement of the movable plate thus becomes a measure of the rotation produced by the column of substance employed.

We have already referred to a very simple arrangement suggested by Professor Poynting, for utilising the principle of the Jellett and Laurent Analyser. Take a plate of quartz, cut at right angles to its axis, and cut it in two along a diameter. Reduce one-half in thickness by a small amount

FIG. 178.

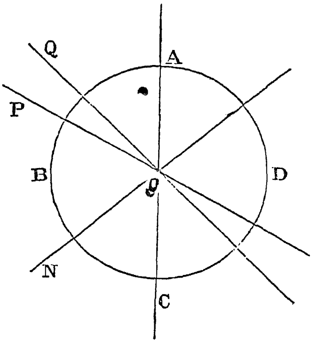
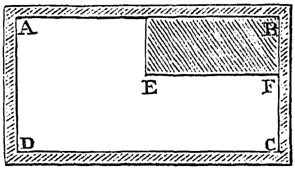


FIG. 179.

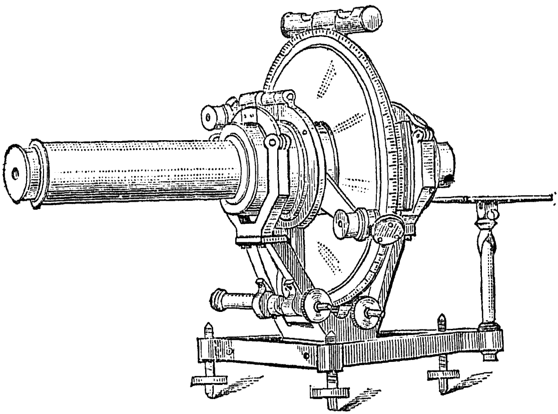


(about one-tenth of a millimètre is sufficient), and place the halves together again with their faces parallel.

Let A B C, A D C (fig. 178), be the two halves, A B C being the thinner; suppose plane polarised light, in which the direction of vibration is O N, falls on the plate; on passing through A B C the direction of vibration is rotated from O N to some other, such as O P. On passing through A D C it is rotated through a somewhat greater angle to the position O Q. Thus after passing through the plate, the planes of polarisation in the two halves of the field are inclined to each other at a small angle, just as in Jellett's and Laurent's instruments. We may accomplish this by a simpler device, as follows:— Let A B C D (fig. 179) be a cell of glass, with plane parallel

faces A B, C D. In the figure we are supposed to be looking down on it from above. Take a piece of glass of about 6 or 8 mm. thickness, and of half the size of the face A B, and place it in the cell up against the face, as E F in the figure, then fill the cell with turpentine or some other active surface. Plane polarised light passing through at right angles to the face A B is rotated by this cell, but the thickness of active

FIG. 180



substance traversed is, in one-half of the field, A D; in the other it is only F C. The planes of polarisation in the two halves are inclined at a small angle.

Fig. 180 shows a graduated circle with verniers and telescope attached, used for holding a Jellett's or Laurent's analyser or a Nicol's prism, and measuring the angle through which the plane of polarisation of light is rotated by any given substance.

CHAPTER XV.

ELECTRO OPTICS.

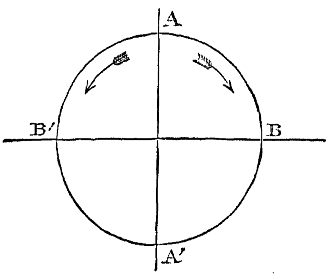
FARADAY discovered that the property of rotating the plane of polarisation of light was acquired by glass and other bodies when placed in a strong magnetic field. This may be shown as follows :—Cylindrical holes are bored through the soft iron poles of a strong electro-magnet, and these are placed so that the axes of the holes are in the same straight line. A ray of plane polarised light passes along this axis and falls on an analyser. Between the poles is placed a piece of borate of lead, a very dense kind of glass, and the analyser adjusted so that the light is quenched, or if a biquartz is used with the analyser, so that the tint is uniform over the whole plate. On passing a current through the coils of the electro-magnet, the light at once appears,—or the two halves change colour,—and can be quenched again by turning the analyser round through a certain angle. The emergent beam is still plane polarised, but the plane of polarisation has been turned round by the action of the magnetic field. If we reverse the current in the electro-magnet, we reverse the direction of rotation of the plane of polarisation.

The amount of rotation produced is proportional to the difference between the values of the magnetic potential at the two points respectively in which the ray enters and leaves the medium. It depends also on the nature of the medium.

Thus, if ω_1, ω_2 be the values of the magnetic potential, ω_2 being the greater, the rotation is equal to $c\,(\omega_2 - \omega_1)$, where c is a constant which for most media is positive. The rotation has been observed since the time of Faraday in almost all diamagnetic substances, and the value of the constant c has been determined for bisulphide of carbon by various observers.

An important difference must be noted between the rotation produced by a magnetic field and that of a solution of sugar or turpentine. If we pass a plane polarised ray through a column of turpentine, and allow it to fall on a mirror of polished glass at the end, and reflect it back, it emerges from the first end of the column polarised in the original plane of polarisation. If, however, instead of passing through the turpentine, it pass through a diamagnetic medium under a strong magnetic force, and is reflected back, the plane of polarisation is rotated by the double passage through twice the angle produced by once traversing the medium. We can see some reason for this if we consider the plane polarised ray as the resultant of two circularly polarised rays. In the quartz or turpentine, the direction of most rapid rotation depends on the direction in which the wave is travelling. Thus if the circle A B A′ B′ (fig. 181) represent the path of an ether particle in a right-handed piece of quartz, in which a wave is travelling through the plane of the paper from above to below, the wave in which the rotation is from A to B travels the more quickly; if, however, the motion be from below upwards, the wave in which the direction of motion is from A to B will be the right-handed wave and will travel the faster.

FIG. 181.



In the case of the magnetic rotation, however, the direction of fastest rotation depends only on that of the magnetic force. If this be such that the wave in which the rotation is from A to B moves the faster when travelling downwards, this wave will also move the faster when the direction of motion is from below upwards; thus as we look at it from above, the plane of polarisation will be

in the same direction, whether the wave be moving towards us. In the case of the quartz, the rotation pear to be in one direction when the wave is moving s, and in the other when it is coming towards us. Becquerel, Professors Kundt, Röntgen, and Lippich bserved this magnetic rotation in gases as well as in nd liquids, while Professor Kundt and Mr. Du Bois vestigated it in thin transparent films of iron.

other close connection between magnetism and light n discovered by Dr. Kerr, of Glasgow. He has found the poles of an electro-magnet be polished so as ct light, and plane polarised light be allowed to fall n, and the reflected beam be received on an analyser determine the position of its plane of polarisation; ition of the analyser depends on the magnetisation electro-magnet. When no current passes through the he reflected beam is plane polarised in one plane. ent is passed through in such a direction that the re- pole is positive, then the plane of polarisation of the d beam is turned in one direction. If the current rsed so as to make the pole negative, the plane arisation is turned in the opposite direction. Dr. as also discovered another link of union between ity and light; he has shown that if a transparent or is subjected to a strong electric stress, and plane d light falls on it, the emergent beam is elliptically d. Two plates of brass were placed parallel to each t a short distance apart, in a cell with parallel glass nd the space between filled with bisulphide of car- The plates were insulated from each other, and con- with the two poles of an electric machine, and plane d light passed through in a direction parallel to the the plane of polarisation of the light being inclined at the brass plates. The emergent light was examined Jamin's analyser, and, on working the electric machine, nds moved across the field. It was found that the

axes of the emergent elliptic vibrations were parallel and perpendicular to the plane of the plates, and that the difference of phase between the two components in these two directions was proportional to the square of the electric force at each point.

The effects may be more easily produced by using instead of two plates two small spheres, or a sphere and a pointed conductor, placed at a small distance apart, in a dielectric medium, and connected with the two poles of the electric machine. It is then seen, on passing plane polarised light through, that the medium has become doubly refracting, and that the axes of refraction are at each point parallel and perpendicular to the direction of the electric force. These phenomena have been still further investigated by Professors Quincke and Röntgen.

All these phenomena teach us that there is some close connection between electricity and light. Now, we have seen that many of the phenomena of light can be explained by the assumption of a medium filling space and endowed with certain properties. Faraday was the first to refer electrical action to the motions and strains of such a medium, and his ideas have been developed by Clerk Maxwell, in his 'Electricity and Magnetism.' The interesting question then arises, are these two media the same, or different?

We will give Maxwell's own account of the matter. He says :—'We have now to show that the properties of the electro-magnetic medium are identical with those of the luminiferous medium.

'To fill all space with a new medium whenever any new phenomenon is to be explained is by no means philosophical; but if the study of two different branches of science has independently suggested the idea of a medium, and if the properties, which must be assigned to the medium to account for electro-magnetic phenomena, are of the same kind as those which we attribute to the luminiferous medium to account for the phenomena of light, the evidence for the

physical existence of the medium will be considerably strengthened.

'But the properties of bodies are capable of quantitative measurement. We, therefore, obtain the numerical value of some property of the medium, such as the velocity with which a disturbance is propagated through it, which can be calculated from electro-magnetic experiments, and also observed directly in the case of light. If it should be found that the velocity of propagation of electro-magnetic disturbances is the same as that of light, and this not only in air, but in other transparent media, we shall have strong reasons for believing that light is an electro-magnetic phenomenon, and the combination of the optical with the electrical evidence will produce a conviction of the reality of the medium similar to that which we obtain in the case of other kinds of matter from the combined evidence of the senses.'

Now the luminiferous ether is the seat of energy, partly potential, partly kinetic, which traverses it in waves with a certain definite velocity. The potential energy arises from the tendency of the medium, when strained, to recover its original state, the kinetic, from the velocity with which the particles vibrate about their position of rest. According to Faraday and Maxwell, the electro-magnetic medium is the seat of two forms of energy, electro-static and electro-kinetic, and these exist not merely in the electrified or magnetised bodies, but in every part of the surrounding space where electric or magnetic force is supposed to act.

Again, when a force acts on the luminiferous ether it is put into a state of strain, and the amount of strain produced by a given shearing stress depends on the rigidity of the ether. If the force be periodic and set up waves of transverse disturbance, the velocity with which these waves traverse the medium is proportional to the square root of the rigidity.

According to Maxwell, the effect of electric force acting on a dielectric or non-conducting medium is to produce electric displacement. In an isotropic medium, the electric

displacement is in the same direction as, and is proportional to, the electric force. The amount of displacement produced by a given force depends on the nature of the medium. That property of the medium which determines the amount of electric displacement produced by a given electric force is called the dielectric constant, or specific inductive capacity of the medium.

For an elastic medium, the rigidity determines the amount of stress required to produce a certain strain, so that we see that the rigidity of an elastic medium corresponds to the reciprocal of the specific inductive capacity of the dielectric.

When a periodic electric force acts on a dielectric, waves of electric displacement are set up. The displacements in these waves are transverse to the direction of propagation; and according to the theory the velocity with which they traverse the medium is proportional to the square root of the reciprocal of the specific inductive capacity. Waves of electric displacement in an isotropic medium resemble those of light in being transverse.

The existence of waves of electric displacement in the air near a body of which the electrical state is changing in a rapid periodic manner has been experimentally verified by the experiments of Hertz, Lodge, and others.

Let us consider a Leyden jar or other electrical condenser, and suppose its surfaces charged to a considerable difference of potential. On bringing the end of a wire connected with one coating of the condenser near to the second coating, a spark passes and the condenser is discharged, the positive electrification of one coating combining with and neutralising the negative of the other. But under proper conditions this discharge is not a simple single transference of the positive charge from one coating to the other. The discharge is oscillatory in its nature, the positive electrification passes over and then some of it surges back again; and this continues for a short time, the one plate being at brief intervals alternately positive and negative, while the other plate is negative and positive. The period

of these oscillations depends on the capacity of the condenser and the form and material of the discharging circuit; their amplitude, that is the quantity of electricity transferred in half a complete period, decreases somewhat rapidly, and after a few oscillations the electricity comes to rest on the two.

The discharge is analogous to the flow of water between two cisterns at different levels. Thus, take a wide glass tube bent to a U-shape, with a wide stop-cock in the horizontal part of the bend. Fill one arm with water, open the stop-cock: the water flows into the second arm, and when the equilibrium condition is attained it stands at the same level in the two; but this does not happen immediately. If the stop-cock be opened widely the water rushes through, flowing into the second tube until the level in that tube has risen above that in the first tube, then after an instantaneous cessation the flow recommences in the other direction, and equilibrium is reached after two or three alternations. The water resembles in this way the electricity of the condenser.

Now the period of the oscillation of such an electrical system can, as Lord Kelvin showed many years ago, be calculated if its capacity and the coefficient of self-induction of the discharge be known. It is given approximately by $2\pi\sqrt{CL}$; C being the capacity and L the coefficient of self-induction in electro-magnetic measure. These electrical oscillations in the discharge set up waves of electric displacement in the medium surrounding the circuit and excite sympathetic vibrations in any neighbouring conducting circuit. As a rule these vibrations are weak, but if it happens that there is in the neighbourhood another circuit with a period of oscillation equal to that of the exciting circuit, the amplitude of the vibrations will be enormously strengthened and small sparks may be produced.

In Hertz' experiments the exciter consists of two plates, each 40 cm. in edge, connected by wires about 30 cm. long to two small spherical knobs. The knobs are gilt, and are placed at a short distance—some 2 or 3 mm.—apart. The discharge of the condenser takes place across this gap, and

is, under these conditions, oscillatory, the complete period being in about 2×10^{-8} seconds.[1]

To examine the effects produced by the vibrations Hertz used a receiver consisting of a copper wire bent into the form of a circle 35 cm. in radius; the ends of the wire are fitted with two small spherical knobs close together, and sparks can pass across the gap between these.

The period of vibration for a wire of this size is the same as that of the primary exciter. If, then, a series of waves falls on the wire oscillations are set up in it, and in consequence of the coincidence of the two periods these oscillations are much greater than they would otherwise be, and sparks may be produced across the gap. There is an important difference, however, between the two conductors, although their period of vibration is the same. In the primary conductor the damping is very large, the waves die away very rapidly and but few are emitted. In the secondary the damping is small, so that oscillations set up in it persist for a long time without much change in amplitude.

In one series of Hertz' experiments the axis of the vibrator, and therefore the direction of the electrical force, was placed so as to be vertical, and the waves were allowed to fall normally on a large plane zinc surface connected to the earth. On placing the receiving circuit with its plane parallel to the zinc plate and the spark gap at the end of a horizontal diameter, it was found that with the receiving circuit close to the zinc plate no sparks were visible, but on moving it away, keeping it parallel to itself, the strength of the sparks increased until at about 1·8 mètres from the plate they reached a maximum, vanishing again at 4 mètres distance, and so on. These effects point to the existence of stationary waves in the space between the exciter and the plate, formed by the interference of the incident waves and those reflected by the plate. At a series

[1] In Hertz' paper, through an error in calculation, this was given as $2{\cdot}8 \times 10^{-8}$ seconds. The mistake, which was pointed out by Poincaré, has been corrected by Hertz in a later edition of his paper.

of points half of a wave-length apart the phases of these two waves will always be exactly opposite and the combined effect zero. Thus in this case the half wave-length is about 4 mètres and the period is 2×10^{-8} seconds. The velocity, therefore, is $2 \times 400/2 \times 10^{-8}$ cm. per second, or 4×10^{10} cm. per second. Now the velocity of light is 3×10^{10} cm. per second. Thus, according to these experiments, the velocity of the electro-magnetic waves is of the same order as that of light, but distinctly greater. There is, however, very considerable difficulty in calculating the period of a circuit such as that used by Hertz, and the discrepancy is due to this, and possibly also to the fact that his results may have been modified by the reflexions from the walls and floor of the room.

In Hertz' first experiments it was supposed that to observe the nodes and loops it was necessary that the receiver should be of the same period as the vibrator, and that the wave-length observed was that of the vibrator. Messrs. Sarasin and De la Rive have shown that this is erroneous, that the effects can be observed with receivers of very different periods, and that the position of the nodes and loops depends mainly on the receiver. Professor J. J. Thomson has pointed out that this is a consequence of the fact that the waves from the vibrator are very rapidly damped, while those of the receiver last for some time. Only one or two waves leave the vibrator at each spark.

Now the resonance effects depend on a series of waves reaching the resonator. If this series be limited to one or two waves there will practically be no resonance. But when a single impulse reaches the receiver it disturbs its equilibrium and sets it vibrating in its own period. The wave thus produced travels on to the reflector, and after reflexion again reaches the receiver. If the phase of the reflected wave and that of the disturbance in the receiver, which in consequence of the small damping has persisted through the interval, agree at the moment when the reflected wave arrives, then the effect is increased and sparks may pass; if, on the other hand, the two phases are opposite,

the effects netralise each other and there are no sparks. Thus, in this form of the experiment the positions of the nodes and loops depend on the receiver.

Again, Hertz examined the velocity with which very rapid electrical effects travel along a wire, and found it to be different from that with which they are propagated through space. He used the same vibrator as for the air waves, and the numerical value is vitiated by the difficulty of calculating the period. Still, his experiments, if they are accepted, would prove the velocity along the wire and through the air to be different.

This result is contrary to Maxwell's theory, and is probably to be explained by the fact that there was interference between the waves travelling down the wire and those reflected from the floor of the rooms.

At any rate, the experiments of Sarasin and De la Rive prove conclusively that the waves travel down the wire at the same rate as they do through free air.

Hertz' experiments on the propagations of a wave along a wire were repeated, with some modifications, by J. J. Thomson, using a vibrator and receiver the same as Hertz'. He found that the wave-length in the wire was the same as Hertz had found it to be in air, and therefore the velocity in the two the same.

The velocity along the wire has also been measured by Lecher and others. Lecher finds it to be equal to the velocity of light, within experimental errors. Hence, combining his results with those of Thomson, Sarasin and De La Rive, and others, we can infer that electro-magnetic waves travel through air with the velocity of light. It has also been shown by various experiments that such waves can be reflected, refracted, and polarised like waves of light.

Again, electrical observations enable us to calculate the specific inductive capacity of a medium such as the air; and hence to infer the velocity with which waves of electric displacement traverse it. We can also observe the velocity of light in the air by methods entirely independent of the elec-

trical measurements. The experiments hitherto made show us that these velocities are, very nearly indeed, the same. This is obvious from the following table, in which the more recent determinations of the two quantities are given :—

Velocity of propagation of electro-magnetic disturbance in centimètres per second.		Velocity of light in air in centimètres per second.	
1884 Klemencic	$3 \cdot 019 \times 10^{10}$	1878 Cornu	$3 \cdot 003 \times 10^{10}$
1888 Himstedt	$3 \cdot 009 \times 10^{10}$	1879 Michelson	$2 \cdot 998 \times 10^{10}$
1889 Kelvin	$3 \cdot 004 \times 10^{10}$	1882 Michelson	$2 \cdot 998 \times 10^{10}$
1889 Rosa	$2 \cdot 999 \times 10^{10}$	1885 Newcomb	$2 \cdot 997 \times 10^{10}$
1890 J. J. Thomson and Searle	$2 \cdot 996 \times 10^{10}$		

It is thus clear that these two quantities are, within the limits of experimental error, the same.

The velocity of light in other transparent media can be found by measuring their refractive index relatively to air; if μ be the refractive index, and the velocity in air be taken as unity, that in the transparent medium will be $1/\mu$. Again, if the specific inductive capacity of the medium relatively to air be K, the velocity with which the electro-magnetic disturbance traverses it will be $1/\sqrt{K}$, so that if the velocity of light in the transparent medium is to be the same as that of the electro-magnetic disturbance, we should have $\mu = \sqrt{K}$; but we know that μ depends on the wave-length of the light considered, and the time occupied in all methods of determining K is infinitely long compared with the period of vibration of any part of he visible spectrum. We take, therefore, the value of μ, which corresponds to waves of infinite period; if we denote this by μ_∞, we are to have $\mu_\infty = \sqrt{K}$. Dr. J. Hopkinson has made a series of experiments to verify this result by determining the refractive indices and specific inductive capacities of various kinds of glass and oils. He finds that for the glasses and animal and vegetable oils μ_∞ is less than $\sqrt{K}$, while for the hydrocarbons μ_∞ is equal to $\sqrt{K}$.

Similar experiments have been made by Curie, Negreano, Cohn and Arons, and others, with the result that for some

substances the agreement is all that could be expected, while for others the discrepancies are very marked. Methods of electrical oscillations have been employed by Thomson, Blondlot, Arons and Rubens, and others to measure the specific inductive capacity of various substances in rapidly changing fields. Thomson and Blondlot both find that the specific inductive capacity of glass is much less in a rapidly changing field than in a steady one, and approaches more nearly to the square of the refractive index, being 2·7, while the value of μ^2 for the glass was 2·3.

The results of the experiments on the whole would seem to show that there is some very intimate connection between light and the electro-magnetic disturbance. At present we understand too little of the theory of dispersion and the manner in which the ether is modified in transparent bodies to explain fully why the two velocities should agree so closely for some media and differ for others.

The explanation which has been given for anomalous dispersion will, however, lead us some way in the direction of the truth.

According to that explanation μ^2 will have a very large positive value for periods slightly greater than that which corresponds to an absorption band; if, then, there be a region of marked absorption very low down in the spectrum, the value of $\mu_\infty{}^2$ will be very much greater than we should anticipate from the observed value in the visible spectrum. Thus the value of K obtained by statical methods would in these cases be largely in excess of the square of the refractive index for visible rays. On the other hand, if there is no marked absorption between the visible portion of the spectrum and the region of the very long infra red rays, the statical value of K would not differ greatly from the square of the refractive index. Again, a substance such as ebonite, which has been shown by J. J. Thomson to be transparent to the long electrical waves, is opaque to light, and this would be caused by the presence of an absorption band a little way below the visible red. Below

this band μ^2 is positive and the waves are transmitted; above it μ^2 is negative and the substance opaque.

At the time of the first edition of the 'Electricity and Magnetism' paraffin was the only substance whose specific inductive capacity had been determined with sufficient accuracy for a comparison: for that we have $\mu_\infty = 1{\cdot}422$, $\sqrt{K} = 1{\cdot}405$. Speaking of these numbers, Professor Maxwell wrote:—'Their difference is greater than can be accounted for by errors of observation, and shows that our theories of the structure of bodies must be much improved before we can deduce the optical from the electrical properties. At the same time, I think that the agreement between the numbers is such that if no greater discrepancy were found between the numbers derived from the optical and electrical properties of a considerable number of substances, we should be warranted in concluding that $\sqrt{K}$, though it may not be the complete expression for the index of refraction, is at least the most important term in it.'

But, in addition to this, Maxwell has shown how to determine the laws of propagation of electro-magnetic waves in a crystal, on the assumption that there are three principal axes of electric induction, along which the co-efficients of specific inductive capacity are different.

He shows that for any plane wave-front there are two, and only two, possible directions of displacement, which are exactly those given by Fresnel's construction; the wave-surface also is strictly a Fresnel's wave-surface, and as, on Fresnel's theory, the velocity of propagation of a given disturbance is inversely proportional to that radius vector of a certain ellipsoid which is parallel to the direction of displacement, the third or normal wave of disturbance does not exist at all in the electro-magnetic waves.

We have seen that Fresnel's construction represents very closely indeed the laws of propagation of light in a crystal, so that we may say that these laws are the same for light and the electro-magnetic disturbance.

Again, a number of electrical facts lead to the conclu-

sion that the kinetic energy in an electro-magnetic field is energy of rotation existing at all points of the field. Starting from this assumption, Maxwell has shown that if a wave of electric displacement traverse a field of magnetic force, the direction of displacement will be rotated as the wave travels on. If the wave be plane polarised in a certain plane on entering the field, it will be plane polarised, but in a different plane, on emergence. And on certain further assumptions as to the nature of the molecular rotation in the magnetic field, he has found an expression for the angle through which the plane of polarisation of a wave of given length will be rotated by its passage through a given field of magnetic force.

This expression agrees fairly with the experimental determinations by Verdet of the rotation of the plane of polarisation of light of various wave-lengths by a magnetic field.

Mr. E. H. Hall, of Baltimore, U.S., has discovered by direct experiment that in a conductor carrying a current in a magnetic field, there exists an electro-motive force, depending probably on the product of the strength of the field and the component of the current at right angles to the magnetic force. If we assume that a similar electro-motive force is called into play by currents of electric displacement in a dielectric, Professor Rowland, of Baltimore, has proved that the direction of the electric displacement would be rotated by the passage of a wave through the magnetic field, and the formula he arrives at is the same as that of Maxwell. I have since proved that this transverse electromotive force is a consequence of the molecular rotation assumed by Maxwell, or, to put it in another way, that this transverse electro-motive force would set up exactly the motion he took as the basis of his calculations.

The problem of the reflexion and refraction of the electro-magnetic waves has been considered by Lorentz, Fitzgerald, J. J. Thomson, Basset, and the author. The electric displacement is found to follow exactly the same laws as those deduced by Neumann, MacCullagh, and Kirchhoff, for light. Fitzgerald also has treated the case in which the

reflecting surface is magnetic, and finds that in consequence there would be a rotation of the direction of vibration, such as was experimentally discovered by Kerr, when light is reflected from a magnet. All these considerations lead us to conclude that the ether and electro-magnetic medium are the same; that, to quote Maxwell's words in the 'Theory of Heat,' 'light is a series of oppositely directed magnetisations and electro-motive forces.' We, perhaps, as yet can hardly affirm that the displacement of the ether in a ray of light is identical with electrical displacement as defined by Maxwell; we may, however, feel quite certain that the link of connection between the two, light and electric displacement, is very close indeed, and fairly hope that no long time will elapse before it becomes known to us.

CHAPTER XVI.

THE VELOCITY OF LIGHT.

IT remains now to describe some of the various methods by which the velocity of light has been determined.

Rœmer, a Dane, was the first to make the measurement.

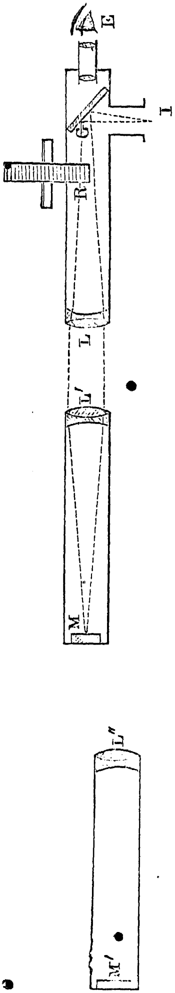
Each of the satellites of Jupiter is eclipsed to us once during its revolution by its passage behind the body of the planet; and the moment at which the eclipse will take place can be calculated from the masses of the planet and satellite, and the dimensions of their orbits.

Suppose, now, that an eclipse is observed when the earth is furthest from Jupiter, and that, starting from this epoch, the times of successive eclipses are calculated on the assumption that the interval between two consecutive eclipses is constant. Then it is found that for the next half-year as the earth is approaching the planet the eclipses happen systematically before the calculated time, and that the difference between observation and calculation increases until when the earth and Jupiter are closest together the

eclipse takes place about $16\frac{1}{2}$ minutes before the calculated time. For the next half-year the difference between observation and calculation is gradually reduced, and when the earth is again at its furthest distance away the two agree. We can account for this if light moves with a finite velocity. Suppose that a number of persons start from a place at equal intervals of five minutes and walk each at the same uniform pace along a road. They will pass a man stationed on the road at intervals of five minutes; but if the man now walks towards the starting place he will meet more persons in a given time than he would if he were stationary, the interval between meeting two people will be less. So it is with the eclipses. The light from the satellite, as it emerges from the shadow, takes time to travel; the earth as it moves towards Jupiter is meeting the light, and therefore the emerging satellite becomes visible to an observer sooner than it would if the earth were at rest. When the earth and the planet are closest together the eclipse will appear to take place about $16\frac{1}{2}$ minutes sooner than it would have done had Jupiter been at its furthest distance from the earth. This interval, then, must be the time it takes the light to travel over the diameter of the earth's orbit. Let us suppose the diameter of the earth's orbit is 186,000,000 miles: this is equal to 299,270 million mètres. To find, then, the velocity of light in mètres per second, we must divide this by $16{\cdot}5 \times 60$, and we find as the result 302,300,000.

Shortly after the time of Rœmer, the fact that light moves with a finite though great velocity was applied by Bradley to explain the phenomenon of aberration of the stars. The direction in which light from a star will appear to come to us will be the direction of its relative motion with respect to the earth, and will thus vary as the earth changes its position in its orbit. In consequence, the star will appear to be displaced from its true position, and we can show that the amount of its displacement will de-

FIG. 182.



pend on the ratio of the velocity of the earth to that of light.

The recent calculations of Gill from the mean of the best modern determinations of this quantity give as its value ·0000994, while the value of the velocity of light deduced from this will depend on the value we take for the solar parallax. If we take this as 8″·80, the velocity of light will be about 299,300,000 mètres per second.

Neither of these methods, depending as they do on the delicate astronomical measurements of the solar parallax and the constant of aberration is, however, susceptible of very great accuracy—at any rate in the individual measurements, though probably the mean of the large number of observations which have been made gives us a result which does not differ much from the truth.

Two other methods for making the determination have been devised by Fizeau and Foucault respectively. A modification of Fizeau's method has lately been used successfully in Scotland by Messrs. Young and J. A. Forbes, while A. A. Michelson, of the United States Navy, has determined the velocity by Foucault's method, with some small alterations of detail. We will consider Fizeau's method first.

L and L′ (fig. 182) are two telescopes at a considerable distance apart, directed to look into each other with their axes parallel. At the focus of L′ is a plane mirror M, at right angles to the axes of the

telescopes, while between the object-glass and focus of L is a plane piece of glass G, inclined at an angle of 45° to the axis. I is a bright source of light, the rays from which pass through an aperture in the tube of the telescope L, and fall on the glass G; they are there reflected along the axis, and I is so placed that its image, formed by reflection from the glass G, coincides exactly with the focus of the telescope L. The rays then emerge from its object-glass parallel to the axis, and after traversing the distance L L′, which, in Fizeau's experiment, was 8,663 mètres, fall on the object-glass of L′, and are refracted by it to its principal focus.

There they are incident on the plane mirror M, and being reflected by it emerge again from L′ as a parallel pencil of rays, and passing through the first telescope L, produce on the eye of the observer E the impression of a bright star of light at M. R is a toothed wheel which revolves about an axis parallel to that of the telescopes, and is so adjusted that as it turns its teeth pass directly between the image of I, formed by the glass G and the mirror M. As each tooth in succession crosses the axis of the telescope L, the light reflected from G is intercepted by it and prevented from reaching M, while the return beam from M is also intercepted and prevented from reaching the eye. Let us suppose that the breadth of each tooth is equal to the distance between two consecutive teeth. If the wheel be moving somewhat slowly, as each space between two teeth begins to cross the axis of L, the light will pass through, and after reflexion at the mirror M will arrive back at the wheel before the whole space has crossed; it will thus reach the eye, and the star M will be visible. Suppose now that the rate of rotation of the wheel is increased. Then it may happen that by the time the light reflected from M again arrives at R a tooth is crossing the axis, and in this case the return beam is stopped and the star of light is eclipsed.

This will happen if the velocity of the tooth be such that

it passes through its own breadth in the time occupied by the light in travelling from L to M and back.

If we double the velocity, by the time that the return beam reaches R the space next to the one through which the light passed will be crossing its path, and we shall see the star bright through this; while with a treble velocity there will be an eclipse again, and so on.

Now, in Fizeau's experiments the wheel had 720 teeth, and the first eclipse took place when the wheel turned 12·6 times per second. Thus the time required to turn through the width of a tooth was $\frac{1}{2} \times \frac{1}{720} \times \frac{1}{12\cdot 6}$ of a second, and this is equal to $\frac{1}{18144}$ of a second.

But in this time the light has travelled twice the distance between M and the wheel, that is, through $2 \times 8{,}663$, or 17,326 mètres. Thus the velocity of light is $17{,}326 \times 18{,}144$ mètres per second, and this, when multiplied out, is rather greater than 314,000,000 mètres per second. Cornu, in 1876, repeated the experiments with improved apparatus, and arrived at the result 300,300,000 mètres per second.

The observations required for this method are difficult to carry out, since it is almost impossible to determine exactly the rate for which the star appears to be eclipsed. This rate may be allowed to vary appreciably without producing vision at all. Until the speed of rotation differs considerably from that required for darkness, the amount of light which reaches the eye will be too small to affect the retina.

In the recent experiments of Messrs. Forbes and Young, a second reflecting telescope L″ (fig. 182) is used behind L′. These two and the telescope L are so adjusted that when the wheel is at rest the observer sees two stars side by side. This second telescope L″ is considerably further from L than L′. As the wheel is rotated L″ is eclipsed before L′. If

the speed of rotation be gradually increased L″ comes into view again, its brightness continually increasing, while L′ continues to get less bright; and a speed can be found for which L′ and L″ appear equally bright. It is easier to determine accurately this speed than to find that at which either light is eclipsed. As we increase the speed L′ disappears, and then comes into view again, getting brighter, while L″ has arrived at its maximum, and is decreasing in intensity, and a second speed can be found for which the brightness of the two again coincide. Thus we can determine a series of speeds at which the two lights are equally bright, and from these speeds can find the velocity of light.

The formula, which gives the result in terms of the quantities actually observed, is somewhat complicated, and for it reference must be made to the original memoir ('Phil. Trans.' Part I. 1882). The value obtained is 301,382,000 mètres per second.

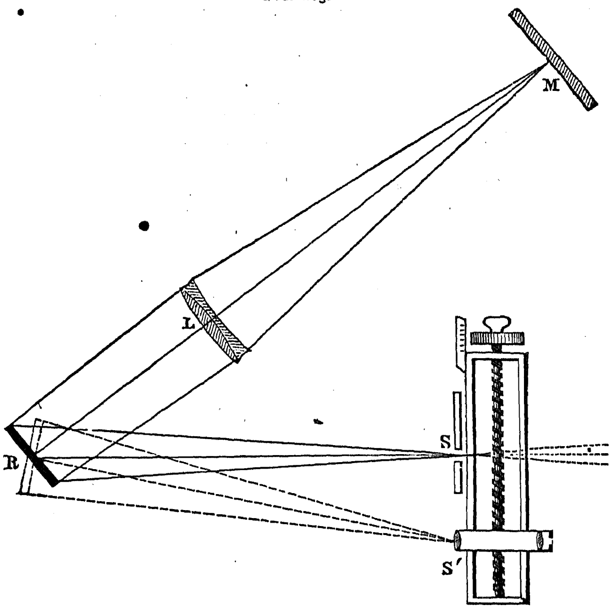
We come next to the experiments of Michelson, who employed a method which is a modification of Foucault's. S (fig. 183) is a slit with its length at right angles to the paper, through which light passes, falling on a plane mirror R, which is capable of rotating about an axis in its own plane, also at right angles to the paper. L is a lens of considerable focal length placed at the distance of its own focal length from R. For certain positions of the mirror R the light reflected there will fall on the lens L, and an image of the slit S will be formed as at M suppose. Another plane mirror is fixed so as to pass through M with its plane at right angles to the line R L.

If the position of the revolving mirror be such that the axis of the pencil reflected from it falls anywhere on the lens L, the light, after passing this lens, will emerge parallel to R L, and fall perpendicularly on the mirror M; it will be reflected back again along the same path to R, and if in the time occupied by the light in travelling from R to M and back, the revolving mirror has not appreciably altered its

position, the return pencil will be reflected back to S, and an image of the slit formed, which will coincide with S.

If, however, the mirror R be revolving rapidly in the direction of the hands of a watch, so that by the time the pencil reflected from M reaches it, its position is that indicated by the dotted line; the image of the slit formed by

FIG. 183.



reflexion from it will no longer coincide with S, but will be displaced to some point S′ to the left of S; and the amount of this displacement will depend on the velocity of the mirror and the time occupied by the light in travelling from R to M and back.

If we measure the displacement S S′, the distances S R and R M and the rate of rotation of the mirror, we can calculate the velocity of light.

Let $SS' = \delta$, $SR = a$, $RM = c$

V = velocity of light

τ = time taken by light to traverse twice the distance RM

θ = angle turned through by mirror in time τ

N = number of complete revolutions of mirror in one second

In one second the mirror turns through an angle of $2\pi N$; therefore in τ seconds it will turn through $2\pi N\tau$.

Thus $\theta = 2\pi N\tau$.

Again, by turning the mirror through an angle θ, the reflected ray has been turned from the position RS to RS'.

$$\therefore \text{angle } SRS' = 2\theta.$$

$$\text{Also } SS' = SR \tan SRS'.$$

$$= a \tan 2\theta.$$

$$\text{so that } \delta = a \tan 4\pi N\tau.$$

But τ = the time taken by light to travel twice the distances RM.

$$\therefore \tau = \frac{2c}{V}$$

$$\therefore \delta = a \tan \frac{8\pi N c}{V}$$

We may simplify the formula if we remember that the angle 2θ being very small, we may write 2θ for $\tan 2\theta$ approximately, and hence

$$\delta = \frac{a \times 8\pi N c}{V}$$

or

$$V = \frac{8\pi N a c}{\delta}$$

In Michelson's experiments

$$a = 8{\cdot}58 \text{ mètres.}$$

$$c = 605 \text{ mètres.}$$

$$\delta = {\cdot}113 \text{ mètres.}$$

$$N = 257$$

approximately.

Substituting in the formula we find $v = 296{,}500{,}000$ mètres per second.

Michelson's final result, taking into account all the necessary corrections, is 299,940,000 mètres per second. Taking as our result the mean of the last three determinations, we get finally for the velocity of light in vacuo, 300,574,000 mètres per second.

We have hitherto assumed that waves of all colours travel through the air at the same rate. Messrs. Young and Forbes believe that their experiments prove that this is not the case. They find that the reflected stars were seen to be coloured—one reddish, the other bluish. The particular colour of a particular star depended on the speed of rotation of the toothed wheel. The star which was increasing in brightness with increase of speed of the toothed wheel was reddish, the one that was diminishing with increase of speed was bluish.

This appearance can be explained by the assumption that blue rays travel more quickly than red. To test this experiments were actually made, using instead of white light the red and blue rays respectively of the spectrum of the electric light formed by a prism, and they find that the difference in velocity between the red and blue rays amounts to about 1·8 per cent. of the whole.

There are, however, numerous considerations which lead us to look for some other explanation of the phenomena observed. Thus, if the above result be true, the image of the slit S formed in Michelson's experiments should not be white, but drawn out into a spectrum of more than two millimètres length. No such effect was observed. The image was white, and the various individual measurements of its position agreed to less than a tenth of a millimètre. Again, if the velocity of light depends on the wave-length, Jupiter's satellites should be coloured just before and just after eclipse: red just before, for the last red rays would arrive later than the blue; and blue just after, for the first blue rays would reach the eye soonest. The same would be

true of temporary stars, while the effect of aberration would be to draw out the star into a small spectrum. None of these consequences have been observed.

In addition to this, Lord Rayleigh has pointed out that the theoretical difficulties in the way of accepting Messrs. Forbes and Young's explanation are very great. He has shown that in cases in which the velocity of wave propagation depends on the wave-length, there is a difference between the velocity of a single wave and the velocity of a group of waves. If we watch the crest of a wave at sea, and find it travels forward at the rate of a mètre per second, we should call this the velocity of the wave. But in the case of light we cannot thus distinguish a particular wave. What we have to do is to impress some peculiarity on a group of waves, and watch the rate at which this peculiarity travels. We call this rate the group velocity, let us denote it by U; V being the wave velocity, or rate of propagation of an individual wave. Whenever V depends on the wave length the mathematical theory shows that U and V are different.

Assuming that Forbes and Young's results are true, and that the relation between velocity and wave-length in air is of the same nature as in glass, Lord Rayleigh has proved that U, the group velocity, would be about 3 per cent. greater than V, the relation connecting the two being

$$V = U\,(1 - \cdot 0273).$$

He has shown, moreover, that the experiments of Fizeau, Cornu, Forbes, and Young, and the observations on Jupiter's satellites, would give us U, the group velocity, for they all depend on impressing a peculiarity—the intermittence of the light—on the train of waves, and measuring the rate at which it travels. The theory of aberration, on the other hand, would give us V, the actual wave velocity; while the result of the experiments of Foucault and Michelson would be V^2/U.

This last result can be proved only by the aid of mathematics.

Hence if v be the velocity determined from Michelson's experiments, we should have

$$v = V^2/U = U\,(1 - \cdot 0546).$$

Thus there should be a difference of nearly 3 per cent. between the values obtained from the theory of aberration and the experiments of Cornu or Forbes and Young, and of more than 5 per cent. between those values and those deduced from the experiments of Foucault or Michelson.

Those considerations constitute an almost irresistible weight of evidence against the views of Forbes and Young.

Our final conclusion then will be that in air, as in a vacuum, waves of all colours travel at the same rate, while the velocity of white light in a vacuum is 299,860,000 mètres per second.

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